Learning, Active Investors, and the Returns of Financially Distressed Firms

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Abstract

I develop a dynamic asset pricing model to analyze expected returns of financially distressed firms in the presence of learning about firm fundamentals and endogenous information acquisition by active investors that acquire large stakes in distressed firms via private investments in public equity. The model reveals that learning and information acquisition critically affect risk exposures close to default and can rationalize low and even negative expected equity returns for firms with high default risk. Similar to Schumpeter's (1934) argument that recessions have a positive, cleansing effect on the economy, the model reveals that equity holders may benefit from the increased speed of learning about insolvent firms in downturns, which increases the value of their abandonment option in these times. Equity holders' option value is further enhanced by the ability to partially free-ride on active investors' acquisition of information on firm fundamentals. Both information channels are shown to affect equity betas, and may account for striking, momentum-type dynamics in risk premia.

Keywords: Expected Returns, Financial Distress, Learning, Information Acquisition, Momentum, Active Investors, Private Investments in Public Equity

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1. Introduction

Measures of default risk predict low future stock returns (Dichev, 1998, Campbell, Hilscher, and Szilagyi, 2008) – this empirical finding has given rise to the so-called "distress anomaly," since standard models suggest that the equity of firms with high default risk is more exposed to aggregate risk and therefore should command higher expected returns. Recently, Park (2011) sheds more light on this finding by documenting that low returns on financially distressed stocks are concentrated among firms that issue discounted equity to new investors via private offerings of public equity that dilute existing shareholders. Although intriguing, these findings do not per se resolve the distress puzzle since existing investors may use default risk measures to predict dilution and thus should appropriately adjust prices for expected dilution. Additionally, proximity to default appears to play an important role for momentum returns, since momentum profits are restricted to high credit risk firms and are nonexistent for firms of high credit quality, as documented by Avramov, Chordia, Jostova, and Philipov (2007). In this paper, I develop a dynamic asset pricing model that proposes two related channels that can reconcile these empirical facts based on investors' rational formation of beliefs about distressed firm's chances of future recovery: first, passive learning from past firm performance, and secondly, active investors' information acquisition and its externality on other investors' risk exposures.

The first channel, learning from firm performance, is related to Schumpeter's (1934) argument that recessions have a positive, "cleansing effect" on the economy. Equity holders benefit in aggregate downturns from a high speed of learning about illiquid firms' chances of future recovery. When exposed to the test of a downturn, truly insolvent firms are likely to show adverse performance that allows equity holders to separate them from solvent firms. Executing their abandonment option then allows equity holders to limit their losses from subsidizing debt holders. Learning about firm fundamentals thus increases equity holders' abandonment option value in aggregate downturns which strongly affects equity holders' exposure to aggregate risk, and may even turn it negative.¹

The second channel constitutes a novel dimension to dynamic asset pricing models: active investors' impact on other claim holders' exposure to aggregate risk via their influence on the firm's refinancing and liquidation decisions. Active investors (for example, activist hedge funds or private equity funds) typically acquire substantial stakes in distressed firms' private placements of public equity. In the model, these specialized investors may exert costly effort to acquire information on distressed firms' chances of future recovery. Management endogenously issues equity to one such investor when the firm becomes illiquid, and investors are uncertain whether the firm is fundamentally solvent.² Providing one investor with a large equity stake helps resolve a standard free-rider problem present among small investors (see, e.g., Grossman and Hart (1980) and Shleifer

¹The quality of information is not to be confused with the physical uncertainty of the underlying earnings process. Increased uncertainty about the underlying naturally increases the value of the option.

²Brophy, Ouimet, and Sialm (2009) document that hedge funds tend to finance companies that have poor fundamentals and pronounced informational frictions, and require substantial discounts.

and Vishny (1986)). A large investor internalizes benefits of costly information acquisition that helps identifying insolvent firms. Existing shareholders can partially free ride on the active investor's effort going forward, and thus are willing to provide the active investor initially with a discounted purchase price, that is, a price lower than the equilibrium market value of the stake obtained. Due to its superior information, the active investor assumes a pivotal role in the firm's future liquidation and refinancing decisions which naturally affect the value of all equity holders.

The active investor's involvement helps resolve debt overhang problems (Myers, 1977) naturally occurring in firms close to default. Better information about a firm's future performance has the positive effect of avoiding unnecessary defaults by firms that are merely temporarily illiquid but fundamentally solvent. Yet better information also has redistributional effects. In particular, debt holders of insolvent firms are worse off, since better informed equity holders quickly abandon insolvent firms, and thus stop subsidizing debt holders via equity injections. The analysis shows that although endogenous information acquisition can have similarly adverse effects on debt holders as risk shifting (Jensen and Meckling, 1976), it does not require any change in the underlying assets or in contracts.

Further, the active investor's information acquisition has redistributional effects for claimants' exposures to aggregate risk, and thus affects expected returns. Since the speed of learning influences valuations across aggregate states of the economy, information production can shift aggregate risk from equity to debt claims. If learning based on a firm's public information is slow in aggregate downturns, the active investor has a natural incentive to increase information production in these times, which allows off-loading systematic risk to debt holders. Since passive shareholders do not incur information acquisition cost on an ongoing basis, their equity stakes are even less exposed to aggregate risk than the active investor's position.

The proposed mechanism provides an explanation why expected returns and the degree of dilution in private offerings are negatively correlated in the data (Park, 2011), and why public companies that raise equity privately from large specialized investors such as hedge funds significantly underperform companies that obtain financing from other investors in the future (Brophy, Ouimet, and Sialm, 2009). Management offers larger discounts to active investors with greater skill – a larger dilution therefore forecasts more effective involvement and thus a stronger reduction in the equity's exposure to aggregate risk. The involvement of an active investor is a state variable that affects distressed firms' conditional betas.

The model further reveals that negative contemporaneous returns for existing share-holders at the time when the active investor acquires shares at a discount are not conclusive evidence that management is acting against shareholders' interests. By conditioning on firms that issue discounted equity to an active investor, the econometrician systematically sorts on firms that just received a negative shock and for that reason approach an active investor. In other words, under the counter-factual of no active investor involvement, existing equity holders' position would lose at least as much value. Since existing

shareholders gain from free riding on the active investor's information production going forward, management is willing to provide the fund attractive terms ex ante – in the limiting case in which the active investor has all the bargaining power, existing shareholders are just as well off with and without discounted equity issuance.

The model also sheds light on the empirical finding by Avramov, Chordia, Jostova, and Philipov (2007) that momentum profits are restricted to high credit risk firms and are nonexistent for firms of high credit quality. The model can generate positive correlations between price changes and expected returns implying that firms with recent stock price declines are also those firms with lower expected returns going forward. A momentum strategy that goes long recent winners and shorts recent losers among distressed firms may therefore generate a large spread in expected returns. Consistent with this theoretical argument, Avramov, Chordia, Jostova, and Philipov (2007) find that both the extreme loser and winner portfolios consist of stocks with the lowest and the next-lowest credit rating, respectively. Additionally, the model's predictions on momentum profits are consistent with the empirical findings by Boguth, Carlson, Fisher, and Simutin (2011) that time-variation in conditional betas leads to inflated estimates of unconditional momentum alphas.

Further, O'Doherty (2012) provides empirical support for the model's predictions by documenting that firms with high default risk have low conditional betas in downturns. This cyclical variation in betas of distressed firms helps explain why unconditional beta estimates from CAPM regressions lead to upward biased betas, causing biased, negative unconditional alpha estimates³ as found by Campbell, Hilscher, and Szilagyi (2008).

1.1. Related Literature

This paper is generally related to a growing literature on learning in financial markets (see Pastor and Veronesi (2009) for a survey), and the relationship between capital structure, asset pricing, and macro-economic cycles (see, e.g., Chen (2010) and Bhamra, Kuehn, and Strebulaev (2010)). Regarding the distress anomaly, alternative theoretical explanations have been proposed in the literature. George and Hwang (2010) argue that firms with high financial distress costs choose low leverage to avoid distress but retain exposure to the systematic risk of bearing such costs in low states, implying that they have higher expected returns than highly levered firms. Garlappi and Yan (2011) provide a model that shows how potential shareholder recovery upon resolution of financial distress (violation of the absolute priority rule) may effectively imply de-levering upon default, which may account for lower expected returns for firms with high default probabilities. Similarly, Garlappi, Shu, and Yan (2008) argue that bargaining between equity holders and debt holders in default may account for low expected equity returns on firms with high default risk given that shareholders can extract high benefits from renegotiation. In contrast to my paper, these theories do not consider the effects of learning on expected equity returns of distressed firms and do not explain the empirical finding that

³See, e.g., Grant (1977) and Jagannathan and Wang (1996).

low returns on financially distressed stocks are concentrated among those firms that issue discounted equity in private placements.

Several empirical papers analyze the relationship between private placements and equity returns. Consistent with Park (2011), Krishnamurthy, Spindt, Subramaniam, and Woidtke (2005) find that shareholders not participating in private placements experience post-issue negative long-term abnormal returns. Based on my model, I argue that these low returns may be rationalized by changes in conditional betas that occur with the change in information quality due to private placements that involve specialized investors such as hedge funds. Hertzel, Lemmon, Linck, and Rees (2002) also document that public firms that place equity privately experience negative post-announcement stock-price performance. Hertzel and Smith (1993) provide empirical evidence that discounts provided in private placements reflect information costs borne by private investors, which is consistent with the mechanism in my model.

Vassalou and Xing (2004) document that the size effect in expected returns exists only in segments of the market with high default risk, and that this is also largely the case for the book-to-market effect. Vassalou and Xing (2004) further find some evidence that distressed stocks with a low distance to default have higher returns, but this evidence comes entirely from small value stocks. Da and Gao (2010) further provide evidence that distressed firms' stock returns in Vassalou and Xing (2004) are biased upwards by 1-month reversal and bid-ask bounce. Griffin and Lemmon (2002) document that among firms with the highest distress risk the difference in returns between high and low book-to-market securities is more than twice as large as that in other firms. Further, the authors find that firms with high distress risk exhibit the largest return reversals around earnings announcements. Sagi and Seasholes (2007) argue that a firm's revenues, costs, and growth options jointly account for dynamics in return autocorrelation, and that account for these effects allows for enhanced momentum strategies. Johnson (2002) provides an alternative rational explanation of momentum effects based on stochastic expected growth rates.

2. The Economy

2.1. Preferences and Technology

The economy is in continuous time and admits a representative household that maximizes stochastic differential utility (Duffie and Epstein, 1992)

$$J_t = E_t \left[\int_t^\infty f(C_\tau, J_\tau) d\tau \right], \tag{1}$$

where f(C, J) is a normalized aggregator of current consumption C and continuation utility J that takes the standard form

$$f(C,J) = \frac{\beta}{\rho} \left((\alpha J)^{1-\frac{\rho}{\alpha}} C^{\rho} - \alpha J \right), \tag{2}$$

with $\rho = 1 - \frac{1}{\psi}$ and $\alpha = 1 - \gamma$, where $\beta > 0$ is the rate of time preference, $\gamma > 0$ is the coefficient of relative risk aversion, and $\psi > 0$ is the elasticity of intertemporal substitution. The normalized aggregator f(C, J) takes the following form when $\psi \to 1$:

$$f(C,J) = \beta \alpha J \left[\log C - \frac{1}{\alpha} \log (\alpha J) \right]. \tag{3}$$

Power utility obtains by setting $\psi = 1/\gamma$.

I consider a Lucas-Breeden economy, where aggregate consumption dynamics are specified exogenously and analyze pricing implications for marginal firms. Let C_t denote the rate of aggregate consumption in the economy at time t, which follows the process

$$\frac{dC(t)}{C(t)} = \theta_C(Z_t) dt. \tag{4}$$

The state variable Z_t governs dynamics in the expected growth rate of aggregate consumption $\theta_C(Z_t)$. I assume that Z_t follows a two-state continuous time Markov chain with $Z \in \{G, B\}$, where G refers to a high growth state and B refers to a low growth state. By considering a larger number of states, the model can in principle capture rich dynamics in consumption growth. Yet, for the purposes of this paper, I will focus on the case of two aggregate states, which suffices to highlight the central points of the argument and increases the transparency of the results. I denote the transition rate between aggregate state Z and Z' by $\lambda(Z)$.

Household maximization implies that a state-pricing process ξ_t may be written as follows.

PROPOSITION 1 (Stochastic discount factor). The stochastic discount factor follows a Markov-modulated jump process,

$$\frac{d\xi_{t}}{\xi_{t-}} = -r_{f}(Z_{t-}) dt - \sum_{Z' \neq Z_{t-}} \kappa(Z_{t-}, Z') \left(dN_{t}(Z_{t-}, Z') - \lambda_{Z_{t-}Z'} dt \right), \tag{5}$$

where $r_f(Z_t)$ is the real risk-free rate, $dN_t(Z_{t-}, Z')$ takes the value 1 if the Markov chain jumps to state Z' and zero otherwise, and $\kappa(Z, Z')$ is defined as follows:

$$\kappa(Z, Z') \equiv -\left(\left(\frac{F(Z')}{F(Z)}\right)^{1-\frac{\rho}{\alpha}} - 1\right). \tag{6}$$

Proof. See Appendix. \blacksquare

2.2. The Firm

Consider a firm that at time t=0 has legacy debt in place with perpetual coupon rate c normalized to 1. Firm-specific Markov states z govern the firm's earnings rates X(z).

Let $\lambda_{z,z'}(Z)$ denote the transition rate of from firm-state z to z' given the economy is in aggregate state Z. For simplicity, I divide the set of states $(z,Z) \in \Omega$ into four subsets: initial liquid states Ω_l , illiquid states Ω_i , and two sets of revealing states denoted by Ω_g and Ω_b . Initially, the firm is assumed to be in liquid states $(z,Z) \in \Omega_l = \{l_g,l_b\} \times \{G,B\}$ in which it generates sufficiently high earnings to cover its interest expenses, specifically, $X(l_g) > X(l_b) > c$. From the liquid states Ω_l the firm can transition into illiquid states $(z,Z) \in \Omega_i = \{i_g,i_b\} \times \{G,B\}$ in which earnings are below the coupon rate c, implying that the firm has to raise equity in order not to default. The information problem at the heart of the analysis arises in the illiquid states since investors have to determine whether the firm is merely illiquid or in fact also insolvent. Specifically, states are defined such that the firm is solvent in state i_g , but insolvent in state i_b , that is, if investors knew the true state of the illiquid firm, they would choose to default in state i_b and provide new equity in state i_g .

To simplify notation,⁴ let Ω_g denote the set of states into which the firm may transition from firm-state $z = i_g$, that is,

$$\lambda_{i_q,z'}(Z) > 0, \forall (z',Z) \in \Omega_q,$$
 (7)

$$\lambda_{i_q, z'}(Z) = 0, \forall (z', Z) \in \bigcup_{j \neq g} \Omega_j,. \tag{8}$$

In firm state $z = i_b$ on the other hand, the firm can only transition into states Ω_b , that is,

$$\lambda_{i_b,z'}(Z) > 0, \forall (z',Z) \in \Omega_b,$$
 (9)

$$\lambda_{i_b,z'}(Z) = 0, \forall (z',Z) \in \bigcup_{j \neq b} \Omega_j.$$
 (10)

In all illiquid states Ω_i the firm's coupon rate is given by x < c. Since the earnings rates are identical across illiquid states Ω_i , investors cannot directly infer the true underlying firm-state z by observing the current earnings rate x. Yet all investors are Bayesian learners and form rational beliefs about the underlying state based on available observables, such as the passage time since the transition into an illiquid state, and potential additional signals. When the firm transitions into states in the sets Ω_b and Ω_g , investors can infer that the firm was previously in state i_b and i_g , respectively. Figure 1 summarizes the state dynamics. If there was no inference problem with regards to the firm-state z and equity holders knew with certainty that the firm was in firm-state i_b , they would immediately trigger default. On the other hand, if equity holders' knew the firm was in firm-state i_g , they would be willing to provide new equity to the firm, as the firm is solvent and just illiquid. Naturally, the inference problem becomes degenerate if the equity value in firm-state i_g is also non-positive, since in that case the illiquid firm is always insolvent.

⁴The specification is without loss of generality since there are no restrictions on the sets Ω_g and Ω_b other than the one that the equity value in state i_g is positive and negative in state i_b . For example, the specification does not preclude the possibility that the sets Ω_b and Ω_g contain subsets of states with identical earnings rates.

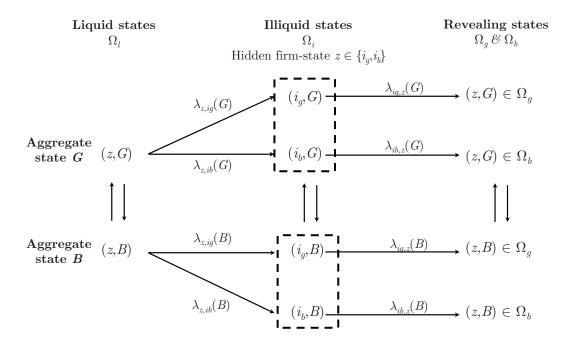


FIGURE I
The figure illustrates the firm's Markov state setup.

2.3. The Active Investor

An investor (also called *active investor* going forward) is endowed with an information production technology that generates a perfectly precise signal s of the firm state ($s_t \in \{i_b, i_g\}$) with Poisson arrival rate

$$h_t = \eta \cdot a_t^{1-\nu},\tag{11}$$

where η is a positive constant and a_t denotes costly effort exerted by the active investor. a_t is not directly observable and not contractible. The active investor incurs cost at rate $a_t \chi$ for exerting effort a_t (with $\chi > 0$).

2.3.1. Contracting

The analysis considers contracts between management and the active investor that provide the active investor with new equity shares that yield an ownership share ω at a purchase price $\bar{\kappa}$. Contracts are limited to one-time provisions of new equity. Management is assumed to act in the interest of existing shareholders and is able to commit not to renegotiate the contract in the future. In particular, any future equity injections by the active investor occur at the same terms as for all other investors. In principle, the contract may be written at any point in time $\bar{t} > 0$. Yet there is no reason to provide the active investor with an equity exposure before the firm enters the illiquid states Ω_i ,

since the active investor's information technology is only useful in states in which the underlying firm-state z is unknown.

2.3.2. Equity exposure

The active investor faces limits to the amount of capital it can allocate to the firm, implying an upper bound on the feasible equity share allocated to the active investor. The upper bound on ω may be due to capital constraints or cost of un-diversification that limit the optimal amount of exposure for the active investor. Since this paper does not attempt to provide an explanation for various economic forces that may limit the investor's exposure, I consider the upper bound on ω as exogenously given and focus on the asset pricing implications for a given upper bound.

2.3.3. Information environment

Adjustments to the active investor's position in the firm are assumed to be publicly observable. The lack of noise in the system implies that the active investor cannot generate profits from trading against less informed investors. Regulation in the United States supports this assumption: investors acquiring more than 5% of a firm's equity with the intent to exert control have to file a 13d with the SEC within 10 days. In addition, investors have to re-file these forms in case of material changes to their positions (1%). Re-financing and liquidation decisions, which are lower-frequency activities, are the key channels in this model, not higher-frequency secondary market trading between informed and uninformed investors. Whereas adding noise to the system (for example by introducing noise traders or liquidity shocks) would be an interesting extension in its own right, it would constitute a distraction in the context of this paper, which aims to address empirical facts on existing shareholders' buy-and-hold returns after a private investment in public equity. A buy-and-hold strategy does not correspond to random or selective buy-and-sell orders that are typical equilibrium outcomes of noisy rational expectations models. Low buy-and-hold returns for existing shareholders cannot be simply explained by losses from trades against informed counterparties. Apart from weakening the focus of the paper, adding noise to the system comes at the cost of reduced analytical tractability and transparency. Instead of proposing a noisy trading environment as a source of incentives for information acquisition, this paper highlights the role of private investments in public equity as a mechanism that allows active investors to internalize rents from information production.

2.3.4. Usage of excess cash

New funds provided by the active investor are invested in marketable securities until the funds are used to either make contractual coupon payments to debt holders, or until management strategically liquidates the firm. In case of strategic liquidation, man-

agement sells non-core assets (marketable securities) and pays out proceeds to equity holders. Thereafter the firm defaults. If, after the first equity injection by the active investor, the firm runs out of cash again, the firm may raise new equity from investors at fair value.

2.3.5. Bargaining power

For simplicity, I consider the case where the active investor has all the bargaining power when the contract is written, allowing it to obtain a purchase price that leaves old shareholders just indifferent between the private placement of shares and the alternative situation in which the active investor does not obtain a stake in the firm. All results are qualitatively robust to cases in which the active investor cannot extract all the rents it generates for equity holders by acquiring information. For the results on expected return dynamics it is only essential that the active investor obtains a stake in the firm that incentivizes it to exert hidden effort to acquire information on firm fundamentals. The distribution of bargaining power only alters degree of initial dilution of existing shareholders. Yet empirically documented dilution of existing shareholders via private placements of public equity suggests that active investors are in fact able to extract part of the rents they generate.

3. Solution

Solving for equity values backwards, I first analyze the revealing states Ω_b , Ω_g , then illiquid states Ω_i , and finally the initial liquid states Ω_i .

3.1. Revealing States (Ω_b and Ω_a)

In the long-run, the firm either has defaulted while being in an illiquid state ($\in \Omega_i$), or has reached the revealing states $\{\Omega_b, \Omega_g\}$. The following proposition characterizes equity prices in the long-run states.

PROPOSITION 2 (Equity value in revealing states). In the states $(z, Z) \in \{\Omega_g, \Omega_b\}$, the firm's equity value is given by

$$V(z_{t}, Z_{t}) = \max_{\tau^{*}} E_{t} \left[\int_{t}^{\tau^{*}} \frac{\xi(Z_{\tau})}{\xi(Z_{t})} (X(z_{\tau}) - c) d\tau \right] = v(z_{t}, Z_{t})^{+},$$
(12)

where the function v(z,Z) solves the following system of equations for all $(z,Z) \in \{\Omega_g, \Omega_b\}$:

$$0 = X(z) - c - r_f(Z) \cdot v(z, Z) + \sum_{z' \neq z} \lambda_{zz'}(Z) \cdot \left(v(z', Z)^+ - v(z, Z)\right) + \sum_{Z' \neq Z} \lambda(Z) \cdot (1 - \kappa(Z, Z')) \cdot \left(v(z, Z')^+ - v(z, Z)\right),$$
(13)

where the superscript $^+$ denotes the standard short-hand for the max $\{.,0\}$ operator.

Proof. See Appendix. ■

Based on the solution to the household value function provided in the proof to Proposition 1 it is straightforward to compute the equity value according to proposition 2. Given the state-contingent equity values, the risk premium in state (z, Z), denoted by rp(z, Z), may be written as follows:

$$rp(z,Z) = -\sum_{Z'\neq Z} \lambda(Z) \kappa(Z,Z') \left(\frac{V(z,Z')}{V(z,Z)} - 1\right), \forall V(z,Z) > 0.$$
 (14)

The risk premium is naturally only defined for states (z, Z) in which the firm is still alive and the equity value V(z, Z) is positive.

3.2. Illiquid States (Ω_i)

Let $V(i_g, Z_t)$ denote the equity value of the firm if investors know with certainty that the firm is in state i_g . I provide a solution to the price $V(i_g, Z_t)$ in the appendix. By definition, the equity value in state i_b is zero $(V(i_b, Z_t) = 0)$, that is, if equity holders knew the firm is in state i_b they would opt to default immediately. Yet, generally, when the firm is illiquid and generates earnings at rate x < c, investors are uncertain about the underlying firm-state $z \in \{i_b, i_g\}$, and therefore may keep the firm alive even though the underlying state is i_b . Let $\pi(t)$ denote the probability that the firm is in state i_g , that is, $\pi(t) \equiv \Pr[z = i_g | \mathcal{F}_t]$. The following proposition characterizes the evolution of posterior beliefs.

LEMMA 1 (Posterior beliefs in illiquid states). The initial value of the probability that the firm is in state i_g at the time of transition into the illiquid state is given by

$$\pi_0 = \frac{\lambda_{l,i_g}\left(Z_0\right)}{\lambda_{l,i_g}\left(Z_0\right) + \lambda_{l,i_b}\left(Z_0\right)}.$$
(15)

Absent a signal or an action by the active investor that reveals the true state, $\pi(t)$ evolves as follows

$$d\pi_t = \varpi(Z) (1 - \pi_t) \pi_t dt, \tag{16}$$

where $\varpi(Z)$ is defined as

$$\varpi(Z) = \sum_{z' \in \Omega_b} \lambda_{i_b, z'}(Z) - \sum_{z' \in \Omega_g} \lambda_{i_g, z'}(Z).$$
(17)

Let $\tau_Z(t)$ denote the time period for which the firm has been illiquid while in aggregate state Z. The posterior probability π_t is given by

$$\pi(t) = \left(1 + e^{\left(-\sum_{\forall Z} \tau_Z(t) \cdot \varpi(Z)\right)} \cdot \frac{1 - \pi_0}{\pi_0}\right)^{-1}.$$
 (18)

Proof. See Appendix. \blacksquare

If $\varpi(Z)$ is constant across aggregate states Z, then the inference problem simplifies in the sense that conditional on observing illiquidity (x < c), there is a one-to-one mapping between the posterior probability π and the time since the last transition into the illiquid state. Otherwise, the posterior probability also depends on the relative time spent in aggregate states B vs. G. Further, if $\varpi(Z) = 0, \forall Z$, then passage time is not informative and does not alter beliefs π_t .

3.2.1. Firm Value Without Active Investor (Reservation Value)

Management acting in the interest of existing equity holders naturally has the outside option not to issue shares to the active investor. The value of the equity under this scenario constitutes a reservation value, V^R , that implies a limit on the price discount management is willing to offer the active investor for shares in the firm. The following proposition characterizes this reservation value V^R .

PROPOSITION 3 (Equity value in illiquid states without active investor). Absent active investor, the firm's equity value in the illiquid state is given by

$$V^{R}(\pi_{t}, Z_{t}) = \max_{\pi^{*}(Z)} E_{t} \left[\int_{t}^{\tau^{*}} \frac{\xi(Z_{\tau})}{\xi(Z_{t})} \left((x - c) d\tau + V(z_{\tau}, Z_{\tau}) \mathbf{1}_{\{z_{\tau} \in \Omega_{g} \cup \Omega_{b}\}} \right) \right] = v^{R}(\pi_{t}, Z_{t})^{+}$$

where $\tau^* = \min \{ \tau : \tau \geq t \land (\pi(\tau) \leq \pi^{R*}(Z_{\tau}) \lor z_{\tau} \in \Omega_g \cup \Omega_b) \}$, and $\mathbf{1}_{\{.\}}$ denotes an indicator function that takes the value 1 if the condition in the subscript is satisfied and 0 otherwise. The function $v^R(\pi, Z)$ solves the following set of ODEs for $Z \in \{G, B\}$:

$$0 = x - c - r_{f}(Z) \cdot v^{R}(\pi_{t}, Z) + v_{\pi}^{R}(\pi_{t}, Z) \frac{d\pi_{t}}{dt} + \sum_{z' \in \Omega_{g}} \pi_{t} \cdot \lambda_{i_{g}, z'}(Z) \cdot (V(z', Z) - v^{R}(\pi_{t}, Z)) + \sum_{z' \in \Omega_{b}} (1 - \pi_{t}) \cdot \lambda_{i_{b}, z'}(Z) \cdot (V(z', Z) - v^{R}(\pi_{t}, Z)) + \sum_{Z' \neq Z} \lambda(Z) \cdot (1 - \kappa(Z, Z')) \cdot (v^{R}(\pi_{t}, Z') \mathbf{1}_{\{\pi \geq \pi^{*}(Z')\}} - v^{R}(\pi_{t}, Z)).$$
 (19)

In states Z for which $\varpi(Z) < 0$, the following conditions are satisfied at $\pi^{R*}(Z)$: $0 = v_{\pi}^{R}(\pi^{R*}(Z), Z), \ 0 = v^{R}(\pi^{R*}(Z), Z).$ If $\varpi(Z) > 0$ for all $Z \in \{G, B\}$, then $\pi^{R*}(Z)$ is set such that $v^{R}(\pi^{R*}(Z), Z) = 0$ and $\lim_{\pi \uparrow 1} v^{R}(\pi, Z) = V(i_g, Z).$ If $\varpi(Z) = 0$, then the ODE for state Z simplifies to a nonlinear equation.

Proof. See Appendix. \blacksquare

The active investor's information production technology can generate rents when investors are uncertain about the firm's underlying state. Providing the active investor

with an equity stake optimally occurs as soon as the firm enters an illiquid state and generates earnings at rate x < c. As discussed in Lemma 1, investors' conditional beliefs at that time are given by $\pi_0(Z)$, implying that the reservation value is given by $V^R(\pi_0(Z_t), Z_t)$.

3.2.2. Active Investor Optimization

Let $V(\pi, Z, \bar{\kappa})$ denote the total equity value at the time when the active investor acquires a stake in the firm at price $\bar{\kappa}$. The issuance of new equity to the active investor implies a cash infusion, generating an initial excess cash position of $\bar{\kappa}$ for the firm. While in the illiquid state, the firm's excess cash balance evolves according to

$$d\kappa = (x - c) dt$$

as long as the firm still has cash $\kappa_t > 0$. The following proposition provides the solution to the active investor's dynamic problem conditional on holding an ω -share of the equity in the firm.

PROPOSITION 4 (Value of the active investor in illiquid states). In the illiquid state, the active investor's value from its exposure to the firm is given by

$$V^{A}(\pi_{t}, Z_{t}, \kappa_{t}) = \max_{\pi^{*}(Z), a} \left\{ E_{t} \left[\int_{t}^{\tau^{*}} \frac{\xi(Z_{\tau})}{\xi(Z_{t})} \left(\omega(x - c) - a_{t}\chi \right) d\tau \right] + \omega \kappa_{t} \right.$$

$$\left. + E_{t} \left[\int_{t}^{\tau^{*}} \frac{\xi(Z_{\tau})}{\xi(Z_{t})} \omega\left(V(z_{\tau}, Z_{\tau}) \mathbf{1}_{\{z_{\tau} \in \Omega_{g} \cup \Omega_{b}\}} + V(i_{g}, Z_{\tau}) \mathbf{1}_{\{s_{\tau} = i_{g}\}} \right) \right] \right\}$$

$$= v^{A}(\pi_{t}, Z_{t})^{+} + \omega \kappa_{t}$$

 $\tau^* = \min \{ \tau : \tau \geq t \land (\pi(\tau) \leq \pi^{A*}(Z_{\tau}) \lor z_{\tau} \in \Omega_g \cup \Omega_b \lor s_{\tau} \in \{i_b, i_g\}) \}.$ The function $v^A(\pi, Z)$ solves the following set of ODEs for $Z \in \{G, B\}$:

$$0 = \omega(x - c) - a(\pi_t, Z_t) \chi - r_f(Z_t) v^A(\pi_t, Z_t) + v_\pi^A(\pi_t, Z_t) \frac{d\pi_t}{dt} + \sum_{z' \in \Omega_g} \pi_t \lambda_{i_g, z'}(Z_t) (\omega V(z', Z_t) - v^A(\pi_t, Z_t)) + \sum_{z' \in \Omega_b} (1 - \pi_t) \lambda_{i_b, z'}(Z_t) (\omega V(z', Z_t) - v^A(\pi_t, Z_t)) + \eta a(\pi_t, Z_t)^{1-v} (\pi_t \omega V(i_g, Z) - v^A(\pi_t, Z_t))) + \sum_{Z' \neq Z} \lambda(Z) (1 - \kappa(Z, Z')) (v^A(\pi_t, Z') \mathbf{1}_{\{\pi \geq \pi^{A*}(Z')\}} - v^A(\pi_t, Z)),$$
 (20)

where

$$a\left(\pi,Z\right) = \left(\frac{\eta\left(1-\upsilon\right)}{\chi}\left(\pi\omega V\left(i_{g},Z\right) - \upsilon^{A}\left(\pi,Z\right)\right)\right)^{\frac{1}{\upsilon}} \mathbf{1}_{\left\{\pi > \pi^{A*}\left(Z\right)\right\}},\tag{21}$$

In states Z for which $\varpi(Z) < 0$, the following conditions are satisfied at $\pi^{A*}(Z)$: $0 = v_{\pi}^{A}(\pi^{A*}(Z), Z_{t}), 0 = v^{A}(\pi^{A*}(Z), Z_{t})$. If $\varpi(Z) > 0$ for all $Z \in \{G, B\}$, then $\pi^{A*}(Z)$

is set such that $v^A(\pi^{A*}(Z), Z) = 0$ and $\lim_{\pi \uparrow 1} v^A(\pi, Z) = \omega V(i_g, Z)$. If $\varpi(Z) = 0$, then the ODE simplifies to a nonlinear equation.

Proof. See Appendix.

The active investor optimally proposes liquidation of the firm (if $\kappa_t > 0$) or abandons the firm (if $\kappa_t = 0$) when the discounted present value of its opportunity cost of information production and net-dividends falls below zero. This solution corresponds to a cutoff strategy in the posterior belief π that depends on the aggregate state Z. Specifically, if the conditional probability that the firm is solvent, $\pi(t)$, drops below a threshold $\pi^{A*}(Z)$, the active investor proposes liquidation or abandons the firm. For $\pi(t) > \pi^{A*}(Z_t)$, it is incentive compatible for the active investor to hold on to the equity share ω and to utilize the information production technology to the extent that it maximizes the active investor's value.

Proposition 4 characterizes the active investor's optimal behavior conditional on a given ω -share in the firm. The optimal share ω is given by the maximum feasible $\omega \in [0,1]$ subject to the active investor's capital constraint. Absent capital constraints or other forces that limit the active investor's exposure to the firm (such as cost of undiversification), the active investor optimally obtains a 100% stake in the firm such that it fully internalizes equity holders' benefits from employing its information production technology. With a 100% stake the active investor maximizes total rents to equity holders and equalizes equity holders' marginal gains from improved information quality with marginal information production cost.

3.2.3. Equity Value in Illiquid States With Active Investor Participation

If the active investor obtains a bad signal $s = i_b$, she will propose a dividend payment if the firm currently has excess cash ($\kappa_t > 0$) and refuse further equity injections. Attempting to sell the equity stake will yield a price of zero, since other investors can infer that the active investor must have received a negative signal if current beliefs π_t are above the threshold $\pi^{A*}(Z_t)$. Since a signal $s = i_b$ implies that the firm is insolvent, other equity holders will support the active investor's proposal to pay a dividend and will also abandon the firm thereafter. Through this channel, equity holders may effectively free ride on the active investor's information production once the active investor is exposed to the firm's equity. The following proposition characterizes the value of the equity in the presence of this externality.

PROPOSITION 5 (Equity value in illiquid states with active investor). The firm's equity value in the illiquid state is given by

$$V(\pi_t, Z_t, \kappa_t) = \max_{\pi^*(Z)} E_t \left[\int_t^{\tau^*} \frac{\xi(Z_\tau)}{\xi(Z_t)} \left((x - c) d\tau + V(z_\tau, Z_\tau) \mathbf{1}_{\{z_\tau \in \Omega_g \cup \Omega_b\}} \right) \right]$$

$$= v(\pi_t, Z_t)^+ + \kappa_t$$
(22)

where $\tau^* = \min \{ \tau : \tau \geq t \land (\pi(\tau) \leq \pi^*(Z_\tau) \lor z_\tau \in \Omega_g \cup \Omega_b) \}$. The function $v(\pi, Z)$ solves the following set of ODEs for $Z \in \{G, B\}$:

$$0 = x - c - r_{f}(Z) v(\pi_{t}, Z) + v_{\pi}(\pi_{t}, Z) \frac{d\pi_{t}}{dt} + \sum_{z' \in \Omega_{g}} \pi_{t} \lambda_{i_{g}, z'}(Z) (V(z', Z) - v(\pi_{t}, Z)) + \sum_{z' \in \Omega_{b}} (1 - \pi_{t}) \lambda_{i_{b}, z'}(Z) (V(z', Z) - v(\pi_{t}, Z)) + \eta a(\pi_{t}, Z_{t})^{1-v} \cdot (\pi_{t} V(i_{g}, Z) - v(\pi_{t}, Z_{t}))) + \sum_{Z' \neq Z} \lambda(Z) (1 - \kappa(Z, Z')) (v(\pi_{t}, Z') \mathbf{1}_{\{\pi \geq \pi^{*}(Z')\}} - v(\pi_{t}, Z))$$
 (23)

where a_t is the optimal solution to the active investor's problem and where $\pi^*(Z) = \pi^{A*}(Z)$. In states Z for which $\varpi(Z) < 0$, we have the condition $\lim_{\pi \downarrow \pi^*(Z)} v(\pi, Z) = 0$. If $\varpi(Z) > 0$ for all $Z \in \{G, B\}$, then $\lim_{\pi \uparrow 1} v(\pi, Z) = V(i_g, Z)$. If $\varpi(Z) = 0$, then the ODE simplifies to a nonlinear equation.

Proof. See Appendix.

Proposition 5 shows that existing shareholders abandon the firm at the same threshold as the active investor, that is at $\pi^{A*}(Z)$. The intuition for this result is as follows. The active investor's value is naturally bounded from below by the value of its equity position under a passive strategy, that is $V^A(\pi, Z) > \omega V^R(\pi, Z)$. This is the case since the active investor could always choose to set its effort to zero $(a_t = 0)$ and simply hold the equity stake. Since V^A and V^R are both increasing functions of the probability that the firm is in the solvent state, $\pi(t)$, the active investor's optimal abandonment cutoff $\pi^{A*}(Z)$ is always weakly lower than the optimal reservation value cutoff, that is, $\pi^{A*}(Z) < \pi^{R*}(Z)$. Further, as soon as the active investor abandons the firm, other equity holders may no longer free-ride on the active investor's information production, implying that their value is simply captured by $(1-\omega)V^R$. The equity value absent active investor involvement, V^R , has to be equal to be zero at $\pi(t) = \pi^{A*}(Z)$ since the active investor value $V^A(\pi, Z)$ is zero, and $V^A > \omega V^R$. On the other hand, for beliefs above the abandonment threshold, $\pi(t) > \pi^{A*}(Z)$, passive shareholders' equity value must be positive since the active investor's value is positive and passive equity holders benefit from free-riding on the fund's information production, that is, they share the same benefits but do not incur information production cost.⁵

⁵As described in the setup, management commits not to renegotiate once the initial contract is written. If renegotiation is allowed, changes to the debt contract could be considered as well, which is outside of the scope of this paper (see, e.g., Garlappi, Shu, and Yan (2008) for analysis of the effects of renegotiation upon default).

3.2.4. Excess Cash κ

Proposition 5 shows that the equity value is additively separable in the excess cash position κ_t and the equity value corresponding to core assets $v(\pi_t, Z_t)^+$. The value function simplifies in this way since excess cash may be paid out at any point in time and investments in marketable securities have a zero NPV. Thus, the abandonment decision is independent of the current level of excess cash κ .

3.2.5. Discussion: The Value of Information in Distress

Information production by the active investor would not be an equilibrium feature if the firm had no debt, since information would not alter decisions and therefore generate no value⁶. The proximity to default makes information valuable to equity holders, and is therefore an integral part of the proposed mechanism that jointly drives active investor activity, diluted equity issuances, and dynamics in expected returns. If the active investor were to learn about the firm state, but there was no state of the world in which default was optimal, the decision value of information would be zero and the information production technology would not be used. This is true despite the fact that the setup features stochastic differential utility: there are no gains to acquiring information on the firm-specific state based on an early-resolution-of-uncertainty motive, since this information does not alter the representative household's information set with regards to aggregate consumption dynamics.

When a firm is in distress, the value of the abandonment option embedded in the equity stake naturally constitutes a particularly large fraction of the total equity value. The option value in turn depends critically on the quality information available to agents that exercise the option. The more precise the information the better are exercising decisions, and the more valuable is the option to the agent. The quality of information is therefore an important determinant of the equity value close to the default boundary. Further, as will be illustrated in the next section, variation in the quality of information therefore also critically affects the co-movement of the firm's equity value with aggregate conditions.

3.2.6. Discussion: Alternative Governance Channels

Alternative channels through which active investors may affect firm value include proxy fights, shareholder proposals to replace management, and the alike. Whereas these alternative types of investor activism could be beneficial whenever the firm faces operating decisions, they do not necessarily relate to firms in financial distress: even in

 $^{^6}$ In the setup the firm's assets always generate positive dividends, implying that the firm should never be shut down – the abandonment option problem would be trivial. Yet if c is interpreted as a maintenance flow cost required to preserve the assets (instead of a coupon payment) even an all-equity firm benefits from information production. In other words, operating leverage would be an alternative interpretation for the setup.

the case of an all-equity firm investor activism of this type could affect the firm's decisions and thereby alter exposures to aggregate risk – in other words, proximity to default would not be an essential ingredient. Yet this paper aims to provide a coherent explanation for the fact that the puzzling empirical regularities addressed are all concentrated among firms with high default risk.

3.3. Liquid States (Ω_l)

If the active investor has all the bargaining power, existing shareholders obtain exactly their reservation value V^R as a result of the equity issuance to the active investor, that is, the purchase price $\bar{\kappa}$ is set such that existing shareholders' stake is worth V^R after the equity issuance. For an ω -share in the firm's post-issuance equity the active investor's purchase price is thus given by

$$\bar{\kappa} = \frac{V^R(\pi, Z)}{(1 - \omega)} - v(\pi, Z)^+. \tag{24}$$

Existing shareholders' ability to free ride on the information produced by the active investor in the illiquid states therefore does not increase shareholder value in the liquid state given that the active investor has all the bargaining power when it purchases the equity stake.

PROPOSITION 6 (Equity value in liquid states). In the liquid states $(z, Z) \in \Omega_l$ the firm's equity value is given by

$$V\left(z_{t}, Z_{t}\right) = E_{t} \left[\int_{t}^{\tau^{*}} \frac{\xi\left(Z_{\tau}\right)}{\xi\left(Z_{t}\right)} \left(\left(X\left(z_{\tau}\right) - c\right) d\tau + V^{R}\left(\pi_{0}\left(Z_{\tau}\right), Z_{\tau}\right) \mathbf{1}_{\left\{z_{\tau} \in \Omega_{i}\right\}} \right) \right], \tag{25}$$

where $\tau^* = \min \{ \tau : \tau \geq t \land z_{\tau} \in \Omega_i \}$ and where the function V(z, Z) solves the following system of equations for all $(z, Z) \in \Omega_l$:

$$0 = X(z) - c - r_{f}(Z) V(z, Z) + (V^{R}(\pi_{0}(Z), Z) - V(z, Z)) \sum_{z' \in \Omega_{i}} \lambda_{zz'}(Z) + \sum_{Z' \neq Z} \lambda(Z) (1 - \kappa(Z, Z')) (V(z, Z') - V(z, Z)).$$
(26)

Proof. See Appendix. \blacksquare

4. The Effects of Learning and Active Investors

In this section, I consider solutions to two parameterizations of the model that aim to provide intuition for the effects generated by learning and endogenous information acquisition by an active investor. In the parameterizations of the model, the firm's earnings process is always positively correlated with aggregate consumption growth. Table 1 in the appendix reports chosen parameter values common to all examples. First, I consider a parameterization that shows how learning based on earnings realizations can cause negative expected returns for firms that are close to default, generating striking dynamics in expected returns as a function of time in the illiquid state. Second, I consider an example that illustrates how endogenous information acquisition by an active investor affects equity risk exposures.

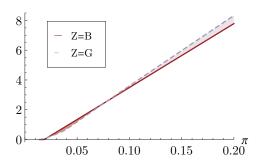
Throughout, I will focus the discussion on the part of the equity value that corresponds to the firm's core assets. The effect of the excess cash position κ on the firm's equity risk premium depends on the types of securities the firm invests in, a choice which is not uniquely pinned down by the model. Investment in any fairly priced security in the economy would be consistent with shareholder value maximization. The expected return premium on the total equity is simply given by the weighted average of the required return premium the firm's core assets and its marketable securities.

4.1. Learning from Past Earnings

Figure 2 plots firm equity values in the illiquid firm state as a function of the aggregate state Z and investor beliefs π . In the left-hand side panel, equity values are plotted over a wider region of beliefs $\pi \in [0,0.2]$. The right-hand side panel of Figure 2 zooms in and provides a more detailed view of equity values close to the default boundary. Whereas the equity value in the good aggregate state Z = G is higher than the one in the bad state Z = B for higher values of π , the opposite is true in a region close to default. Here the equity value in the bad aggregate state is higher, implying negative co-movement of equity values with aggregate conditions.

In the given parameterization, the firm's earnings fundamentals are strongly positively correlated with aggregate growth, that is, a solvent firm (in firm-state $z=i_g$) is more likely to transition back into a liquid state while the economy is in the good aggregate state than in the bad aggregate state (see caption of figure 2). Conversely, an insolvent firm (in firm-state $z = i_b$) is more likely to transition to a state with even lower earnings in the bad aggregate state, and is less likely to do so in the good aggregate state. These transition rates imply that in the good aggregate state (Z=G), the absence of changes in the firm's earnings rate lowers investors' beliefs about the firm's quality, since good firms quickly transition back into a liquid state $(\in \Omega_q)$. In the bad aggregate state (Z = B) on the other hand, passage time is good news, since bad firms (those in firm-state $z=i_b$) quickly reveal themselves by switching into states with even worse earnings ($\in \Omega_b$). Since firms that manage to not deteriorate even further in bad times are likely to be good firms, posterior beliefs are increasing in passage time when the economy is in aggregate state B. In terms of the graph in Figure 2, firms without news in bad times (Z=B) drift upwards on their equity value function. In the good state (Z=G) on the other hand, the equity value drifts downwards absent news and smoothly approaches a value of zero, where the firm is abandoned by the active investor and equity holders.





Equity Value Close To Default

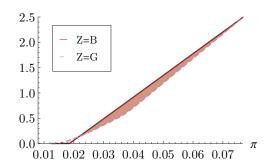
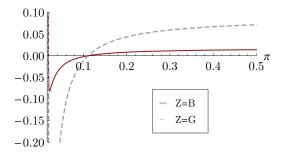


FIGURE II

The graphs plot the equity value of the firm as a function of investors' beliefs about the firm's hidden state. The graph in the left-hand side panel plots the equity value over a range of beliefs $\pi \in [0,0.2]$. The right-hand side panel provides an enlarged picture of the equity value for beliefs π close to the default thresholds. Transition rates for good types are given by: $\lambda_{i_g,\Omega_g}(B) = 0.25$; $\lambda_{i_g,\Omega_g}(G) = 1.2$. Transition rates for bad types are given by: $\lambda_{i_b,\Omega_b}(B) = 1.2$; $\lambda_{i_b,\Omega_b}(G) = 0.25$.

The considered parameterization yields effects that are related to the notion that recessions have a "cleansing effect" on the economy (Schumpeter (1934)): downturns help investors identify truly insolvent firms $(z = i_b)$ more quickly since these firms are less likely to be able to pool with good types $(z = i_g)$ in terms of earnings performance when aggregate conditions are poor. This effect of speeding up learning about insolvent types via the "test of a downturn" improves the quality of information used to exercise equity holders' abandonment option. Since this option value constitutes are large fraction of the total equity value when the firm is close to default, information quality and learning speed also have a large effect on the variation in equity values across aggregate states.

Equity Risk Premia And Beliefs



Equity Risk Premia And Passage Time

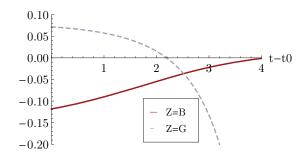


FIGURE III

The figure illustrates equity risk premia as a function of beliefs π (left-hand side) and passage time absent events such as changes in the earnings rate (right-hand side). In the graph on the right-hand side, starting values for beliefs π are 0.5 and 0.15 in aggregate states G and B respectively.

Expected Returns and Momentum The negative comovement between aggregate conditions and firm equity values close to default imply negative risk premia in a region of beliefs π_t close to the default threshold. Figure 3 illustrates expected equity returns as functions of beliefs π (left-hand side graphs) and passage time without events (righthand side graphs). Whereas in good times (Z = G) passage time is associated with declining expected returns, the opposite is true for bad times (Z=B). Further, as discussed above, in the good aggregate state passage time is associated with decreasing equity prices, whereas in the bad aggregate state passage time increases prices. The positive correlation between recent price changes and expected returns implies that firms with recent stock price declines ("recent losers") are also those firms with lower expected returns going forward. On the other hand, distressed firms that recently had an increase in their equity value ("recent winners") have lower expected returns going forward. A momentum strategy that goes long recent winners and shorts recent losers may therefore generate a large spread in expected returns. This result is consistent with the empirical finding of Avramov, Chordia, Jostova, and Philipov (2007) that both the long and the short portfolio of momentum strategies consist of stocks with high default risk. Distressed firms are good candidates for portfolio strategies that sort on past returns, since their past returns are strongly correlated with changes in expected returns going forward.

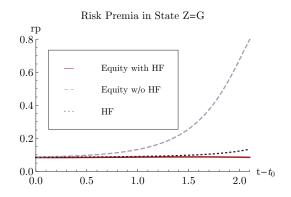
A limitation of the 2 state Markov chain setup for the aggregate state Z is the fact that risk premia on a claim to aggregate consumption are pro-cyclical and relatively small in the given parameterization (see table 1). Whereas two aggregate states have the benefit of higher transparency, extensions to three or more states (e.g. by adding a "disaster state") would improve the model's ability to fit these other dimensions.

4.2. Private Placements and Information Acquisition

In the parameterization used for figures 2 and 3 learning from past earnings was strong enough in the bad aggregate state B to induce negative equity risk premia close to default. Relative to this parameterization, the following example, illustrated in figure IV, only changes the values of two transition rates (see caption of figure IV for details). These different transition rates imply that bad firm types are now less likely to reveal themselves by switching to earnings rates corresponding to states in the set Ω_b . In the previous parameterization (for figures 2 and 3) this self-revelation mechanism was strong in the bad aggregate state Z = B. Since learning based on past earnings is weaker in the parameterization considered in figure IV, it cannot reverse the standard result that equity risk premia increase as the firm approaches the default boundary. Further, the different transition rates also imply that passage time is associated with deteriorating beliefs in both aggregate states ($\frac{d\pi(Z)}{dt} < 0$ for all Z). Yet the example reveals that a private placement of a 20% public equity stake to an active investor can significantly reduce equity holders' expected returns.

Figure IV plots the firm's equity risk premium with and without active investor participation in state G (left-hand side) and state B (right-hand side) as a function of passage time, starting from initial beliefs of $\pi_0 = 0.5$. Further, the figure plots the

expected returns on the active investor's position, which includes the active investor's information acquisition cost. The example reveals that the participation of an active investor greatly reduces equity risk premia of other investors, and extends the time period for which equity holders are willing to hold on to the firm absent negative news. Due to the presence of the active investor and its information acquisition, equity holders' option value is increased and default is optimally triggered later. The active investor has a higher exposure to aggregate risk than other shareholders, since it endogenously incurs higher information acquisition cost in the bad aggregate state Z = B.



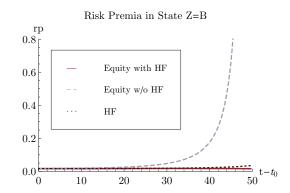


FIGURE IV

The figure plots risk premia for passive equity investors with and without participation of an active investor ("with HF", "w/o HF") and for the active investor ("HF") in aggregate states Z = G (left-hand side panel) and Z = B (right-hand side panel) as a function of passage time without events, starting from a prior belief of $\pi = 0.5$. The parameterization is identical to the one in figures 2 and 3, except for the transition rates of bad types in the illiquid state. Transition rates for bad types are given by: $\lambda_{i_b,\Omega_b}(B) = 0.2$; $\lambda_{i_b,\Omega_b}(G) = 0.1$. All other parameters are identical to the ones in figures 2 and 3.

The example illustrates the stark difference in the dynamics of risk premia with and without the participation of a active investor. This result is directly supported by the empirical finding by Park (2011) that low returns for distressed firms are concentrated among those firms that issue public equity in private placements. The theoretical results of the model in fact also predict that the degree of dilution of existing shareholders is determined by the effectiveness of the active investor and therefore should be negatively related to expected returns going forward.

Robustness: Endogenous vs. Exogenous Information It is worth noting that the active investor's participation can also reduce equity risk exposures if the active investor cannot adjust its effort a_t over the business cycle. Specifically, the effect of the active investor's participation is still present if the investor is simply endowed with a constant signal hazard rate $h_t = \bar{h}$. Yet since it is reasonable that active investors choose the extent to which they focus on distressed companies over the business cycle, it is plausible that there is substantial endogenous variation in effort targeted at distressed

firms. The implications of an endogenous choice of h_t and the robustness of the results to this endogenous choice are therefore useful to analyze.

5. Conclusion

This paper has provided a tractable dynamic asset pricing model to analyze the effects of learning and information production by active investors on expected return dynamics of financially distressed firms. The model reveals that learning can rationalize low and even negative expected equity returns for illiquid firms, and that issuances of privately placed public equity to active investors may constitute an important factor influencing expected returns on passive equity holders' positions. Further, the model can explain the empirical finding by Avramov, Chordia, Jostova, and Philipov (2007) that momentum profits are restricted to high credit risk firms and are nonexistent for firms of high credit quality. The analysis suggests that information acquisition by specialized intermediaries may generate substantial externalities on other shareholders and may account for intriguing empirical facts related to distressed firms' expected returns.

6. Appendix

6.1. Proof of Proposition 1

In equilibrium, the representative household consumes the aggregate consumption flow C_t . The value function is given by

$$J(C_t, Z_t) = E_t \left[\int_t^\infty f(C_\tau, J_\tau) d\tau \right]. \tag{27}$$

The Hamilton-Jacoby-Bellman (HJB) equation in state Z (for all $Z \in \Omega$) is therefore

$$0 = f(Y, J(Y, Z)) + J_Y(Y, Z) Y \mu_Y(Z) + \sum_{Z' \neq Z} \lambda_{ZZ'} (J(Y, Z') - J(Y, Z)).$$
 (28)

Conjecture the solution for J takes the standard form

$$J(Y,Z) = F(Z)\frac{C^{\alpha}}{\alpha}.$$
 (29)

Substituting this conjecture into the HJB equation yields the following system of nonlinear equations for F(Z), for all Z:

$$0 = \left(\frac{\beta \alpha}{\rho} \left(F\left(Z\right)^{-\frac{\rho}{\alpha}} - 1\right) + \alpha \theta_{C}\left(Z\right)\right) F\left(Z\right) + \sum_{Z' \neq Z} \lambda\left(Z\right) \left(F\left(Z'\right) - F\left(Z\right)\right). \tag{30}$$

Duffie and Epstein (1992) show that household maximization implies that a state-pricing process ξ_t may be written as follows:

$$\xi_t \equiv \exp\left[\int_0^t f_J(C_\tau, J_\tau) d\tau\right] f_C(C_t, J_t). \tag{31}$$

Using the value function $J(C_t, Z_t) = F(Z_t) \frac{C_t^{\alpha}}{\alpha}$ we obtain

$$\xi_t = C_t^{\alpha - 1} \beta F(Z_t)^{1 - \frac{\rho}{\alpha}} e^{\left\{ \int_0^t \left(\frac{\beta(\alpha - \rho)}{\rho} F(Z_\tau)^{-\frac{\rho}{\alpha}} - \frac{\beta\alpha}{\rho} \right) d\tau \right\}}.$$
 (32)

Applying Itô's lemma to obtain $r_{f}\left(Z\right)=-\frac{E_{t}\frac{d\xi_{t}}{dt}}{\xi_{t-}}$ yields

$$r_f(Z) = \frac{\beta \alpha}{\rho} - \frac{\beta (\alpha - \rho)}{\rho} F(Z)^{-\frac{\rho}{\alpha}} - (\alpha - 1) \mu_Y(Z)$$
(33)

$$-\sum_{Z'\neq Z} \lambda_{ZZ'} \left(\left(\frac{F(Z')}{F(Z)} \right)^{1-\frac{\rho}{\alpha}} - 1 \right). \tag{34}$$

and

$$\frac{d\xi_{t}}{\xi_{t-}} = -r_{f}(Z_{t}) dt + \sum_{Z' \neq Z_{t-}} \left(\left(\frac{F(Z')}{F(Z_{t-})} \right)^{1-\frac{\rho}{\alpha}} - 1 \right) \left(\mathbf{1}_{\{Z_{t-}, Z'\}} - \lambda_{Z_{t-}Z'} dt \right). \tag{35}$$

where $\mathbf{1}_{\{Z_{t-},Z'\}}$ takes the value one when there is a jump from state Z_{t-} to state Z'.

6.2. Proof of Proposition 2

In the states $(z, Z) \in {\Omega_g, \Omega_b}$, the firm's equity value is given by

$$V(z_{t}, Z_{t}) = E_{t} \left[\int_{t}^{\tau^{*}} \frac{\xi(Z_{\tau})}{\xi(Z_{t})} (X(z_{\tau}) - c) d\tau \right] = v(z_{t}, Z_{t})^{+},$$
 (36)

where $\tau^* = \min \{ \tau : \tau \ge t \land V(z_\tau, Z_\tau) \le 0 \}$. The corresponding HJB equation implies that the function v(z, Z) solves the following system of equations for all $(z, Z) \in \{\Omega_q, \Omega_b\}$:

$$0 = X(z) - c - r_f(Z) \cdot v(z, Z) + \sum_{z' \neq z} \lambda_{zz'}(Z) \cdot \left(v(z', Z)^+ - v(z, Z)\right) + \sum_{Z' \neq Z} \lambda(Z) \cdot (1 - \kappa(Z, Z')) \cdot \left(v(z, Z')^+ - v(z, Z)\right),$$
(37)

where the superscript $^+$ denotes the standard short-hand for the max $\{.,0\}$ operator.

6.3. Equity Value in Solvent and Illiquid States (i_g, Z)

The firm's equity value in states (i_q, Z) is given by

$$V(i_g, Z_t) = E_t \left[\int_t^{\tau^*} \frac{\xi(Z_\tau)}{\xi(Z_t)} (x - c) d\tau + V(z_\tau, Z_\tau) \cdot \mathbf{1}_{\{z_\tau \in \Omega_g \cup \Omega_b\}} \right], \tag{38}$$

where $\tau^* = \min \{ \tau : \tau \ge t \land z_\tau \in \Omega_g \cup \Omega_b \}$. The corresponding HJB equation implies that the function $V(i_g, Z)$ solves the following system of equations for all Z:

$$0 = x - c - r_f(Z) V(i_g, Z) + \sum_{z' \neq z} \lambda_{zz'}(Z) (V(z', Z) - V(i_g, Z))$$
$$+ \sum_{Z' \neq Z} \lambda(Z) (1 - \kappa(Z, Z')) (V(i_g, Z') - V(i_g, Z)). \tag{39}$$

The values V(z',Z) for $(z',Z) \in \Omega_g \cup \Omega_b$ are provided in proposition 1.

6.4. Proof of Lemma 1

The active investor reveals its signal independent of the underlying firm state i_g or i_b . Thus, the hazard rate of revelation is identical across firm types and cancels out. The rest of the result follows from Bayes law.

6.5. Proof of Proposition 3

Absent active investor involvement, the firm's equity value in the illiquid state is given by

$$V^{R}(\pi_{t}, Z_{t}) = \max_{\pi^{*}(Z)} E_{t} \left[\int_{t}^{\tau^{*}} \frac{\xi(Z_{\tau})}{\xi(Z_{t})} \left((x - c) d\tau + V(z_{\tau}, Z_{\tau}) \mathbf{1}_{\{z_{\tau} \in \Omega_{g} \cup \Omega_{b}\}} \right) \right]$$
(40)

$$+V\left(\pi_{\tau},Z_{\tau}\right)\mathbf{1}_{\left\{Z_{\tau}\neq Z_{t}\right\}}\right)]\tag{41}$$

$$= v^R \left(\pi_t, Z_t\right)^+ \tag{42}$$

where $\tau^* = \min \{ \tau : \tau \geq t \land (\pi(\tau) \leq \pi^{R*}(Z_{\tau}) \lor z_{\tau} \in \Omega_g \cup \Omega_b \lor Z_{\tau} \neq Z_t) \}$, and $\mathbf{1}_{\{.\}}$ denotes an indicator function that takes the value 1 if the condition in the subscript is satisfied and 0 otherwise. The corresponding HJB equation yields the following set of ODEs that the function $v^R(\pi, Z)$ solves for $Z \in \{G, B\}$:

$$0 = x - c - r_{f}(Z) \cdot v^{R}(\pi_{t}, Z) + v_{\pi}^{R}(\pi_{t}, Z) \frac{d\pi_{t}}{dt} + \sum_{z' \in \Omega_{g}} \pi(t) \cdot \lambda_{i_{g}, z'}(Z) \cdot (V(z', Z) - v^{R}(\pi_{t}, Z)) + \sum_{z' \in \Omega_{b}} (1 - \pi(t)) \cdot \lambda_{i_{b}, z'}(Z) \cdot (V(z', Z) - v^{R}(\pi_{t}, Z)) + \sum_{Z' \neq Z} \lambda(Z) \cdot (1 - \kappa(Z, Z')) \cdot \left(v^{R}(\pi_{t}, Z')^{+} - v^{R}(\pi_{t}, Z)\right)$$
(43)

Boundary Conditions If $\varpi(Z) < 0$ in state Z, then $v^R(\pi_t, Z)$ satisfies the smooth pasting and value matching conditions,

$$0 = \varpi(Z) \cdot (1 - \pi^{R*}(Z)) \cdot \pi^{R*}(Z) \cdot v_{\pi}^{R}(\pi^{R*}(Z), Z), \tag{44}$$

$$0 = v^{R}(\pi^{R*}(Z), Z). \tag{45}$$

If $\varpi(Z) > 0$, then $\pi^{R*}(Z)$ has to be chosen such that $v^R(\pi^{R*}(Z), Z) = 0$ and $v^R(1, Z)$ matches $V(i_g, Z)$. If $\varpi(Z) = 0$, then the ODE for state Z simplifies to a nonlinear equation. To verify these boundary conditions, let $\bar{V}^R(\pi_t, Z_t, \tau^R)$ denote the equity value given beliefs π_t , aggregate state Z_t , and given that the agent follows a strategy of abandoning the firm at time $t + \tau^R$ if no jump to any other state occurs in the time between t and $t + \tau^R$. The first-order necessary condition for τ^R yields

$$\frac{\partial \bar{V}^R \left(\pi_t, Z_t, \tau^R \right)}{\partial \tau^R} \bigg|_{\tau^R = \tau^{R*}} = 0.$$

A change in variables yields alternatively

$$\left. \left(\frac{\partial \tilde{V}^R \left(\pi_t, Z_t, \pi^R \right)}{\partial \pi^R} \cdot \frac{d \pi^R \left(\tau^R, \pi_t, Z_t \right)}{d \tau^R} \right) \right|_{\tau^R = \tau^{R*}} = 0,$$

where I define the function $\pi^{R}\left(\tau^{R},\pi_{t},Z_{t}\right)$ as follows:

$$\pi^{R}\left(\tau^{R}, \pi_{t}, Z_{t}\right) = \left(1 + e^{\left(-\tau_{Z_{t}}^{R} \cdot \varpi(Z_{t})\right)} \cdot \frac{1 - \pi_{t}}{\pi_{t}}\right)^{-1},$$

implying that for all $\pi_t \in (0,1)$ we have $\frac{d\pi^R}{d\tau^R} > 0$. Thus, for $\pi_t \in (0,1)$ the first-order necessary condition may also be written as

$$\frac{\partial \tilde{V}^{R}\left(\pi_{t}, Z_{t}, \pi^{R}\right)}{\partial \pi^{R}} \bigg|_{\pi^{R} = \pi^{R*}(Z_{t})} = 0,$$

where I define

$$\pi^{R*}\left(Z_{t}\right) \equiv \pi^{R}\left(\tau^{R*}, \pi_{t}, Z_{t}\right).$$

Notice that for $\varpi(Z) > 0$, any $\tau^R \ge 0$ will correspond to $\pi^R \ge \pi_t$, since waiting time τ^R increases the conditional probability π_t . Let $V^R(\pi_t, Z_t)$ denote the value function from the optimal solution of the equity holders' problem. Given the assumption that the equity value is strictly positive at π_t , i.e. $V^R(\pi_t, Z_t) > 0$, we obtain

$$V^{R}\left(\pi^{R}\left(\tau^{R}, \pi_{t}, Z_{t}\right), Z_{t}\right) \geq V^{R}\left(\pi_{t}, Z_{t}\right) > 0$$
, for all $\tau^{R} \geq 0$,

given that $\frac{\partial V^R(\pi, Z_t)}{\partial \pi} \geq 0$ for all $\pi \in [\pi_t, 1]$, since $\pi^R(\tau^R, \pi_t, Z_t) \geq \pi_t$ for all $\tau^R \geq 0$. $\frac{\partial V^R(\pi, Z_t)}{\partial \pi} \geq 0$ follows from the assumption that $V(i_g, Z_t) > V(i_b, Z_t) = 0$. Thus, given that $V^R(\pi_t, Z_t) > 0$ and $\varpi(Z) > 0$, the equity value after any positive waiting time is

also positive, and thus it is optimal not to abandon the firm as long as it stays in the current state, that is, it is optimal to set $\tau^{R*} = \infty$. Since

$$\lim_{\tau^R \to \infty} \pi^R \left(\tau^R, \pi_t, Z_t \right) = 1,$$

the ODE simplifies in the limit $\tau^R \to \infty$ to the non-linear equation that the function $V(i_q, Z_t)$ solves, implying that

$$\lim_{\pi \to 1} V^R \left(\pi_t, Z_t \right) = V \left(i_g, Z_t \right).$$

On the other hand, for $\varpi(Z) < 0$, it follows that $\frac{d\pi^R}{d\tau^R} < 0$, implying that waiting time corresponds to lower conditional probabilities π_t . By assumption, at $\pi = 0$ the equity value is zero $(V(i_b, Z) = 0)$, and the firm is abandoned. Further, by assumption, we have $V^R(1, Z) = V(i_g, Z) > 0$. It is optimal to abandon at π^{R*} where π^{R*} satisfies

$$V^R\left(\pi^{R*}, Z\right) = 0,$$

and where the smooth pasting condition

$$\left. \frac{\partial V^{R}\left(\pi, Z_{t} \right)}{\partial \pi} \right|_{\pi_{t} = \pi^{R*}} = 0,$$

is satisfied. If smooth pasting was not satisfied then there could be an optimal cutoff $\pi^{R^{\hat{}}}$ where the resulting value function $V^{R^{\hat{}}}$ satisfies $V^{R^{\hat{}}}$ ($\pi^{R^{\hat{}}}$, Z) = 0 and

$$\left. \frac{\partial V^{R^{\hat{}}}(\pi, Z_t)}{\partial \pi} \right|_{\pi_t = \pi^{R^{\hat{}}}} > 0.$$

Yet, then heuristically, at $\pi_t = \pi^{R^{\hat{}}}$, the agent benefits from waiting another instant Δt and abandoning the firm afterwards, since the expected income flow is positive:

$$(x - c + \sum_{z' \in \Omega_g} \pi(t) \cdot \lambda_{i_g, z'}(Z) \cdot V(z', Z)) \cdot \Delta t$$

$$+ \sum_{z' \in \Omega_b} (1 - \pi(t)) \cdot \lambda_{i_b, z'}(Z) \cdot V(z', Z) \cdot \Delta t$$

$$+ \sum_{Z' \neq Z} \lambda(Z) \cdot (1 - \kappa(Z, Z')) \cdot v^R(\pi_t, Z')^+ \cdot \Delta t$$

$$= -V_{\pi}^{R^{\hat{c}}}(\pi_t, Z) \frac{d\pi_t}{dt} \cdot \Delta t$$

$$> 0. \tag{46}$$

This contradicts that $\pi^{R^{\hat{}}}$ is an optimal cutoff. Since $\frac{\partial V^{R^{\hat{}}}(\pi,Z_t)}{\partial \pi}\Big|_{\pi_t=\pi^{R^{\hat{}}}} > 0$ violates optimization and since $\frac{\partial V^{R}(\pi,Z_t)}{\partial \pi}$ is weakly positive over the whole domain $\pi \in [0,1]$, it follows that $\frac{\partial V^{R^{\hat{}}}(\pi,Z_t)}{\partial \pi}\Big|_{\pi_t=\pi^{R*}(Z)} = 0$ must hold at the optimal cutoff $\pi^{R*}(Z)$, given that $\varpi(Z) < 0$.

6.6. Proof of Proposition 4

In the illiquid state, the active investor's value from its exposure to the firm is given by

$$V^{A}(\pi_{t}, Z_{t}) = \max_{\{\pi^{*}(Z_{t}), a_{t}\}} \left\{ E_{t} \left[\int_{t}^{\tau^{*}} \frac{\xi(Z_{\tau})}{\xi(Z_{t})} (\omega(x - c) - a_{t}\chi) d\tau \right] + E_{t} \left[\int_{t}^{\tau^{*}} \frac{\xi(Z_{\tau})}{\xi(Z_{t})} \omega(V(z_{\tau}, Z_{\tau}) \mathbf{1}_{\{z_{\tau} \in \Omega_{g} \cup \Omega_{b}\}} + V(i_{g}, Z_{\tau}) \mathbf{1}_{\{s_{\tau} = i_{g}\}}) \right] \right\}$$

$$= v^{H}(\pi_{t}, Z_{t})^{+}$$

$$(47)$$

 $\tau^* = \min \{ \tau : \tau \geq t \land (\pi(\tau) \leq \pi^{A*}(Z_{\tau}) \lor z_{\tau} \in \Omega_g \cup \Omega_b \lor s_{\tau} \in \{i_b, i_g\}) \}$. The corresponding HJB equation yields the following set of ODEs that the function $v^A(\pi, Z)$ solves for $Z \in \{G, B\}$:

$$0 = \omega(x - c) - a_{t}\chi - (r_{f}(Z_{t})v^{A}(\pi_{t}, Z_{t}) + v_{\pi}^{A}(\pi_{t}, Z_{t})\frac{d\pi_{t}}{dt} + \sum_{z' \in \Omega_{g}} \pi(t)\lambda_{i_{g},z'}(Z_{t})(\omega V(z', Z_{t}) - v^{A}(\pi_{t}, Z_{t})) + \sum_{z' \in \Omega_{b}} (1 - \pi(t))\lambda_{i_{b},z'}(Z_{t})(\omega V(z', Z_{t}) - v^{A}(\pi_{t}, Z_{t})) + \eta \cdot a_{t}^{1-v} \cdot (\pi(t)\omega V(i_{g}, Z) - v^{A}(\pi_{t}, Z_{t}))) + \sum_{Z' \neq Z} \lambda(Z)(1 - \kappa(Z, Z'))(v^{A}(\pi_{t}, Z')^{+} - v^{A}(\pi_{t}, Z)).$$

$$(48)$$

The first order necessary condition for a_t yields

$$a_{t} = \left(\frac{\eta \cdot (1 - \upsilon) \cdot \left(\pi(t) \cdot \omega \cdot V\left(i_{g}, Z\right) - \upsilon^{A}\left(\pi_{t}, Z_{t}\right)\right)\right)}{\psi}\right)^{\frac{1}{\eta}}.$$
(49)

Boundary Conditions As in proposition 3, smooth pasting and value matching applies in aggregate states Z for which $\varpi(Z) < 0$,

$$0 = \varpi(Z) (1 - \pi^{A*}(Z)) \cdot \pi^{A*}(Z) \cdot v_{\pi}^{A}(\pi^{A*}(Z), Z_{t}), \tag{50}$$

$$0 = v^{A}(\pi^{A*}(Z), Z_{t}). (51)$$

If $\varpi(Z) > 0$, then $\pi^{A*}(Z)$ has to be chosen such that $v^{A}(\pi^{A*}(Z), Z) = 0$ and $v^{A}(1, Z)$ matches $\omega V(i_g, Z)$. If $\varpi(Z) = 0$, then the ODE simplifies to a nonlinear equation.

6.7. Proof of Proposition 5

The firm's equity value in the illiquid state is given by

$$V(\pi_{t}, Z_{t}) = \max_{\pi^{*}(Z)} E_{t} \left[\int_{t}^{\tau^{*}} \frac{\xi(Z_{\tau})}{\xi(Z_{t})} \left((x - c) d\tau + V(z_{\tau}, Z_{\tau}) \mathbf{1}_{\{z_{\tau} \in \Omega_{g} \cup \Omega_{b}\}} \right) \right] = v(\pi_{t}, Z_{t})^{+}$$
(52)

where $\tau^* = \min \{ \tau : \tau \ge t \land (\pi(\tau) \le \pi^{A*}(Z_\tau) \lor z_\tau \in \Omega_g \cup \Omega_b) \}$. The corresponding HJB equation yields the following set of ODEs that the function $v(\pi, Z)$ solves for $Z \in \{G, B\}$:

$$0 = x - c - r_{f}(Z) v(\pi_{t}, Z) + v_{\pi}(\pi_{t}, Z) \frac{d\pi_{t}}{dt} + \sum_{z' \in \Omega_{g}} \pi(t) \lambda_{i_{g}, z'}(Z) (V(z', Z) - v(\pi_{t}, Z)) + \sum_{z' \in \Omega_{b}} (1 - \pi(t)) \lambda_{i_{b}, z'}(Z) (V(z', Z) - v(\pi_{t}, Z)) + \eta \cdot a_{t}^{1-v} \cdot (\pi(t)V(i_{g}, Z) - v(\pi_{t}, Z_{t}))) + \sum_{Z' \neq Z} \lambda(Z) (1 - \kappa(Z, Z')) \left(v(\pi_{t}, Z')^{+} - v(\pi_{t}, Z)\right).$$
 (53)

Equity holders take the active investor's optimal decisions $(a_t \text{ and } \pi^{A*}(Z))$ as given (see proposition 4).

Boundary Conditions (Case 1 $\varpi(Z)$ < 0): For states Z with $\varpi(Z)$ < 0, the function $v(\pi,Z)$ matches a value of zero at $\pi=\pi^{A*}$, that is $v(\pi^{A*},Z)=0$ (smooth pasting holds as well $\frac{\partial v(\pi,Z)}{\partial \pi}|_{\pi=\pi^{A*}}=0$. At $\pi=\pi^{A*}$, the active investor abandons the firm, implying that the equity holders are left with the reservation value $V^R(\pi,Z)$. Since the active investor's value is naturally bounded from below by the value of its equity position under a passive strategy $(a_t=0)$, optimization implies the relation $V^A(\pi,Z) \geq \omega V^R(\pi,Z)$. Since $\frac{\partial V^A(\pi,Z)}{\partial \pi} \geq 0$ and $\frac{\partial V^R(\pi,Z)}{\partial \pi} \geq 0$, it is also the case that the active investor's optimal abandonment cutoff $\pi^{A*}(Z)$ is weakly lower than the equity holders' optimal reservation value cutoff, that is, $\pi^{A*}(Z) \leq \pi^{R*}(Z)$. The equity value absent active investor involvement, V^R , has to be equal to be zero at $\pi(t) = \pi^{A*}(Z)$ since the active investor value $V^A(\pi,Z)$ is zero, and $\omega V^R(\pi,Z) \leq V^A(\pi,Z)$. For $\pi(t) > \pi^{A*}(Z)$, it be that $V(\pi_t,Z_t) > 0$, since $V^A(\pi,Z) > 0$ and passive equity holders obtain the same benefits as the active investor from holding the equity (up to the scaling factor ω), but do not bear the flow cost of information acquisition going forward. The smooth pasting condition

$$\lim_{\pi_{t} \mid \pi^{A*}(Z)} V_{\pi}\left(\pi, Z_{t}\right) = \lim_{\pi_{t} \uparrow \pi^{A*}(Z)} V_{\pi}\left(\pi, Z_{t}\right) = 0,$$

applies at $\pi_t = \pi^{A*}(Z)$. If it were the case that $V_{\pi}(\pi, Z_t)|_{\pi_t = \pi^{A*}(Z)} > 0$, and $v(\pi_t, Z) = 0$, then equity holders would benefit from waiting for another instant since active investor

abandonment implies $a_t = 0$, implying equity holders would obtain the positive flow

$$(x - c + \sum_{z' \in \Omega_g} \pi(t) \lambda_{i_g, z'}(Z) (V(z', Z))) \cdot \Delta t$$

$$+ \sum_{z' \in \Omega_b} (1 - \pi(t)) \lambda_{i_b, z'}(Z) (V(z', Z)) \cdot \Delta t$$

$$+ \sum_{Z' \neq Z} \lambda(Z) (1 - \kappa(Z, Z')) \left(v(\pi_t, Z')^+ \right) \cdot \Delta t.$$

$$= -v_{\pi}(\pi^{A*}(Z), Z) \frac{d\pi_t}{dt} \cdot \Delta t$$

$$> 0. \tag{54}$$

Yet this would imply that $\pi = \pi^{A*}(Z)$ is not an optimal cutoff. $\pi^{A*}(Z)$ is thus not an optimal abandonment cutoff for equity holders unless $V\left(\pi^{A*}(Z), Z\right) = 0$, and $V_{\pi}(\pi, Z_t)|_{\pi_t = \pi^{A*}(Z)} > 0$.

Boundary Conditions (Case 2 $\varpi(Z) > 0$): For $\varpi(Z) > 0$, then v(1, Z) matches the value $V(i_g, Z)$. If $\varpi(Z) = 0$, then the ODE simplifies to a nonlinear equation. The value function for state Z, denoted by $v(\pi, Z)$, generally jumps discontinuously from zero to a positive value at $\pi = \pi^{A*}(Z)$. By definition, the value of the active investor's position at $\pi = \pi^{A*}(Z)$ zero, that is, $V^A(\pi^{A*}(Z), Z) = 0$. If the active investor has not abandoned the firm yet, it must be that the equity value is strictly positive, that is, $V(\pi^{A*}(Z), Z) > 0$, since equity holders obtain the same benefits as the active investor from holding the equity (up to the scaling factor ω), but do not bear the flow cost of information acquisition going forward. Thus, $\omega V(\pi^{A*}(Z), Z) > V^A(\pi^{A*}(Z), Z) = 0$.

	Variable Descriptions	Notation	Values	
			Z = B	Z = G
1.	Rates of transition between aggregate states	$\lambda(Z)$	0.20	0.20
2.	Local drift of aggregate consumption	$\theta_C(Z)$	0.00	0.04
3.	Risk-free rate	r_f	0.01	0.01
4.	Risk premium of a claim to aggregate consumption	rp^C	0.01	0.03
5.	Scaling factor of the active investor's hazard rate function	η	0.30	
6.	Curvature of the active investor's hazard rate function	v	0.50	
7.	Active investor's information production cost parameter	χ	0.02	
7.	Active investor's equity share	ω	0.20	
8.	Earnings rate in the illiquid states Ω_i	x	0.05	Ó
9.	Coupon rate of the debt contract	c	1.00	
10.	Rate of time preference	β	0.01	
11.	Elasticity of intertemporal substitution	ψ	2.00	
12.	Coefficient of relative risk aversion	γ	28.00	

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