

Banks' Risk Exposures*

Juliane Begenau
Stanford

Monika Piazzesi
Stanford & NBER

Martin Schneider
Stanford & NBER

October 2013

Abstract

This paper studies US banks' exposure to interest rate risk. We exploit the factor structure in interest rates to represent many bank positions as portfolios in a small number of bonds. This approach makes exposures comparable across banks and across the business segments of an individual bank. We also propose a strategy to estimate exposure due to interest rate derivatives from regulatory data on notional and fair values together with the history of interest rates.

*PRELIMINARY & INCOMPLETE. Email addresses: begenau@stanford.edu, piazzesi@stanford.edu, schneidr@stanford.edu. We thank John Cochrane, Darrell Duffie, Isil Erel, Bob Hall, Lars Hansen, Anil Kashyap, Jean-Charles Rochet, Chris Sleet, John Taylor, Harald Uhlig and seminar participants at Arizona State, Chicago Booth Econometrics and Statistics Workshop, Chicago Money & Banking, Duke, the 4th Banque de France–Bundesbank Conference in Paris, the Conference in Honor of Sargent and Sims at the Federal Reserve Bank of Minneapolis, the Conference on Expectational Coordination at the College de France, CREI, the Econometric Society Meeting in Seoul, Federal Reserve Board, the Federal Reserve Banks of Chicago, San Francisco, and New York, NYU, MIT, Ohio State, Princeton University, the ShovenFest at Stanford, SITE, Society of Economic Dynamics in Cyprus, Stanford, Stanford Conference in Quantitative Finance, UC Davis, Yale and Wharton.

1 Introduction

The economic value of financial institutions depends on their exposure to market risk. A traditional bank borrows short term via deposits and lends long term via risky loans. Modern institutions have increasingly borrowed short term in the money market, for example via repurchase agreements and lent long term via holding securities such as mortgage bonds. Modern banks also play a prominent role in derivatives markets. The value of positions taken as a result of these activities changes if market prices change, for example because of news about future monetary policy or default rates.

Measuring banks' risk exposure is not only important for regulation, but also matters for economic analysis more broadly. Institutions are the main players in markets for fixed income instruments. For example, many short term instruments (such as commercial paper) are not traded directly by households. Moreover, banks choose risk exposures that are different from each other and therefore have different experiences when conditions change (for example, Lehman versus JP Morgan during the 2007-2009 financial crisis.) This has motivated a literature that aims to explain asset prices from the interaction of heterogeneous institutions. To quantify such models, we need to know banks' risk exposures.

It is difficult to discern exposures from banks' *reported* credit market positions. Indeed, common data sources such as annual reports and regulatory filings record accounting measures on a large and diverse number of credit market instruments. Accounting measures are not necessarily comparable across positions. For example, the economic value of two loans with the same book value but different maturities will react quite differently to changes in interest rates. At the same time, many instruments are close substitutes and thus entail essentially the same market risk. For example, a 10 year government bond and a 9 year high-grade mortgage bond will tend to respond similarly to many changes in market conditions.

This paper constructs comparable and parsimonious measures of banks' exposure to market risk by representing their positions as portfolios in a small number of bonds. We start from balance sheet data from the US Reports on Bank Conditions and Income ("Call Reports"). We show how to construct, for any bank and for each major class of credit market instruments, replicating portfolios of bonds that have approximately the same conditional payoff distribution. We then compare portfolios across positions as well as across banks.

Our findings suggest that the overall position of the major dealer banks is a portfolio which is long in long-term bonds and short in cash. We also find that these banks have large net positions in interest-rate derivatives. This net derivative position comes close in magnitude to the net position in other fixed income business. We document that,

during much of our sample, the net-interest rate derivative position does not hedge other balance-sheet positions. Instead, banks *increase* their risk exposure through derivatives.

Because of its large size, it is important to account for the net position in interest-rate derivatives when measuring exposure. The key difficulty in measuring the exposure in interest-rate derivatives is that banks do not report the *sign* of their position – whether they represent bets on interest rate increases (e.g., pay-fixed swaps) or decreases (e.g., pay-floating swaps.) Moreover, there is no detailed information about the maturities of these net (as opposed to gross) derivatives positions or the start day of these derivatives (and thus their associated locked-in interest rates).

To deal with the lack of reported information, we propose a novel approach to obtain the exposure contained in the net position in interest-rate derivatives. We specify a state space model of a bank’s derivatives trading strategy. We then use Bayesian methods to estimate the bank’s strategy using the joint distribution of interest rates, bank fair and notional values as well as bid-ask spreads. Intuitively, the identification of the bank’s strategy relies on whether the net position (per dollar notional) gains or loses in value over time, together with the history of rates. If rates go up and the bank’s derivative position experiences gains, the Bayesian estimation puts more probability on a derivative position with a pay-fixed interest rate.

Our approach is motivated by the statistical finding that the market value of fixed income instruments exhibit a low-dimensional factor structure. Indeed, a large literature has documented that the prices of many types of bonds comove strongly, and that these common movements are summarized by a small number of factors. It follows that for any fixed income position, there is a portfolio in a few bonds that approximately replicates how the value of the position changes with innovations to the factors.

For loans and securities, the replication portfolio is derived from detailed information on the maturity distribution provided by the call reports. For loans reported at book value, we follow Piazzesi and Schneider (2010) and represent loan portfolios as bundles of zero coupon bonds. For securities reported at market value, we use those market values together with the properties of zero coupon bond prices. For derivatives, the replication portfolio becomes an observation equation for a state space system, which has unobservable replication weights that can be estimated.

Related literature

The current regulatory framework is known as Basel II. The regulation distinguishes between credit risk due to borrower default and market risk due to price changes. Regulator ask banks to estimate default probabilities of the securities that they are holding either with (external) credit ratings or with internal models. Based on the default probabilities, regulators compute capital requirements for the various positions. This approach

treats the positions one by one. Our portfolio approach treats credit and market risk jointly – exploiting the fact that borrowers tend to default when prices move and vice versa. Moreover, we make positions comparable with each other.

A popular approach to measuring the interest-rate risk exposure of a bank is to run regressions of the bank's stock return on a risk factor, such as an interest rate. The regression coefficient on the interest rate – often referred to as the interest-rate beta, is a measure of the bank's average exposure to interest rate changes over the sample period considered (Flannery and James 1984a). More recently, Landier, Sraer and Thesmar (2013) take the left-hand side variable to be changes in interest income or earnings as a fraction of assets. Interest rate betas do not tell us where the bank's exposure comes from, that is, what positions generate it. This issue has been investigated by relating interest rate betas to summary statistics of bank positions. For example, interest rate betas have been related to banks' maturity gaps, that is, the difference between bank assets and liabilities that mature within a specified horizon (Flannery and James 1984b). Moreover, changes in bank equity values have been related to off-balance sheet statistics that indicate derivative use (Venkatachalam 1996). A key feature of this line of work is that exposure measures are by construction *constant over time* and cannot speak to how exposures change. Recent extensions have attempted to incorporate time-varying interest rate betas, but those have proven difficult to estimate (for example, Flannery, Hammed, and Harjes 1997, Hirtle 1997). Our replication approach is designed to provide time series of exposure. Moreover, since we work with positions data, we can report for each date what positions are generating what exposure.

Our Bayesian approach estimates a time-varying exposure from banks' gains and losses on their interest-rate derivative positions. This approach builds on early work by Gorton and Rosen (1995) who did not have data on market values, because few banks reported them before the adoption of fair value accounting in the mid 1990s. Instead, Gorton and Rosen use data on "replacement costs" from the Call Reports, which refers to the value of derivatives that are assets to the bank (not netting out the liabilities). Under the assumption that the positions have constant maturity and *constant* interest-rate exposure, these data can be used to compute the market value of interest-rate derivatives.

We find that banks mostly take pay-floating positions in interest-rate derivatives, which are positions that gain in value from a surprise fall in interest rates. Some of the counterparties to these positions are nonfinancial corporations, who use pay-fixed positions in swaps to insure themselves against surprising interest-rate increases. Hentschel and Kothari (2001) and Chernenko and Faulkender (2011) document these positions empirically. Jermann and Yue (2012) use a theoretical framework to understand why non-financial corporations have a need for pay-fixed swaps. Minton, Stulz, and Williamson

(2009) document which financial corporations use credit derivatives.

Since the financial crisis, there has been renewed interest in documenting the balance-sheet positions of financial institutions. We share the important goal of this literature: to come up with data on positions that will inform the theoretical modeling of these institutions, as called for by Franklin Allen in his 2001 AFA presidential address. Adrian and Shin (2011) investigate the behavior of Value-at-Risk measures reported by investment banks. They document that VaR per dollar of book equity stayed constant throughout the last decade, including the financial crisis, when these institutions were deleveraging. He, Khang, and Krishnamurthy (2010) document the behavior of book values of balance sheet positions of various financial institutions. These positions do not include derivatives.

Our estimated exposures in the form of replicating portfolios provide broad risk measure for financial institutions. Other risk measures focus on tail risk (e.g., VaRs, Acharya, Pedersen, Phillipon, and Richardson 2010, Kelly, Lustig, and van Nieuwerburgh 2011) or on stress tests (Brunnermeier, Gorton, Krishnamurthy 2012, Duffie 2012). The advantage of replicating portfolios is that they describe the entire distribution, not just tail risks or individual scenarios. Moreover, our portfolios are additive, so that they can be compared across positions within a bank as well as across banks.

2 Banks' fixed income portfolios

Our goal is to understand financial institutions' fixed income strategies. In particular, we would like to compare strategies across institutions and relate different components of an individual institution's strategy, such as its loan portfolio and its derivatives trading business. To characterize the risk exposure of these strategies, we want to describe the period t portfolio holdings and their loadings on one-step risks. We are thus interested in measuring the entire one-step ahead conditional value distribution of these portfolios.

To do that, we fix a probability space $(S^\infty, \mathcal{S}, \mathcal{P})$. Here S is the state space: one element $s \in S$ is realized every period. Denote by s^t the history of state realizations. It summarizes all contingencies relevant to institutions up to date t , including not only aggregate events (such as changes in interest rates), but also events specific to an individual institution, such as changes in the demand for loans and deposits, or the order flow for swaps.

We think of a fixed income instrument as a history-contingent payoff stream that is denominated in dollars. The simplest example is a safe zero coupon bond issued at some date t that pays off one dollar for sure at the maturity date $t + n$, say. More generally, payoffs could depend on interest rates – for example, an interest rate swap

or an adjustable rate mortgage promise payoff streams that move with a short term interest rate – or on other events, such as customers’ decisions to prepay or default on a mortgage.

We assume that every instrument of interest can be assigned a fair value. Following GAAP accounting rules, we view the fair value as the price at which the instrument could be sold “in an orderly transaction”. For instruments traded in a market, fair values can be read off market prices. For nontraded instruments, such as loans, fair values have to be constructed from the payoffs of comparable instruments.

The starting point for our analysis is the fact that fair values of fixed income instruments tend to move together. In particular, the overwhelming majority of movements in bond prices is due to the “overall level” of interest rates. The latter can be summarized by any particular interest rate, for example a riskless nominal short rate. Since fixed income instruments are fairly predictable payoff streams, it is natural that changes in discount rates drive their value. For bonds that are issued by borrowers in the private sector (such as mortgages or business loans), default risk is also very important. To describe the movements of private sector bond prices, it is thus important to capture changes in credit risk.

We thus assume that fair values of all relevant fixed income instruments can be written as functions of calendar time and a small number of random variables that we refer to as factors. Let f_t denote a $(K \times 1)$ -vector-valued stochastic process of factors. Here each f_t is a random variable that depends on the history s^t , but we mostly suppress this dependence in what follows.

The fair value $\pi(f_t, t)$ of a fixed income instrument depends on the factors and calendar time t , which is important because the maturity *date* is part of the description of the payoff stream. As an example, let the payoff stream correspond to a riskfree zero coupon bond with maturity date $t + m$ that was issued at date t or earlier. Let $i_t^{(m)}$ denote the yield to maturity on an m -period zero coupon bond quoted in the market at date t . The price at date t is $\exp(-i_t^{(m)}m)$. At any later date $t + j$ before maturity date (so $j < m$), the price is $\exp(-i_{t+j}^{(m-j)}(m - j))$. The payoff stream thus satisfies our assumption as long as the interest rate depends on the factors.

We assume further that the distribution of the factors is given by a process with a conditional Gaussian distribution. We thus represent the distribution of f_t under \mathcal{P} by a process that satisfies

$$f_{t+1} \sim \mathcal{N}(\mu(f_t, t), \sigma_t^2) \tag{1}$$

with a conditional mean that depends on the factor and a volatility that may depend on time t to capture time-varying uncertainty. We assume that the riskless one period

interest rate is a linear function of the factors:

$$i_t = \delta_0 + \delta_1^\top f_t.$$

We approximate the change in the fair value of the instrument as a linear function in the shocks $\sigma_t \varepsilon_{t+1}$. If time were continuous, Ito's lemma would deliver this result exactly, given conditional normality and the smoothness of the value π . Here we use a second-order Taylor expansion and the properties of normal distributions. We write

$$\begin{aligned} \pi(f_{t+1}, t+1) - \pi(f_t, t) &\approx \frac{d\pi}{df}(f_t, t)(f_{t+1} - f_t) + \frac{d\pi}{dt}(f_t, t) + \frac{1}{2} \sigma_t \frac{d^2\pi}{df df^\top}(f_t, t) \sigma_t^\top \\ &=: a_t^\pi + b_t^\pi (f_{t+1} - \mu_t(f_t, t)), \end{aligned} \quad (2)$$

where the first (approximate) equality uses the fact that the third moments of a normal distribution are zero and higher moments are an order of magnitude smaller than the first and second moments. The coefficient a_t^π is the expected change in fair value. If we divide a_t^π by the current fair value, $\pi(f_t, t)$, we get the expected return. The $1 \times K$ slope coefficients b_t^π is the exposure of the fair value to the factor risks, ε_{t+1} .

We are now ready to replicate the payoff stream of any instrument by $K + 1$ simple securities. That is, we define, for each date t , a portfolio of $K + 1$ securities that has the same value as the instrument in every state of the world at date $t + 1$. We always take one of the securities to be the riskless one period bond; let θ_t^1 denote the number of short bonds in the portfolio at date t . Since θ_t^1 is also the face value of the one-period riskless bonds, we will refer to θ_t^1 as *cash*. For the payoff stream corresponding to a short bond, the coefficients in (2) are given by $a_t^\pi = i_t e^{-i_t}$ and $b_t^\pi = 0$. Consider K additional "spanning" securities that satisfy

$$\hat{P}_{t+1} - \hat{P}_t = \hat{a}_t + \hat{b}_t \varepsilon_{t+1} \quad (3)$$

The $K \times 1$ vector $\hat{\theta}_t$ denotes the holdings of spanning securities at date t . In our one factor implementation below the only spanning security will be a long bond (so that $\hat{\theta}_t$ will be a scalar).

For each period t , we equate the change in the value of the position and its replicating portfolio. This means that for every possible realization of the shocks ε_{t+1} , the holdings of cash θ_t^1 and spanning securities $\hat{\theta}_t$ solve

$$\begin{pmatrix} a_t^\pi & b_t^\pi \end{pmatrix} \begin{pmatrix} 1 \\ \varepsilon_{t+1} \end{pmatrix} = \begin{pmatrix} \theta_t^1 & \hat{\theta}_t^\top \end{pmatrix} \begin{pmatrix} i_t e^{-i_t} & 0 \\ \hat{a}_t & \hat{b}_t \end{pmatrix} \begin{pmatrix} 1 \\ \varepsilon_{t+1} \end{pmatrix}. \quad (4)$$

These are $K + 1$ equations in $K + 1$ unknowns, the holdings $(\theta_t^1, \hat{\theta}_t^\top)$ of cash and longer spanning bonds. If the matrix on the left hand side is nonsingular then we can find portfolio holdings $(\theta_t^1, \hat{\theta}_t^\top)$ that satisfy this equation.

If the market prevents riskless arbitrage, then the value of the replicating portfolio at date t should be the same as the value of the payoff stream $\pi(f_t, t)$. Suppose to the contrary that the value of the replicating portfolio, $\hat{\pi}(f_t, t)$ say, was lower than $\pi(f_t, t)$. Then one could sell short one unit of the payoff stream, buy one unit of the replicating portfolio and invest the difference $\hat{\pi}(f_t, t) - \pi(f_t, t)$ in the riskless asset. Since the change in value for y and the replicating portfolio is identical, this strategy delivers a riskfree profit that consists of the interest earned on $\hat{\pi}(f_t, t) - \pi(f_t, t)$. It follows that one period ahead a position in the payoff can be equivalently viewed as a position in the replicating portfolio: it has the same value at date t as well as in each state of the world at date $t + 1$. Alternatively, we can divide equation (4) by the value $\hat{\pi}(f_t, t) = \pi(f_t, t)$ and see that the returns on these two strategies are equalized state by state.

Once positions are represented as portfolios, we can measure risk by considering how the value of the position changes with the prices of the long term spanning securities \hat{P}_t , or equivalently with the factor innovations ε_{t+1} . If the short interest rate is the only factor, then the exposure of the position is closely related to duration, which is defined as (minus) the derivative of a bond's value with respect to its yield. In this case, the holdings of the spanning bonds $\hat{\theta}_t$ are the delta of the position, and the change in value (4) can be used for Value at Risk (VaR) computations that determine the threshold loss that occurs with a certain probability. For example, we might determine that a given bond has a one-quarter 5% VaR of 90 cents. This would correspond to a 5% probability that the bond's price will fall by more than 90 cents over the quarter.

The advantage of the portfolio representation (4) over VaR is that it fully describes the conditional distribution of risk in the instrument, not just the probability of a certain tail event. Another advantage is that the replicating portfolios of various fixed-income positions are additive, making these positions easy to compare. The same is not true for VaR computations of complex positions. Moreover, our approach can easily incorporate factors in addition to the short rate, such as liquidity factors.

2.1 Data on fixed income portfolios

Our data source for bank portfolios are the Bank Reports of Conditions and Income, or "call reports", filed quarterly by US commercial banks and bank holding companies (BHCs). The call reports contain detailed breakdowns of the key items on an institution's balance sheet and income statement. The breakdowns are for most items more detailed than what is contained in corporations' SEC filings for banks. At the same time, the call reports contain all banks, not simply those that are publicly traded. They also contain additional information that helps regulators assess bank risk. Of particular interest to us are data on the maturity distribution of balance sheet items such as loans and borrowed

money, as well as on the notional value and maturity of interest rate derivative contracts.

Table 1 shows the balance sheet for JP Morgan in year 2011. JP Morgan has 2.3 Trillion Dollars of total assets and liabilities in 2011. Much of its balance sheet is traditional bank business. For example, half of its liabilities are deposits and a third of its assets are loans and some cash. JP Morgan also borrows 10% from other banks in the Federal Funds Market and lends out 17% of its assets to other banks. The more modern bank business is contained in the 16% securities and 20% trading assets. These positions contain a myriad of fixed-income securities that are traded on markets. Appendix A shows Schedule HC-B which has positions on Treasuries, U.S. agency bonds, bonds issued by states and municipalities, mortgage-backed securities, asset-backed securities, and other structured financial products.

TABLE1: JP MORGAN'S BALANCE SHEET IN YEAR 2011

Assets		Liabilities	
Cash	6%	Deposits	50%
Securities	16%	Other borrowed money	15%
Trading assets	20%	Trading liabilities	6%
Fed funds + Repos	17%	Fed funds + Repos	10%
Loans	31%	Other liabilities	11%
Other assets	10%	Equity	8%

The trading assets contain assets, such as Treasuries, for which JP Morgan is a market maker. It also contains derivatives. These are mostly interest-rate derivatives. In particular, JP Morgan has 60 Trillion Dollar in notionals of swap positions. We want to understand the net exposures in these various types of positions.

Figure 1 gives a first impression of the value of the various positions over time. The dotted dark blue line shows the net fair value of interest rate derivatives. The value seems smaller in magnitude compared to its traditional fixed income business. However, this net fair value is not very informative about the risk exposure that this position represents. For example, if JP Morgan enters a pay-floating swap contract, its net fair value is zero (up to bid-ask spreads.) This position still represents a leveraged bet on future interest rate declines which will lose in value if rates go up unexpectedly.

The solid dark blue line describes a net fixed income position without interest rate derivatives: it comprises loans plus securities plus net trading assets less deposits and other debt. To put these numbers in perspective the red line is (book) equity over assets. Finally, the light blue line labeled "net other" is a residual defined so that all three blue

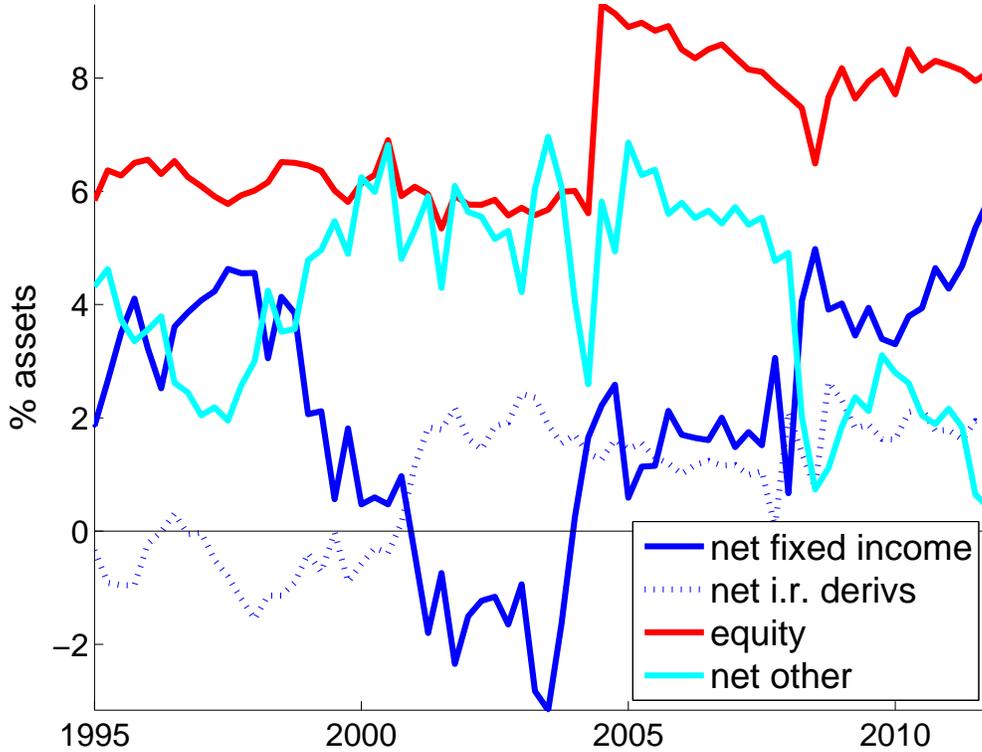


Figure 1: Balance sheet positions of JP Morgan Chase

lines together add up to equity. The remainder of this paper is about understanding the risk exposure inherent in the leveraged fixed income positions represented by the dark blue lines (both dotted and solid.)

More formally, the fair values in Figure (1) only represent the overall value of their replicating portfolio

$$\pi(f_t, t) = \theta_t^1 e^{-it} + \hat{\theta}_t^\top \hat{P}_t$$

in the notation of Section 2. To learn about the risk exposure of the portfolio, we would need to know the portfolio weights $\hat{\theta}_t^{(k)} \hat{P}_t^{(k)} / \pi(f_t, t)$ on each of the $k = 1, \dots, K$ risky spanning securities and how these weights change over time. In particular, these portfolio weights may involve large long-short positions that represent large exposures to risk factors ε_{t+1} but with an overall small fair value $\pi(f_t, t)$. The spanning securities depend on the risk in the factors ε_{t+1} through the loadings \hat{b}_t in equation (3). Therefore, once we know the portfolio weights for each period t , we know how the overall portfolio

depends on the risk factors. In the rest of the paper, we will compute the replicating portfolio $(\theta_t^1, \hat{\theta}_t^\top)$ for each of the fair values in Figure 1 and for each of the U.S. banks.

3 A Portfolio View of Bank Call Reports

In this section we replicate major bank positions in the call reports by portfolios in two “spanning” zero coupon bonds – a one quarter bond (which we often refer to as "cash") as well as a five year bond. Zero coupon bonds are useful because most instruments can be viewed as collections of such bonds, perhaps with adjustments for default risk. For example, a loan or a swap can be viewed as collection of zero coupon bond positions of many different maturities – one for every payment. We now describe a pricing model that gives rise to a linear representation of fair values as in (2) as well as the pricing of zero coupon bonds for that model.

3.1 Summarizing interest rate dynamics

We consider an exponential affine pricing model that describes the joint distribution of riskfree nominal government bonds and risky nominal private sector bonds. The nominal pricing kernel process M_{t+1} represents one step ahead dollar state prices (normalized by conditional probabilities) for dollar payoffs contingent on the factor innovation ε_{t+1} . In particular, for any payoff $x(s^t, \varepsilon_{t+1}(s^t))$ the date t price is $E[M_{t+1}(s^{t+1}) x(s^t, \varepsilon_{t+1}(s^t)) | s^t]$.

We choose the functional form

$$\begin{aligned} M_{t+1} &= \exp\left(-i_t - \frac{1}{2}\lambda_t^\top \lambda_t - \lambda_t^\top \varepsilon_{t+1}\right) \\ \lambda_t &= l_0 + l_1 f_t \end{aligned} \tag{5}$$

The price of a certain payoff of one is the one period zero coupon bond price $P_t^{(1)} = \exp(-i_t)$. Consider next the risky payoff $\exp(\varepsilon_{t+1,k} - 1/2)$: it has mean one and its price is $\exp(-i_t - \lambda_{t,k})$. We can interpret $\lambda_{t,k}$ as the price discount induced by risk contained in the k th innovation $\varepsilon_{t+1,k}$ – it is often referred to as the “market price” of the risk introduced by that innovation. Equivalently, it represents the expected excess return, or risk premium, on the risky payoff. These risk premia can in general vary over time with the factors.

We assume that the factor dynamics (1) are an AR(1), so that

$$\mu(f_t, t) = \phi f_t.$$

The autoregressive parameter ϕ and the factor volatility σ may depend on calendar time in some of our estimations below as well as the risk premia parameters l_0 and l_1 . To keep the notation in this section simple, we will omit this time dependence.

Riskfree government bonds

The price of an m -period riskless zero coupon bond is given recursively by $P_t^{(0)} = 1$

$$P_t^{(m)}(s^t) = E \left[M_{t+1}(s^{t+1}) P_{t+1}^{(m-1)}(s^t, \varepsilon_{t+1}(s^t)) | s^t \right]$$

For $m = 1$, this equation just restates the definition of the one period bond price $P_t^{(1)}$. More generally, the one-period-ahead payoff of an m -period zero coupon bond is the price of the bond next period, when the bond has matured to have maturity $m - 1$.

Our functional form assumptions ensure that all prices take the form

$$P_t^{(m)} = \exp(A_m + B_m^\top f_t) \tag{6}$$

where the coefficients A_m and B_m satisfy a system of difference equations. Log prices are therefore affine in the factors. This property is convenient when computing log returns: for example the one period holding return on the m -period bond $\log P_{t+1}^{(m-1)} - \log P_t^{(m)}$ is affine in f_{t+1} and f_t .

The difference equations for the coefficients A_m and B_m are given by the system

$$\begin{aligned} A_{m+1} &= A_m - B_m^\top \sigma l_0 + \frac{1}{2} B_m^\top \sigma \sigma^\top B_m - \delta_0 \\ B_{m+1}^\top &= B_m^\top (\phi - \sigma l_1) - \delta_1^\top \end{aligned}$$

with boundary conditions $A_1 = -\delta_0$ and $B_1 = -\delta_1$ (Ang and Piazzesi, 2003).

The recursion illustrates how the model incorporates the expectations hypothesis of the term structure. Indeed, consider first risk neutral pricing ($\lambda_t = 0$). The Jensen's inequality terms involving the variance $\sigma \sigma^\top$ tend to be small relative to the other terms. Up to Jensen terms, we can write the log price as minus the sum of expected future short rates

$$\log P_t^{(m)} = -\delta_0 m - \delta_1 \sum_{j=0}^{m-1} \phi^j f_t + \text{Jensen term} = -E_t \sum_{j=0}^{m-1} i_{t+j}^{(1)} + \text{Jensen term}.$$

Roughly speaking, long-term interest rates $i_t^{(m)} = -\log P_t^{(m)}/m$ are then short-term interest rates that are expected to prevail over the lifetime of the bond. The coefficient recursion for A_m and B_m computes this expectation since the factor f_t has AR(1) dynamics. With risk adjustment, the mechanics remain the same; the only difference is

that expectations are taken under a risk-adjusted probability. After risk adjustment, expectations are formed using different AR(1) coefficients $\phi - \sigma l_1$ for the factors and a different long-run mean ($-\sigma l_0$ rather than 0.)

The affine model leads to simple formulas for the coefficients a_t^π and b_t^π in (2):

$$\begin{aligned}
P_{t+1}^{(m-1)} - P_t^{(m)} &\approx P_t^{(m)} \left(B_{m-1}^\top (f_{t+1} - f_t) + A_{m-1} - A_m + (B_{m-1} - B_m)^\top f_t + \frac{1}{2} B_{m-1}^\top \sigma \sigma^\top B_{m-1} \right) \\
&= P_t^{(m)} \left(B_{m-1}^\top (\phi - 1) f_t + A_{m-1} - A_m + (B_{m-1} - B_m)^\top f_t + \frac{1}{2} B_{m-1}^\top \sigma \sigma^\top B_{m-1} \right) \\
&\quad + P_t^{(m)} B_{m-1}^\top \sigma \varepsilon_{t+1} \\
&= P_t^{(m)} (i_t + B_{m-1}^\top \sigma \lambda_t + B_{m-1}^\top \sigma \varepsilon_{t+1}) \\
&= a_t^m + b_t^m \varepsilon_{t+1},
\end{aligned} \tag{7}$$

where the second equality uses the coefficient difference equations.

After dividing the expected change in the bond price (7) by the current price of the bond $P_t^{(m)}$, we can see that the expected excess returns on a riskfree m -period bond held over one period is $B_{m-1}^\top \sigma \lambda_t$. The amount of risk in the excess return on a long bond is $B_{m-1}^\top \sigma$, which is a vector describing the amount of risk due to each of the shocks ε_{t+1} . The vector λ_t of market prices of risk captures a contribution to expected excess returns that is earned as a compensation for a unit exposure to each shock.

Suppose that there is a single factor ($K = 1$) which is positively related to the riskless short rate, that is $\delta_1 > 0$. A large positive shock ε_{t+1} means an increase in the short rate, which lowers the one-period bond price $P_{t+1}^{(1)}$. If $\lambda_t < 0$, a higher short rate represents a bad state of the world. The reason is that, with a negative λ_t , the pricing kernel M_{t+1} depends positively on ε_{t+1} . Since bond prices are exponential-affine (6) and the coefficient B_{m-1} is negative, they have low payoffs in bad states and are thus unattractive assets to investors. To hold such assets, investors want to be compensated with positive expected excess returns. In other words, on average $B_{m-1}^\top \sigma \lambda_t$ is positive.

The conditional standard deviation of the return on the long-term bond is (approximately) equal to $-B_{m-1} \sigma$. This suggests an alternative interpretation of $-\lambda_t$ in equation (7) as the (positive) Sharpe ratio of the bond, its expected excess return divided by the return volatility. The Sharpe ratio is positive if long bonds have low payoffs in bad states – in which case they are unattractive assets that need to compensate investors with a positive premium.

Risky private sector bonds

Private sector bonds are subject to credit risk. For each dollar invested in risky

bonds between t and $t + 1$, there is some loss from default. We treat a risky bond as a claim on many independent borrowers, such as a mortgage bond or an index of corporate bonds. For every dollar invested in the risky bond at date t , there will be some loss from default between dates t and $t + 1$ due to the law of large numbers. This loss will depend on the pool of borrowers and, in particular, their credit rating. For example, a pool of junk-rated borrowers will generate higher expected losses than a pool of AAA-rated borrowers. Moreover, the loss can be larger or smaller depending on the state of the economy at date t , captured by the factors f_t , as well as the state of the economy at date $t + 1$, captured by f_{t+1} (or equivalently, given knowledge of f_t , by the innovation ε_{t+1}).

To retain the tractability of the affine model, we follow Duffie and Singleton (1999) and assume that the recovery value on a bond in default is proportional to the value of the bond. In particular, suppose $\tilde{P}_t^{(m)}$ is the value of an m -period zero coupon risky bond trading at date t with a certain credit rating. As of date t , investors anticipate the value of the bond at $t + 1$ to be $\Delta_{t+1} \tilde{P}_{t+1}^{(m-1)}$, where the loss factor

$$\Delta_{t+1} = \exp \left(-(\tilde{i}_t - i_t) - \frac{1}{2} \left(\tilde{\lambda}_t^\top \tilde{\lambda}_t - \lambda_t^\top \lambda_t \right) - \left(\tilde{\lambda}_t - \lambda_t \right)^\top \varepsilon_{t+1} \right)$$

captures jointly the probability of default and the recovery value. We assume that the risky rate \tilde{i}_t is linear and the process $\tilde{\lambda}_t$ is linear as well

$$\begin{aligned} \tilde{i}_t &= \tilde{\delta}_0 + \tilde{\delta}_1^\top f_t, \\ \tilde{\lambda}_t &= \tilde{l}_0 + \tilde{l}_1 f_t. \end{aligned}$$

Each credit rating will have its own associated group of parameters $\tilde{\delta}_0, \tilde{\delta}_1, \tilde{l}_0$ and \tilde{l}_1 .

The prices of risky bonds are determined recursively as risk adjusted present values:

$$\tilde{P}_t^{(m)}(s^t) = E \left[M_{t+1}(s^{t+1}) \Delta_{t+1}(s^{t+1}) \tilde{P}_{t+1}^{(m-1)}(s^t, s_{t+1}) | s^t \right]$$

As for riskless bonds, there is an exponential affine solution

$$\tilde{P}_t^{(m)} = \exp \left(\tilde{A}_m + \tilde{B}_m^\top f_t \right),$$

where \tilde{A}_m and \tilde{B}_m satisfy a system of difference equations

$$\begin{aligned} \tilde{A}_{m+1} &= \tilde{A}_m - \tilde{B}_m^\top \sigma \tilde{l}_0 + \frac{1}{2} \tilde{B}_m^\top \sigma \sigma^\top \tilde{B}_m - \tilde{\delta}_0 \\ \tilde{B}_{m+1}^\top &= \tilde{B}_m^\top \left(\phi - \sigma \tilde{l}_1 \right) - \tilde{\delta}_1^\top \end{aligned} \tag{8}$$

with boundary conditions $\tilde{A}_1 = -\tilde{\delta}_0$ and $\tilde{B}_1 = -\tilde{\delta}_1^\top$. The exposures \tilde{B}_m of bond prices to the factor will vary across credit ratings.

The expected excess return on a risky short bond is

$$E_t \log \Delta_{t+1} + \frac{1}{2} \text{var}_t (\log \Delta_{t+1}) - \log \tilde{P}_t^{(1)} - i_t = -\lambda_t^\top (\tilde{\lambda}_t - \lambda_t)$$

With risk neutral pricing, the right-hand side is zero. Therefore, the spread between risky and riskfree interest rate $\tilde{i}_t - i_t$ reflects only the expected loss per dollar invested which can vary over time with f_t . More generally, the spread can be higher or lower than the risks-neutral spread because of risk premia. In particular, $\lambda_t < 0$ means that high factor realizations are a bad state of the world (so that M_{t+1} is high when ε_{t+1} is large.) If losses per dollar invested are higher when the factor is high, risky bonds have low payoffs in bad states. Investors then demand a positive expected excess return on the one-period risky bond over the short rate. Whether or not losses are high when interest rates are high depends on how payoffs Δ_{t+1} depend on ε_{t+1} . If $\tilde{\lambda}_t > \lambda_t$, high ε_{t+1} leads to high losses and thus low payoffs Δ_{t+1} . In this case, investors want positive expected excess returns, which we can also see from the right-hand side of the equation.

Replication of risky zero coupon bonds

The affine model leads to simple formulas for the coefficients a_t^π and b_t^π in (2) if the payoff stream is a risky zero coupon bond. Taking default into account, the change in the portfolio value between t and $t + 1$ is

$$\begin{aligned} \Delta_{t+1} \tilde{P}_{t+1}^{(m-1)} - \tilde{P}_t^{(m)} &\approx \tilde{P}_t^{(m)} \left(-(\tilde{i}_t - i_t) + \tilde{A}_{m-1} + \tilde{B}_{m-1}^\top (\phi - 1) f_t - \tilde{A}_m \right) \\ &\quad + \tilde{P}_t^{(m)} \left(\left(\tilde{B}_{m-1} - \tilde{B}_m \right)^\top f_t + \frac{1}{2} \tilde{B}_{m-1}^\top \sigma \sigma^\top \tilde{B}_{m-1} \right) \\ &\quad + \tilde{P}_t^{(m)} \left(\tilde{B}_{m-1}^\top \sigma - \left(\tilde{\lambda}_t - \lambda_t \right)^\top \right) \varepsilon_{t+1} \\ &= \tilde{P}_t^{(m)} \left(i_t + \left(\tilde{B}_{m-1}^\top \sigma - \left(\tilde{\lambda}_t - \lambda_t \right)^\top \right) \lambda_t + \left(\tilde{B}_{m-1}^\top \sigma - \left(\tilde{\lambda}_t - \lambda_t \right)^\top \right) \varepsilon_{t+1} \right) \\ &= \tilde{a}_t^m + \tilde{b}_t^m \varepsilon_{t+1}, \end{aligned}$$

where again the first equality uses the coefficient difference equations.

Note that the parameters $\tilde{\delta}_0$ and $\tilde{\delta}_1$ affect the replication of the change in value only through the coefficients \tilde{A}_{m-1} and \tilde{B}_{m-1} . This is because they represent predictable losses from default which affect the value of the risky position, but not its change over time. The coefficient $\tilde{a}_t^{(m)}/\tilde{P}_t^{(m)}$ is the expected return on the risky bond, which is equal to the riskless short rate plus the risk premium $\left(\tilde{B}_{m-1}^\top \sigma - \left(\tilde{\lambda}_t - \lambda_t \right)^\top \right) \lambda_t$. The

risk premium has two terms as it compensates investors for both time variation in the bond price at $t + 1$ as well as the default loss between t and $t + 1$. For the riskless bond, this risk premium is just equal to $B_{m-1}^\top \sigma \lambda_t$ as computed above. The coefficient $\left(\tilde{B}_{m-1}^\top \sigma - \left(\tilde{\lambda}_t - \lambda_t \right)^\top \right) / \tilde{P}_t^{(m)}$ is the volatility of the return on the bond between t and $t + 1$. In the one factor case, the market price of risk λ_t is again the Sharpe ratio.

Replication with a single factor

Suppose we have a single factor, so that we can replicate any instrument using cash θ_t^1 and a public bond $\hat{\theta}_t$ with spanning maturity m . To replicate a private bonds with maturity n , we equate the changes in value

$$P_t^{(1)} \theta_t^1 i_t + P_t^{(m)} \hat{\theta}_t (i_t + B_{m-1} \sigma (\lambda_t + \varepsilon_{t+1})) = \tilde{P}_t^{(n)} \left(i_t + \left(\tilde{B}_{n-1} \sigma - \left(\tilde{\lambda}_t - \lambda_t \right) \right) (\lambda_t + \varepsilon_{t+1}) \right).$$

The replicating portfolio weight is given by

$$\frac{P_t^{(m)} \hat{\theta}_t}{\tilde{P}_t^{(n)}} = \frac{\tilde{B}_{n-1} \sigma - \left(\tilde{\lambda}_t - \lambda_t \right)}{B_{m-1} \sigma}$$

on the m -period public bond. To translate this portfolio weight into holdings θ_t , we also match the value $\tilde{P}_t^{(n)}$. An important feature of the portfolio weight is that it can be computed entirely with risk-adjusted parameters. These parameters that determine \tilde{B}_n , σ , $\tilde{\lambda}_t$, and λ_t can be estimated from repeated cross sections of bonds for every period separately. This allows the procedure to accommodate changes in risk σ over time as well as in risk premia parameters l_0, l_1, \tilde{l}_0 and \tilde{l}_1 . Below, we will report results for both the case where these parameters are constant and for the case where the parameters change over time.

If the bond we are replicating is riskless, the portfolio weight is constant over time and has the simpler formula

$$\frac{P_t^{(m)} \hat{\theta}_t}{P_t^{(n)}} = \frac{B_{n-1}}{B_{m-1}}$$

Intuitively, if $m = n$, the portfolio weight is equal to 1. Moreover, the B s are negative and their absolute value increases in maturity, so we will find a larger portfolio weight if $n > m$ and smaller otherwise. If $n = 1$, then $B_0 = 0$, and the portfolio weight on the long riskless bond is zero, because the replicating portfolio consists only of cash.

A risky, private sector bond is like a riskless bond with a different duration. Whether it is shorter or longer depends on the parameters of Δ . There are two effects. First, the replicating portfolio captures exposure to the interest rate induced by losses from default between t and $t + 1$. The direction of this effect depends on the sign of $\tilde{\lambda}_t - \lambda_t$.

If $\tilde{\lambda}_t > \lambda_t$, then there is more default (or a lower payoff in default) when interest rates are high. As a result, a riskier bond will have more exposure to changes in interest rates and is thus more similar to a longer riskless bond.

The second effect comes from the difference between the coefficients B and \tilde{B} . From (8), this effect depends not only on \tilde{l}_1 , but also on $\tilde{\delta}_1$. If $\tilde{\delta}_1 > \delta_1$, then there are larger *expected* losses from default if interest are high. This means that risky bond prices are more sensitive to interest rates than riskless bond prices of the same maturity, so that again risky bonds work like longer riskless bonds. The opposite result obtains if $\tilde{\delta}_1 < \delta_1$.

3.2 The estimated one factor model

We estimate the government and private sector yield curves using quarterly data on Treasury bonds, swap rates and corporate bond rates over the sample 1995:Q1-2011:Q4. We have data on corporate bonds with 14 different credit ratings: AAA, AA, A+, A, A-, BBB+, BBB, BBB-, BB+, BB, BB-, B+, B and B-. Swap contracts are not much affected by credit risk because of collateral provisions. The floating side of the swap is set to LIBOR, which does reflect the credit risk of banks. As a result, swap rates are very close to AAA-rated bond yields.

The government bond yields are the solid lines in Figure 2, while swap yields with the same maturity are the dashed lines in the same color. In a principle component analysis, a large fraction of the variation in these yield data, 93%, is explained by a single factor. In Figure 2, this is reflected by the fact that all rates vary around together. The movements in the longer maturity rates (towards the top in the figure) are somewhat dampened versions of the movements in the shorter maturity interest rates (towards the bottom.) The gray shaded area is the TED spread, defined as the difference between the 3-month LIBOR rate and the 3-month T-bill rate. This spread is higher in times when interest rates are high—right before recessions. This is why these credit spreads are commonly used as leading recession indicators. During the financial crisis of 2007-2008, the TED spread increased further when interest rates fell unexpectedly.

As our single factor, we choose the BB-rated interest rate, which will capture both interest rate risk as well as credit risk. To measure the exposure of public and private bonds to changes in this risky rate, we need (i) its time series dynamics (1), (ii) coefficients that describe the dependence of short public (δ_0, δ_1) and private $(\tilde{\delta}_0, \tilde{\delta}_1)$ short-term rates depend on the factor and (iii) coefficients that capture risk premia for public (l_0, l_1) and private bonds $(\tilde{l}_0, \tilde{l}_1)$.

The estimation of the government yield curve is in several steps. First, we estimate the parameters ϕ and σ with OLS on the (demeaned) factor dynamics (1). Then we

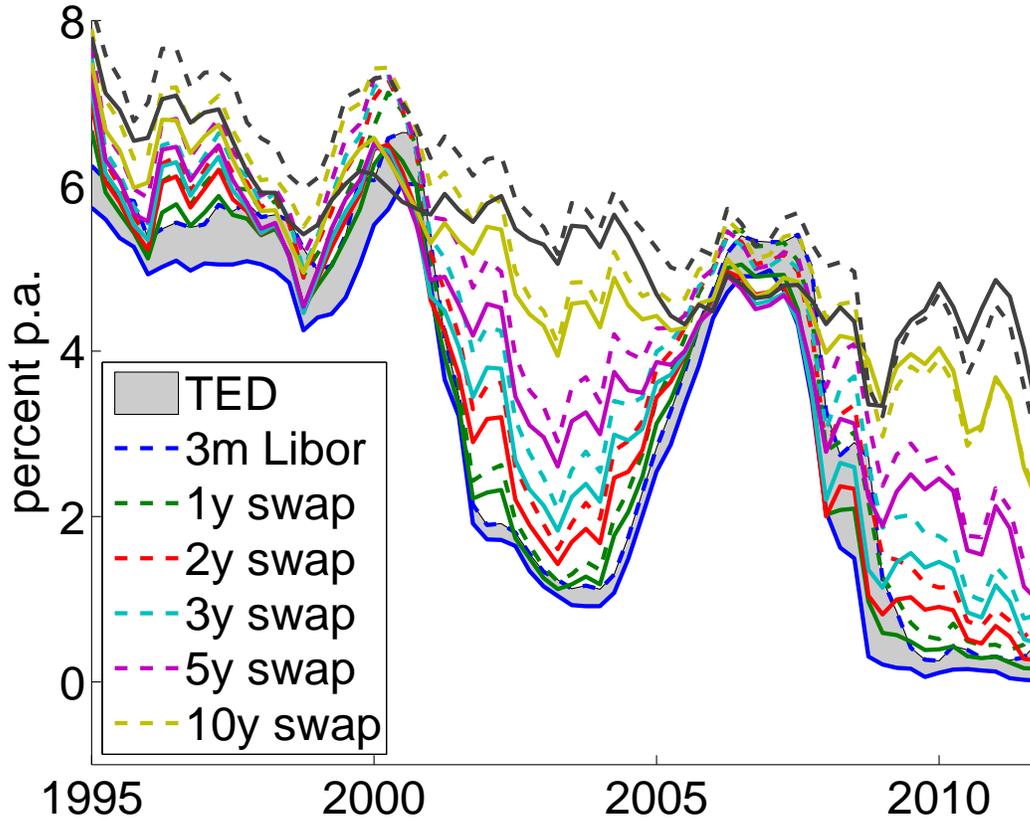


Figure 2: Public and private sector zero-coupon interest rates with the same maturity. Solid lines are public, dashed lines are private. The gray shaded area is the TED spread, which is the difference between the 3-month libor rate and the 3-month Treasury bill rate.

estimate the parameter δ_0 as the mean of the riskless short rate and δ_1 with an OLS regression of the short rate on the factor. Finally, we estimate the parameters l_0 and l_1 by minimizing the squared errors from the model

$$\min_{l_0, l_1} \sum_{t,m} \left(i_t^{(m)} - \hat{i}_t^{(m)} \right)^2 \quad (9)$$

where

$$\hat{i}_t^{(m)} = -\frac{A_m}{m} - \frac{B_m^\top}{m} f_t.$$

For every credit rating, we also estimate a group of parameters $\tilde{\delta}_0$, $\tilde{\delta}_1$, \tilde{l}_0 , and \tilde{l}_1 . The coefficients $\tilde{\delta}_0$ and $\tilde{\delta}_1$ are obtained with an OLS regression of the short credit-risky rate

on the demeaned factor. The estimates of the parameters \tilde{l}_0 and \tilde{l}_1 minimize the squared errors

$$\min_{\tilde{l}_0, \tilde{l}_1} \sum_{t,m} \left(\hat{i}_t^{(m)} - \tilde{i}_t^{(m)} \right)^2 \quad (10)$$

where the model-implied private yields are

$$\tilde{i}_t^{(m)} = -\frac{\tilde{A}_m}{m} - \frac{\tilde{B}_m^\top}{m} f_t.$$

For the portfolio weights, it is enough to estimate the risk-adjusted parameters. We do this by solving the nonlinear-least squares problems (9) and (10) for every period t separately. We also tried estimating the factor dynamics and the $(\delta, \tilde{\delta})$ parameters for each period. These estimations did not give portfolio weights that differed much from what we report below.

Panel A in Table 2 contains the estimation results together with Monte Carlo standard errors. The parameter δ_0 times four is the average short rate, 3.08%. The riskless short rate has a loading below one on the factor, $\delta_1 = 0.86$. The factor is highly persistent with a quarterly autoregressive coefficient of 0.92. The market prices of risk are on average negative, $l_0 = -0.29$ (since the factor has a mean of zero), implying that high nominal interest rates represent bad states of the world. Investors want to be compensated for holding assets – such as private or public nominal bonds – that have low payoffs (low prices) in those states. These average prices of risk are, however, imprecisely estimated in small samples. The market prices of risk are significantly smaller (“more negative”) in bad times, $l_1 = -27$. In other words, risk premia are significantly countercyclical.

Losses per dollar invested in risky bonds are estimated to be larger when the factor realization is large $\tilde{\lambda}_t > \lambda_t$. Therefore, investors demand compensation for holding risky bonds; expected excess returns on risky bonds are positive. Spreads between risky and riskless rates covary positively with the level of interest rates—as suggested by the large TED spread during periods of high rates in Figure 2.

Panel B in Table 2 shows average absolute fitting errors between 45 basis points and up to 1.19 percentage points. With one factor that captures both the level of interest rates and credit risk, the estimation distributes these errors across the various maturities and credit ratings. Since the factor is the BB-rated short rate, the errors are largest for the safest bonds, which have short maturities and higher credit ratings.

TABLE 2: YIELD CURVE ESTIMATIONS

PANEL A: PARAMETER ESTIMATES				
	ϕ	σ		
Factor dynamics	0.9153 (0.0053)	0.0016 (0.0001)		
	δ_0	δ_1	l_0	l_1
public yields	0.0077 (0.1425)	0.8648 (0.1153)	-0.2940 (0.9836)	-27.7352 (7.1847)
	$\tilde{\delta}_0$	$\tilde{\delta}_1$	\tilde{l}_0	\tilde{l}_1
private yields				
swaps	0.0090 (0.1623)	0.9851 (0.1452)	-0.2392 (0.9808)	-18.8120 (9.4895)
A-rated	0.0095 (0.1665)	1.0103 (0.1385)	-0.2697 (0.9813)	-16.8991 (9.0964)
BBB-rated	0.0106 (0.1599)	0.9704 (0.1616)	-0.2802 (0.9816)	-18.5919 (9.3845)

PANEL B: MEAN ABSOLUTE ERRORS (% PER YEAR)

maturity n (in qrts)	1	4	8	12	20	40
public yields $i_t^{(n)}$	1.19	1.31	1.25	1.15	0.95	0.72
private yields $\tilde{i}_t^{(n)}$						
swaps	1.19	1.25	1.16	1.07	0.93	0.80
A-rated	1.04	1.07	1.00	0.86	0.70	0.51
BBB-rated	0.78	0.83	0.77	0.65	0.54	0.45

Note: Panel A reports parameter estimates and small sample standard errors. The data are quarterly zero coupon yields from Treasuries, swaps, and corporate bond rates with AAA, AA, A+, A, A-, BBB+, BBB, BBB-, BB+, BB, BB-, B+, B and B-.ratings for the sample 1995:Q1-2011:Q4. The single factor is the BB-rated short rate. The sequential estimation procedure is described in the text. The small sample standard errors are computed from 10,000 Monte Carlo simulations with the same sample length as the data. Panel B reports mean absolute fitting errors for public interest rates $i_t^{(m)}$ and private rates $\tilde{i}_t^{(m)}$ for the indicated credit rating in annualized percentage points.

3.3 Bank sample

Our analysis considers domestic top-tier bank holding companies (BHC) during 1995:Q1-2011:Q4. Call Reports are filed by both for bank holding companies (BHC) and commercial banks. A commercial bank is often owned by a BHC and a BHC itself can be a subsidiary of a larger BHC. We consider only BHCs that are the "top tier" company in their BHC, and eliminate any BHC that is owned another BHC. We also eliminate all BHC that have a foreign parent.

Most of the data series we are interested in are available directly at the consolidated BHC level. However, for some series, in particular for positions on loans, securities and borrowed money broken down by maturity, detailed information is available only at the commercial bank level. To use this information, we match commercial banks to their top tier parent BHC. We then aggregate positions by maturity over all commercial banks with the same parent. We verify that total positions from bank and BHC files are consistent.

We use information on merger and acquisition activities over our sample from the Federal Reserve Bank of Chicago. This data has date of merger, the identity number of the non-surviving and the acquiring bank and their respective bank holding company identity number.

3.4 Loans and securities

The major categories of fixed income instruments on the asset side of the bank balance sheet are cash, loans and securities. In order to arrive at a portfolio representation, we first express balance sheet positions in terms of zero coupon bonds. For example, an installment loan is as a bundle of many zero coupon bonds – one for each installment – each with a credit rating derived from the risk weight assigned to the loan in the Call Report. We now describe how data on balance sheet positions, interest rates, maturities and risk weights is used for this step.

Accounting data

Under traditional accounting rules, loans and securities are recorded on balance sheets at face value. The face value of a loan is usually the amount of money disbursed when the loan is taken out (although there can be small differences, for example, when a mortgage borrower buys points.) The face value of a security position is the par value of the security. Recent statements by the Financial Accounting Standard Board (FASB) have moved US GAAP rules towards marked-to-market (MTM) accounting.

Statement FAS 115, issued in 1993, introduced a three way split of positions into

"held to maturity" (HTM), "available for sale" (AFS), and "held for trading" instruments. The latter two categories are recorded at fair value on the balance sheet, while HTM instruments are recorded at face value.¹ The call reports provide a breakdown into the three categories for both loans and securities. Over our sample, the majority of positions in loans is recorded at face value, while the majority of positions in securities is recorded at fair value.

Maturity data in the call reports are in the form of maturity (or repricing) buckets. The five buckets contain positions of maturity less than one quarter, 1-4 quarters, 1-3 years, 3-5 years, 5-15 years and more than 15 years. The call reports also provide, for both loans and securities, risk weights that are used to calculate risk-based capital requirements. Guidelines for filling out call reports relate risk categories to credit ratings.

Short term assets

For short term assets and liabilities, book value and fair value are typically very similar. Here "short term" refers not to the maturity date, but rather to the next repricing date. For example, a 30 year adjustable rate mortgage that resets every quarter will also have a fair value close to its book value. We treat all assets with repricing date less than one quarter as a one quarter bond, applying a private sector or government short rate depending on the issuer.

Loans

We view loans as installment loans that are amortized following standard formulas. We derive a measure of market value for loans by first constructing a payment stream corresponding to a loan portfolio, and then discounting the payment stream using the yield curve. The resulting measure is not necessarily the market price at which the bank could sell the loan. Indeed, banks might hold loans on their portfolios precisely because the presence of transaction costs or asymmetric information make all or parts of the portfolio hard to sell. At least part of the loan portfolio should thus best be viewed as a nontradable "endowment" held by the bank. Nevertheless, our present value calculation will show how the economic value of the endowment moves with interest rates.

The first step is to find, for each date t , the sequence of loan payments by maturity expected by the bank. Let x_t^m denote the loan payment that the expected as of date t by the bank in $t+m$, (x^m) . To construct payment streams, we use data on the maturity distribution of loan face values (N_t^m) together with the yield to maturity on new loans by maturity (r_t^m). For the first period in the sample, we assume that all loans are new. We thus determine the payments (x_t^m) by a standard annuity formula: N_t^m must equal

¹The difference between AFS and trading assets is how changes in fair values affect earnings: trading gains and losses directly affect net income, whereas gains and losses on AFS assets enter other comprehensive income (OCI), a component of equity.

the present value of an annuity of maturity m with payment x_t^m and interest rate r_t^m .

We can also determine how much face value from the initial vintage of loans remains in each following period, assuming that loans are amortized according to the standard schedule. We then calculate recursively for each period the amount of new loans issued, as well as the expected payments and evolution of face value associated with that period's vintage. In particular, for period t we compute new loans as the difference between total loan face values observed in the data and the partially amortized "old loans" remaining from earlier periods.

This procedure produces a complete set of payment streams for each date and maturity. The market value can then be calculated by applying the appropriate private or public sector prices to the payment streams. We use risk weights to select the yield curves with which to value each loan portfolio. We thus use the AAA, A and BBB yield curves to evaluate payment streams with different risk weights as suggested by the call report guidelines.

Securities & trading assets

Suppose there is a pool of securities for which we observe fair values by maturity (FV_t^m). Without information on face values, it is difficult to construct directly the payment stream promised by the securities. As a result, the construction of the replicating portfolio from payoff streams is not feasible. However, we can use the maturity information to view securities as zero coupon bonds that can be directly replicated. Consider the case of riskless bonds – here we count both government bonds and GSE-insured mortgage bonds. We assume that the fair value FV_t^m is the market value of $\theta_t^{(m)} = FV_t^m / P_t^{(m)}$ riskless zero coupon bonds.

We then replicate these bonds according to (4). Similarly, for private sector bonds – all private sector bonds that are not GSE-insured – we can find $\theta_t^{(m)} = FV_t^m / \tilde{P}_t^{(m)}$ and then replicate the resulting portfolio of private sector zero coupon bonds. As for loans, the call reports provide maturity buckets for different types of securities. We again proceed under the assumption that the maturity is uniform conditional on the bucket and that the maximal maturity is 20 years.

For securities held for trading, detailed data on maturities is not available. This item consists of bonds held in the short term as inventory of market making banks. We proceed under the assumption that the average maturity is similar to that of securities not held for trading. From the breakdown of bonds held for trading into different types we again form private and public bond groups and replicate with the respective weights.

Liabilities

Since the large banks we study have low default risk, we treat their liabilities as riskless. We treat short term liabilities as short term bonds. For long term debt, we

follow a similar procedure for constructing vintages of coupon rates as for loans and then value the resulting payment streams. The difference between bonds and loans is that long term debt is treated as coupon bonds issued at a par value equal to the face value. As a result, the payment stream consists of a sequence of coupon payments together with a principal payment at maturity, and the face value is not amortized.

3.5 Interest rate derivatives

The data situation for interest rate derivatives is different than that for loans and securities. In particular, we do not observe the direction of trades, that is, whether a bank wins or loses from an increase in interest rates. We can, however, infer the direction of trade from the joint distribution of the net fair value in interest rate derivatives together with the history of interest rates. Intuitively, if the bank has a negative net fair value and interest rates have recently increased, we would expect that the bank has position that pays off when interest rates fall, for example it has entered in pay-fixed swaps or it has purchased bonds forward. The strength of this effect should depend on the bid and ask prices that the bank deals at.

Our goal is to approximate the net position in interest rate derivatives by a replicating portfolio. We work under the assumption that all interest rate derivatives are swaps. In fact, swaps make up the majority of interest rate exposures, followed by futures which behave similarly as they also have linear payoffs in interest rates. A more detailed treatment of options, which have nonlinear payoffs, is likely to be not of primary importance and in any case is not feasible given our data.

Fair values of interest rate swaps

A plain vanilla interest rate swap is an agreement by two parties to exchange interest payments at regular intervals. The interest payments are proportional to a notional amount N . One party pays a fixed interest rate, the swap rate s , while the other party pays a floating short term market rate. The payments are made at a certain frequency up to a given maturity m . The stream of fixed interest rate payments together with the notional value paid at maturity, is referred to as the “fixed leg” of the swap. The stream of floating payments together with the notional value at maturity is called the “floating leg”. Although the notional values cancel exactly, including them as part of the fixed and floating leg streams is helpful in calculations.

The fixed leg is equivalent to a portfolio of bonds, specifically an m period annuity that pays sN per period, together with an m period zero coupon bond that pays N . The value of the fixed leg – the present value of the payment stream – thus depends positively on bond prices; it declines when interest rates rise and bond prices fall. The value of the floating leg is always equal to N dollars in cash. Indeed, the stream of

interest payments that accrues to the owner of the floating leg is the same as the stream earned by someone who rolls over the notional at the short term rate.

To see the dependence of fair values on the yield curve, consider a pay-fixed swap of maturity m and notional value of one dollar that promises fixed payments at the swap rate s and receives floating payments at the short rate $- \log \tilde{P}_t^{(1)}$. Define $\tilde{C}_t^{(m)}$ as the date t price of an annuity that promises one dollar every period up to date $t + m$. Using $\tilde{P}_t^{(m)}$ to denote a zero coupon bond of maturity m , the fair value can be written as the difference between the floating and fixed legs

$$F_t(s, m) = 1 - \left(s\tilde{C}_t^{(m)} + \tilde{P}_t^{(m)} \right) \quad (11)$$

There is always a swap rate that makes the fair value zero – this is how the swap rate is set in a frictionless market. For a given locked in swap rate, an increase in interest rates that lowers bond prices also increases the fair value of a pay fixed swap. Put differently, the fair value of a pay fixed (receive floating) swap increases when rates rise since higher floating rates are then received.

The fair value of a pay-floating swap with notional value of one dollar is equal to $-F_t(s, m)$. We can view a pay-floating swap as a leveraged position in long bonds which gains as interest rates fall. Put differently, the fair value of a pay-floating (receive fixed) swap increases when rates fall, as lower floating rates must be paid. In what follows we denote the direction of trade by d , with $d = 1$ for pay fixed and $d = -1$ for pay floating. A swap position with notional value one is then identified by a triple (d, m, s) and has fair value $dF(s, m)$. More generally, fair value is proportional to notional, i.e. for notional value N the fair value is $dF(s, m)N$.

Inference strategy

The basic idea behind our estimation exercise can be explained using (11). Suppose we knew that a bank has exactly one swap position with known maturity m and swap rate s . Suppose further that we observe the maturity m , the yield curve (as reflected in the bond prices $P_t^{(m)}$ and $C_t^{(m)}$, the notional N and the fair value, FV say. However, we do not know the direction of trade or the swap rate. The valuation formula says that

$$\frac{FV}{N} = dF(s, m) \quad (12)$$

It follows that for each direction $d \in \{1, -1\}$ there is a unique swap rate that rationalizes the observed fair value. For example, if the fair value is positive then a pay-fixed position can be justified only by a locked-in swap rate that is relatively low compared to the current interest rate. In contrast, a pay-floating position can be justified only by a locked-in swap rate that is relatively high. Since the actual swap rate locked by

the bank must have been locked in in the recent past, we can then use the history of interest rates to assess which possibility is more likely. While we can never be certain about the direction and swap rate, we may be able to become rather confident about it since combinations (d, s) can appear implausible in light of history.

Bank positions in the call reports represent aggregates over many individual swap positions, each with its own direction, remaining maturity, locked-in swap rate. The triples (d, m, s) differ for example because of differences in clients or the dates at which rates were locked. Let $N_t^{d,m,s}$ denote the date t amount of notionals in swaps with direction, maturity and swap rate (d, m, s) . The total net fair value of the bank's position is the sum over all individual positions:

$$FV_t = \sum_{d,m,s} N_t^{d,m,s} dF_t(s, m). \quad (13)$$

We now describe a series of assumptions such that the idea above can be used at the bank level. We then formulate an observation equation for bank swap positions that is based on (12) evaluated at the average maturity, plus measurement error and compute posteriors for for directions and swap rates.

Swap books and bid ask spreads

In a textbook frictionless market, swap rates are determined at the inception date (when the swap contract is written) so as to set the initial fair value equal to zero. In practice, initial swap rates incorporate bid ask spreads. Indeed, most swaps are traded over the counter. As for many classes of bonds, a few large dealers make the market. In particular, dealers intermediate between two parties by initiating, say, a pay fixed swap with the first party as well as an offsetting pay floating swap with the second party. Often one of the parties is another dealer. The concentration of the market is illustrated in Figure 3. It shows the total notionals of interest rate derivatives held for trading, for all BHCs as well as for the top three BHCs in terms of interest rate derivatives held for trading. Here we exclude Goldman-Sachs and Morgan Stanley, firms that became BHCs only after the financial crisis.

The swap dealer makes money by adjusting the swap rates to incorporate a spread. In particular, the swap rate on a pay-fixed (pay-floating) swap is typically lower (higher) than the rate that makes the fair value zero. Moreover, the both swaps remain in the accounts of the dealer and contribute to the reported numbers for notional and fair values. The income on the swap is earned only period by period as the swap payments are made and are recorded as income when they received.² Bidask spreads in the swap market are small – 2 basis points on average over our sample.

²Swap dealing and bond dealing therefore affect a dealer's position rather differently. A bond dealer

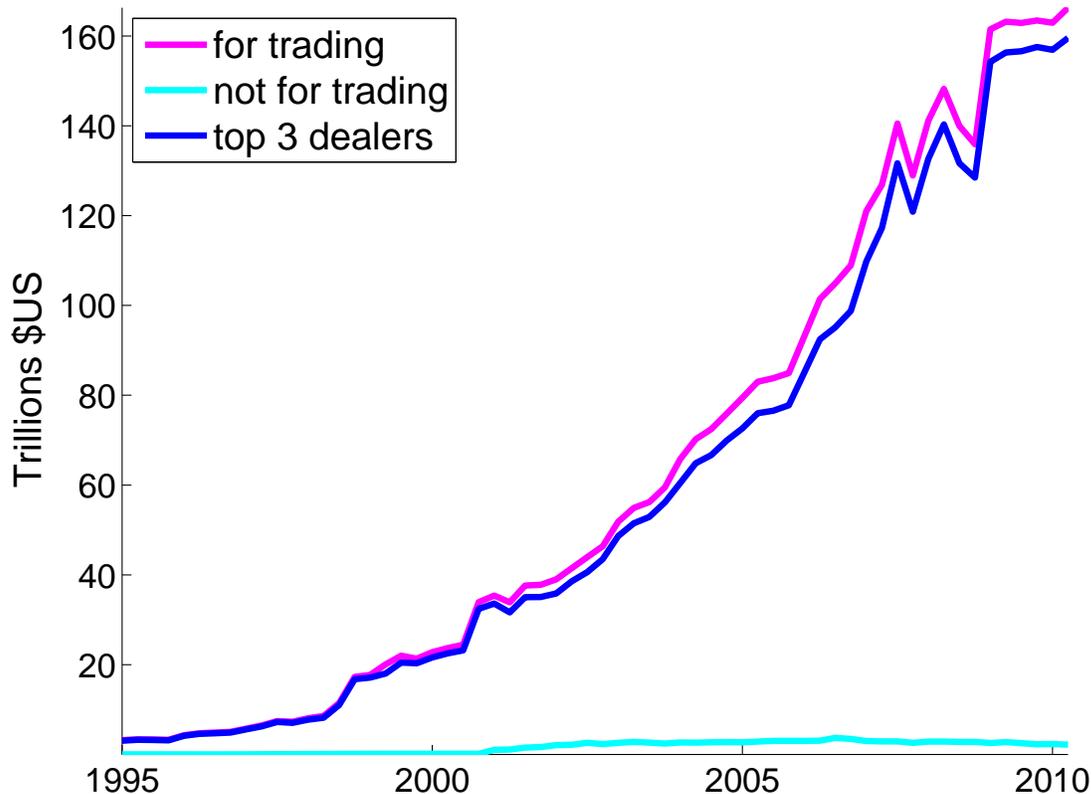


Figure 3: Total notional amounts in interest-rate derivatives of US banks. The notional amounts are for trading, not for trading, and the top three dealer banks.

Consider swap positions of the same maturity m started at date t . We assume that the rate on swaps of direction d takes the form $s - dz$, where s satisfies $F_t(s, m) = 0$ and z is one half the bid-ask spread. Swaps started at date t thus contribute to the fair value (13) only if there is a positive bid-ask spread. In contrast, swap positions started at earlier dates may also contribute to the fair value because $F_t(s, m) \neq 0$. We refer to the latter positions as "active". Let $\hat{N}_t^{d,m}$ denote the sum of all maturity m notional amounts of active positions with direction d and let N_t^m denote the sum of all (active or new) notional amounts for maturity m .

Without loss of generality, we can assume that the swap rate on all active notional amounts

makes the market by buying and selling bonds. He makes money because he buys at a lower bid price and sells at a higher ask price. The inventory of bonds currently held is recorded on the dealer's books as trading assets (or trading liabilities if the dealer allows a short sale). Once the dealer sells a bond, it is no longer on the dealer's balance sheet. The bid-ask spread enters as income once it is earned.

with the same maturity and direction is the same. Indeed, the valuation formula (11) implies that two positions with different swap rates and notionals, but the same direction and maturity are equivalent to a single position with a notionals-weighted swap rate. By an analogous argument, we can assume that the bid-ask spread on all swap positions of the same maturity is the same.

The net fair value (13) of the bank's position can then be rewritten as

$$FV_t = \underbrace{\sum_{d,m} \hat{N}_t^{d,m} dF_t(s_t^{d,m}, m)}_{FV_t^{own}} + \underbrace{\sum_m N_t^m z_t^m C_t^{(m)}}_{FV_t^{rent}}, \quad (14)$$

For every maturity, the fair value naturally decomposes into two parts. The first sum FV_t^{own} is the net fair value due to the bank trading on its own account, valued at the "midmarket rate" s . Its sign depends on the size and direction of the bank's trade as well as the relationship between swap rate and yield curve captured by F_t . The second term FV_t^{rent} consists of the present value of bid-ask spreads, which scales directly with total notionals.

Consider next the gross value of all swap positions: the ratio of gross to net value is informative about the bank's intermediation activity. We assume that the sign of a position with nonzero value is driven by the midmarket rate. Formally, positions are such that $F_t(s_t^m - z_t^m, m) > 0$ if and only if $-F_t(s_t^m + z_t^m, m) < 0$ and that $F_t(s_t^m - z_t^m, m) < 0$ if and only if $-F_t(s_t^m + z_t^m, m) > 0$. This condition is satisfied if bid-ask spreads are small relative to movements in interest rates.

Summing up over both sides of the balance sheet, we obtain a measure of gross fair value

$$\begin{aligned} GV_t &= \sum_{d,m,s} N_t^{d,m,s} |dF_t(s, m)| \\ &= \sum_{d,m} \hat{N}_t^{d,m} |F_t(s^{d,m}, m)| + \sum_m |N_t^{1,m} - N^{-1,m}| z_t^m C_t^{(m)} \end{aligned} \quad (15)$$

Again we have a two part decomposition. The first sum collects the gross positions that would arise if all trades had taken place at midmarket rates. The second term is a correction due to the presence of bid-ask spreads. Gross and net fair value coincide if the bank trades exclusively in one direction for each maturity. More generally, gross fair value is much larger than net fair value if there is a lot of intermediation.

Data

FAS 133 requires that all derivatives are carried on the balance sheet at fair value. Banks thus compute for every derivative position whether the fair value is positive or

negative. Positions with positive (negative) fair value are included on the asset (liability) side of the balance sheet. In the call reports, schedule HC-L provides both fair values and notional values for derivatives by type of exposure. We therefore have measures of both net and gross fair value FV_t and GV_t . The call reports also distinguish between derivatives “held for trading purposes” or “not held for trading”. The difference lies in how changes in fair value affect income, as in the case of nonderivative assets. However, the meaning of “held for trading” is broader for derivatives than for loans and securities and does not only cover short term holdings.³

Independently of whether a derivative is designated as “for trading”, FAS 133 provides rules for so-called hedge accounting. The idea is to allow businesses to shelter earnings from changes in the fair value of a derivative that is used to hedge an existing position (a “fair value hedge”) or an anticipated future cash flow (a “cash flow hedge”). In both cases, there are stringent requirements for demonstrating the correlation between the hedging instrument and the risk to be hedged. If the derivative qualifies as a “fair value hedge”, then the fair value on the hedged position may be adjusted to offset the change in fair value of the derivative. This is useful if the hedged position is not itself marked to market, for example if it is fixed rate debt and the derivative is a pay-floating swap. If the derivative qualifies as a “cash flow hedge”, then a change in its fair value can initially be recorded in OCI, with a later adjustment to earnings when the hedged cash flow materializes.

Current accounting rules imply that call reports cannot be used to easily distinguish hedging, speculation and intermediation. In particular, there is no clean mapping between “held for trading” and short term holdings due to intermediation or short term speculation, and there is no clean mapping between “not held for trading” and hedging. It is therefore not possible to map these categories into motives for trading such as “speculation” or “hedging” of other positions. On the one hand, “held for trading” derivatives could contain long term speculative holdings, but also hedges, in principle even qualifying accounting hedges. On the other hand, derivatives “not held for trading” could contain speculative holdings, as long as they are not short term.

Accounting rules do suggest that the majority of positions due to market making activity are held “for trading”. For this category, we therefore assume that the relevant swap rates incorporate spreads. We then separately estimate the rents from market making FV_t^{rent} and the value from trading for own account FV_t^{own} . In contrast, we

³The broad scope of the term “held for trading” is clarified in the Federal Reserve Board’s Guide to the BHC performance report: “Besides derivative instruments used in dealing and other trading activities, this line item [namely, derivatives held for trading purposes] covers activities in which the BHC acquires or takes derivatives positions for sale in the near term or with the intent to resell (or repurchase) in order to profit from short-term price movements, accommodate customers’ needs, or hedge trading activities”. In contrast, derivatives “not held for trading” comprise all other positions.

assume that all derivatives held “not for trading” are due to trading on own account.

We obtain information on bid-ask spreads in the swap market by maturity from Bloomberg. Bid-ask spreads for contracts between dealers tend to be lower than those in contracts between dealers and clients. To assess the share of contracts a bank does with other dealers, we use data on net credit exposure in derivatives to various counterparties, available in the call reports since 2009. Finally, we have data on the maturity of notionals in three broad maturity buckets: less than one year, 1-5 years and more than 5 years.

Market maker rents from derivatives held for trading

We use data on bid-ask spreads, maturities, notionals and the yield curve to obtain an estimate of

$$FV_t^{rent} = \sum_m N_t^m z_t^m C_t^{(m)}.$$

For every maturity m , the spread factor z_t^m in (14) reflects (one half) the average bid-ask spread for all the swaps currently on the bank’s books. To the extent that bid-ask spreads change over time, its magnitude depends on how many current swaps were initiated in the past when bid-ask spreads were, say, higher. To capture this effect, we construct a vintage distribution of swap notionals analogously to the vintage distributions of loans and long term debt discussed above. We use data on bid-ask spreads on new swaps to find, for each maturity and period, the total bid-ask spread payment earned by swaps of that maturity in that period.

More specifically, suppose we know the distribution (N_t^m) of total notional values by maturity as well as the distribution of bid-ask spreads on new swaps by maturity, that is, the sequence ($2z_t^m$). We assume that in the first sample period, all swaps are new, and we record the stream of bid-ask spread payments (z_1^m) on those swaps. We then proceed recursively: for each period and maturity, new swaps are defined as the difference between total notionals for that period and “old” notionals that remain from the previous period, taking into account that the old swaps have aged by one period. We then use the current bid-ask spreads to add to the stream of payments for all future periods.

Trading on own account

To analyze the bank’s trading on its account, we assume that, at a given point in time, the maturity distribution of notionals as well the locked in midmarket rate are the same for pay-fixed and pay-floating swaps. This assumption is motivated by our view of the fixed income market as to a large degree by trades on a single factor. Formally, we have $s_t^{m,-1} = s^{m,1} = s_t^m$, say. Moreover, there is a set of weights w_t^m as well as totals of active pay-fixed and pay-floating notionals \hat{N}_t^1 and \hat{N}_t^{-1} such that, for all m , $\hat{N}_t^{m,1} = w_t^m \hat{N}_t^1$ and $\hat{N}_t^{-1,m} = w_t^m \hat{N}_t^{-1}$. It follows that the bank’s active net position

goes in the same direction $d_t = \text{sign}(\hat{N}_t^1 - \hat{N}_t^{-1})$ for each maturity. We can also define $\Omega_t = |\hat{N}_t^1 - \hat{N}_t^{-1}| > 0$ as the bank's total notionals dedicated to trading on its own account.

The net and gross fair values can now be rewritten as

$$\begin{aligned} FV_t &= \Omega_t d_t \sum_{m,\sigma} w_t^m F_t(s_t^m, m) + FV_t^{rent}, \\ GV_t &= \hat{N}_t \left| \sum_{d,m} w_t^m F_t(s_t^m, m) \right| + \frac{\Omega_t}{\hat{N}_t} FV_t^{rent}. \end{aligned}$$

For large dealer banks, the observed ratios FV_t/GV_t is a small percentage (below 2% on average for the 4 largest dealers in our sample). In addition, the market rent is at most of similar order of magnitude as the net fair value. It follows that the share of notionals Ω_t/\hat{N}_t used for trading on the bank's own account must also be small and the second term in GV_t is an order of magnitude smaller than the first.

Combining the two equations and ignoring the second term in GV_t , we can relate the multiple μ_t of gross fair value to notionals to the bank's per dollar cumulative gains from trading on its own account:

$$\mu_t := \text{sign}(FV_t - FV_t^{rent}) \frac{GV_t}{\hat{N}_t} \approx d_t \sum_m w_t^m F_t(s_t^m, m) \quad (16)$$

The equation holds exactly if there is no intermediation. Indeed, in this case the share of notionals dedicated to own account trading can be read off directly from the ratio of net to gross fair value, that is, $\Omega_t/\hat{N}_t = FV_t/GV_t$. Since the net notional position is small relative to total notionals, the equation still works even when there is intermediation. However, in order to correctly determine the sign of the own account position, we must first remove the positive rents from market making.

The multiple μ_t defined in (16) can be interpreted as fair value per dollar of *net* notional used by the bank to trade on its own account. We use this multiple as the observable series from which we infer the bank's trading strategy. In principle, dynamics of μ_t depends on the entire distribution (w_t^m), a potentially complicated state variable. We restrict attention to first order effects due to changes in the average maturity $\bar{m}_t = \sum_m w^m m$. Our observation equation is

$$\mu_t = d_t F(\bar{m}_t, s_t) + \varepsilon_t; \quad s_t = \sum_m w^m s_t^m \quad (17)$$

where the average swap rate s_t and the direction of trading d_t are unknown and where the

measurement error is iid normal with mean zero and variance σ_ε^2 .⁴ Up to measurement error, this equation has the same form as (12).

Estimation

To calculate the multiple μ_t for quarter t , we use FV and GV_t from the end of quarter call reports. In the case of derivatives held for trading, we also use our estimate FV_t^{rent} . We identify active notionals \hat{N}_t with all notionals outstanding at the end of the previous quarter $t-1$. It would require a knife edge realization of the swap rate for a swap started at any previous date not to be active at date t . The only inactive swaps are therefore those newly issued at date t . The bond prices that enter the valuation factor F_t come from the swap curve.

We take a Bayesian approach to estimation starting from a prior over the joint sequence of directions and average swap rates (d_t, s_t) . We set the variance of the measurement error σ_ε^2 to 10% of the variance of the multiple. The prior over the direction σ_t is a symmetric two state Markov chain with a 10% probability of switching the sign of d_t . The simplest exercise would treat each date as entirely separate, and simply specify a prior. However, this approach would ignore the information that swap books at different dates must be dynamically connected.

Our choice of prior for the swap rate is instead based on the following considerations about the dynamics of swap books. First, swap rates locked-in as of time t should be drawn from the empirical distribution of rates that occurred in the last couple of years. It makes no sense, for example, to expect that near-zero interest rates that prevailed in 2011 could have been locked in in 2007. Second, the average swap rate should be relatively closer to the current swap rate if notionals have been growing quickly recently. This is because new notionals always reflect current rates. Finally, a rate that was quoted at some date $\tau < t$ – and therefore could be locked as of date t – should carry little weight if there are intermediate dates $s \in (\tau, t)$ such that that rate looks very implausible.

To describe the prior for the swap rates, we sketch a stylized swap trading strategy. We view the bank as entering the period with active maturity \bar{m}_t swaps. We know that at the end of the period the bank will have a portfolio of maturity $\bar{m}_{t+1} + 1$ swaps. As

⁴If (i) the bond prices $P_t^{(m)}$ and $C_t^{(m)}$ were linear when viewed as functions of maturity m and (ii)

the swap rate s_t^m is independent of the bond prices $P_t^{(m)}$ and $C_t^{(m)}$ when all are viewed as random variables with distribution given by the weights w_t^m , then the approximation of the average valuation factor is exact, i.e.

$$\sum_m w_t^m F_t(s_t^m, m) = F(\bar{m}_t, s_t).$$

a first step we replicate the initial swaps in terms of maturity $\bar{m}_{t+1} + 1$ swaps and cash, using a replication argument as in (4). For the fixed leg of a swap of maturity \bar{m}_t , there exist coefficients a_t^s and b_t^s such that the fixed leg is replicated by b_t^s units of the fixed leg of a swap of maturity $\bar{m}_{t+1} + 1$ together with a_t^s dollars in cash. We thus derive a position of $d_t b_t^s$ maturity $\bar{m}_{t+1} + 1$ swaps. We assume that the cash position goes to income; it is typically small since maturities change in small steps.

Consider now the trades the bank can make, that is, how it moves from the beginning of period position of $b_t^s N_{t-1}$ swaps to the end of period position of N_t swaps. There are two possibilities. On the one hand, the bank can either increase or decrease its exposure to swaps. If the bank increases its exposure, it combines N_{t-1} swaps with the old locked in rate s_t with $N_t - b_t^s N_{t-1}$ new swaps that are issued at the current market rate s_t^* say. The payment stream of the combined swaps is equivalent to holding $b_t^s N_{t-1} + N_t$ swaps at the adjusted swap rate

$$s_{t+1} = \frac{b_t^s N_{t-1}}{N_t} s_t + \frac{N_t - b_t^s N_{t-1}}{N_t} s_t^*.$$

On the other hand, the bank can decrease its exposure to long swaps by canceling some of the old swaps. In practice, cancellation is often accomplished by initiating an offsetting swap in the opposite direction. If the current swap rate for the relevant maturity is different from the original locked-in rate, the cancellation will also involve a sure gain or loss. We assume that this gain or loss is directly booked to income and does not appear as part of the fair value after cancellation. The remaining long swaps then retain the same locked-in swap rate, that is $s_t = s_{t-1}$.

Our stylized bank makes moves between positions in the simplest possible way. In particular, if the direction d of its exposure remains the same, then it makes only one of the above trades – it either increases or decreases its exposure. the only exception to this rule is the case where the bank changes direction: in this case we assume that it cancels all existing long swaps and issues all new swaps in the opposite direction. Given these assumption, a sequence of directions d_t together with an initial conditions for s_1 implies a unique history of swap rates.

To complete the description of the prior, we need a distribution for the initial swap rate s_1 . To allow for a flexible functional form, we discretize the swap rate on a fine grid. We then set the initial prior equal to the empirical distribution from the first 5 years of data in our sample. All updating is therefore done with discrete distributions and posteriors are therefore available in closed form.

Estimation results

To illustrate how the estimation works, Figure 4 shows the trading positions for JPMorganChase. The top left panel shows the evolution of notional values. These

numbers are large because of the lack of netting of interdealer positions in the call reports: the notionals of each bank by itself amounts to several times US GDP. The blue line in the top right panel shows the multiple μ_t , that is the net fair value as a share of net notionals used for trading on the bank's own account.

The bottom panels display the estimation results. The bottom left panel is the posterior probability of a pay-fixed position ($d_t = 1$). In the bottom right panel, the blue line shows the swap rate at the average maturity \bar{m}_t . The red and green solid line represent the conditional posterior median swap rates given that the direction is pay fixed or pay floating, respectively. The dotted red and green lines show the posterior conditional interdecile range for the swap rate. The red line labeled "estimate" in the top right picture represents an estimate of the multiple obtained by evaluating $d_t F(\bar{m}_t, s_t)$ from (17) at the regime with higher probability and the median swap rate conditional on that regime.

The estimation infers pay-fixed positions during times of temporarily rising interest rates. For example, during 2005-07, the fair value is positive and it is not plausible to find recent high enough interest rates to justify it. The code thus infers that a low rate had been locked in as part of a pay-fixed strategy. In contrast, the spectacular gains enjoyed by JPMorgan Chase in both the 2001 recession and the Great recession when interest rates fell strongly suggest a pay-floating strategy.

4 Replication results

Figure 5 illustrates the results of the replication exercise for JP Morgan Chase. The solid lines represent the replicating portfolio for the bank's "traditional" net fixed income position, defined as loans plus securities less deposits and other borrowings. The solid green line shows the face values of 5 year zero coupon bonds, and the solid red line shows the face value of short bonds. The dotted line shows the replicating portfolio for the total net position in interest rate derivatives. Finally, the dashed line presents the replicating portfolio for bonds in the bank's trading portfolio. This position is broken out separately in part because the replication results are more uncertain for this item due to the lack of information on maturities.

Figure 6 shows the replicating portfolio for four top dealer banks. The top left panel replicates Figure 5, and the other panels show Bank of America, Wells Fargo and Citibank.

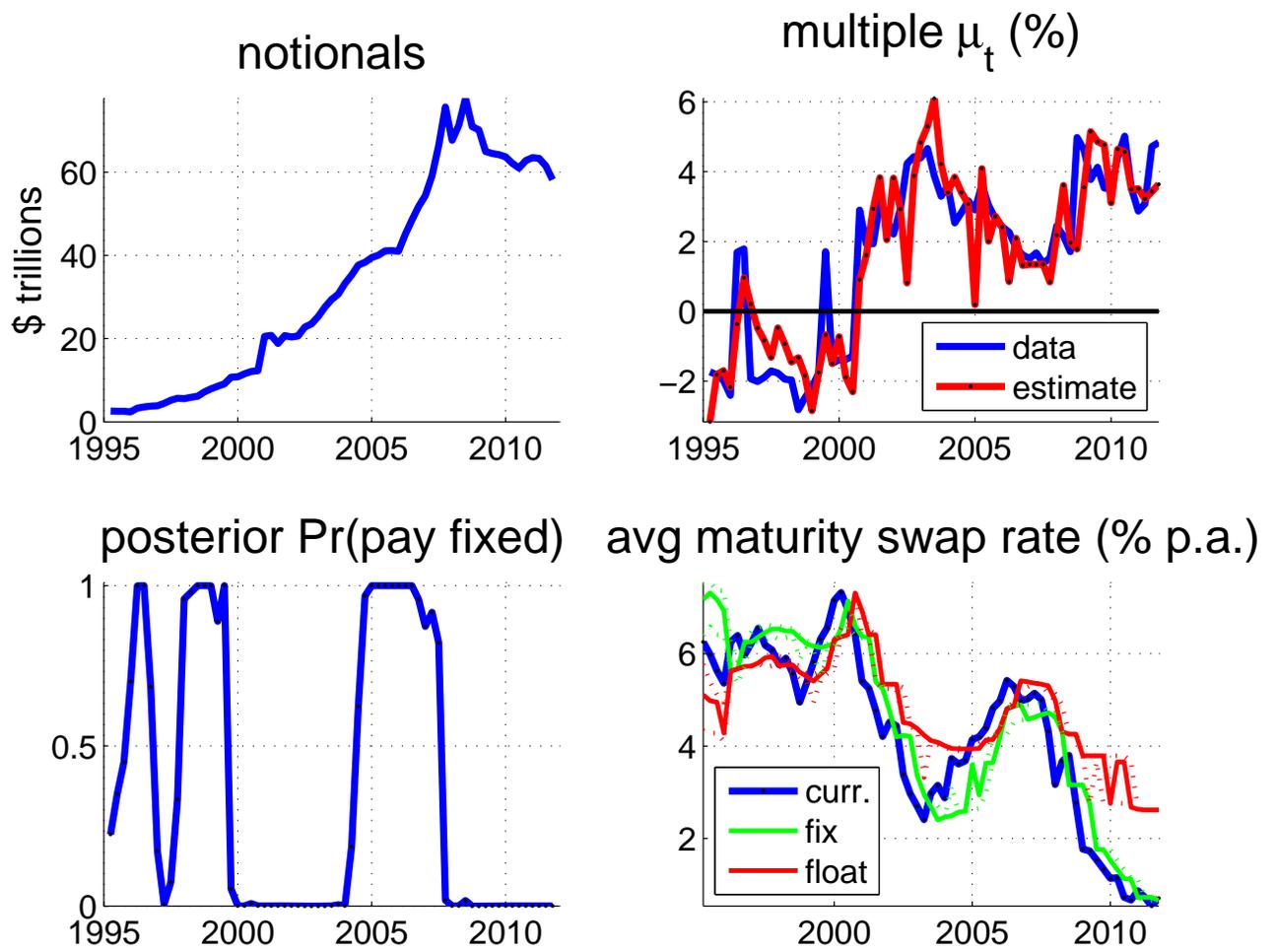


Figure 4: Trading positions for JP MorganChase.

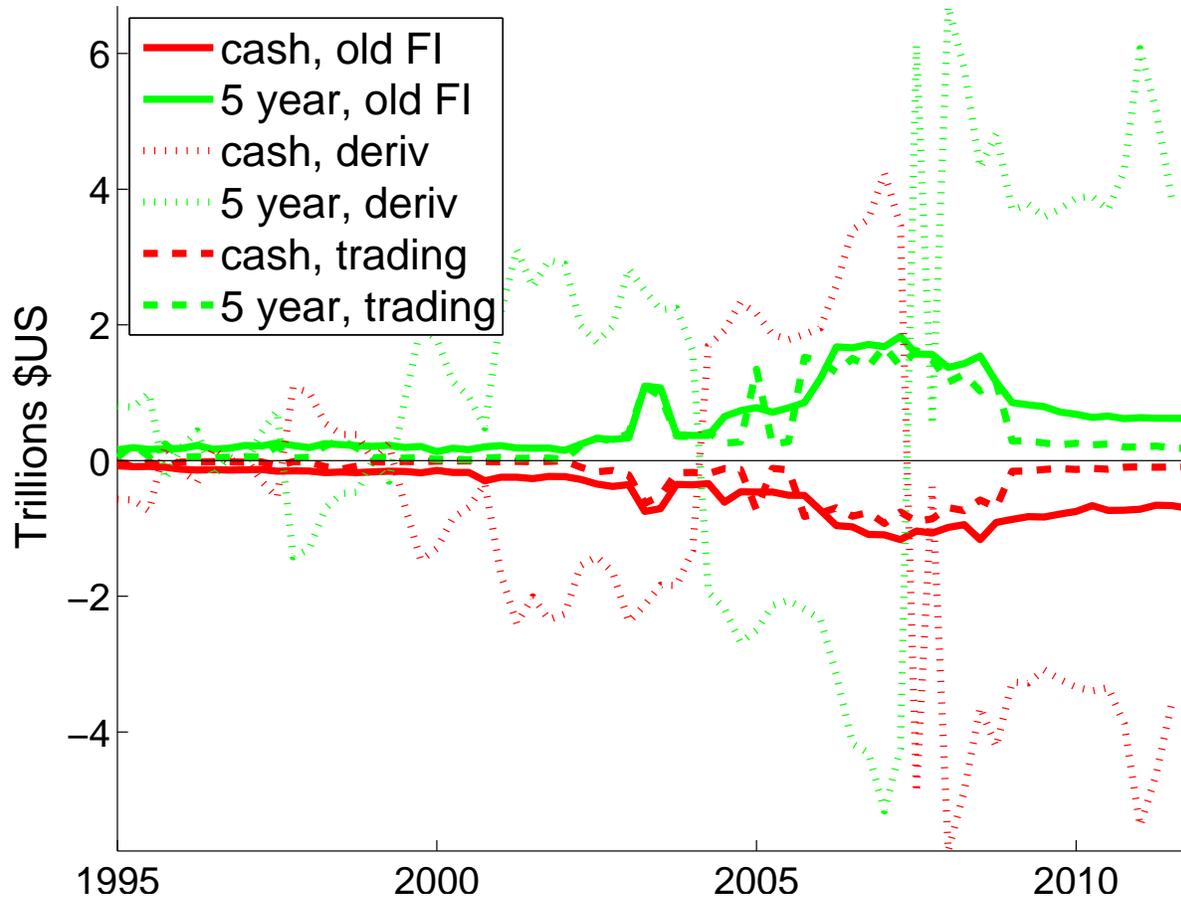


Figure 5: Replication portfolios for JP Morgan Chase. The portfolios are holdings of cash (in red) and a 5-year riskless zero coupon bond (in green). Solid lines are replicating portfolios for the traditional fixed income position, while dotted lines are for derivatives and dashed lines are for bonds held for trading.

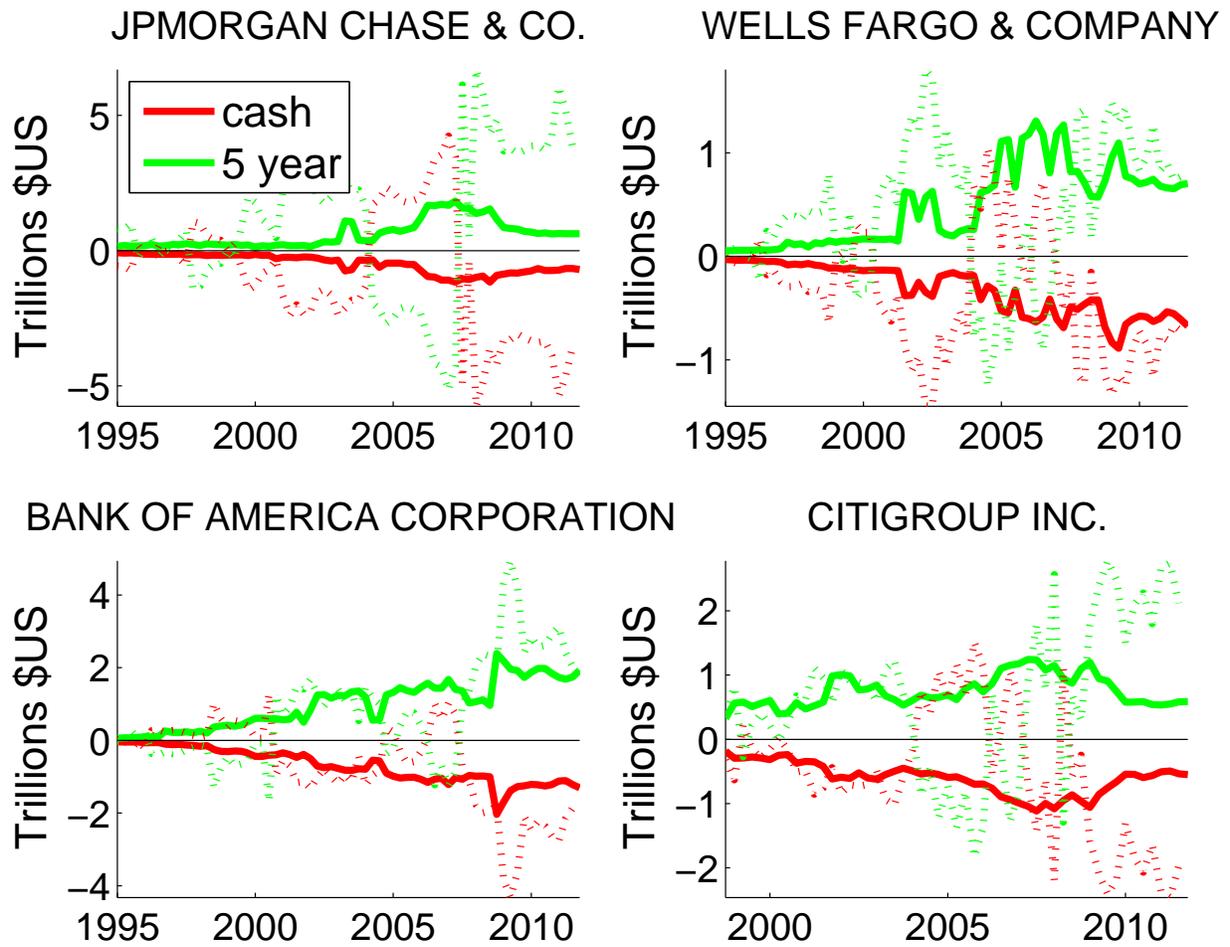


Figure 6: Replication portfolios of four top dealer banks. The portfolios are holdings of cash (in red) and a 5-year riskless zero coupon bond (in green). Solid lines are replicating portfolios for the traditional fixed income position, while dotted lines are for derivatives and dashed lines are for bonds held for trading.

References

- Acharya, Viral, Pedersen, Lasse, Thomas Phillipon, and Matt Richardson 2010. "Measuring Systemic Risk." Working Paper, NYU.
- Acharya, Viral, Matt Richardson, Stijn van Nieuwerburgh, and L. White 2011. "*Guaranteed to Fail: Freddie, Fannie, and the Debacle of U.S. Mortgage Finance*." Princeton University Press.
- Adrian, Tobias and Hyun Song Shin 2011. "Procyclical Leverage and Value at Risk." Working Paper, Princeton.
- Allen, Franklin 2001. "Do Financial Institutions Matter?" *Journal of Finance*, 56, pp. 1165-1175.
- Ang, Andrew and Monika Piazzesi 2003. "A No-Arbitrage Vector Regression of Term Structure Dynamics with Macroeconomic and Latent Variables." *Journal of Monetary Economics*, 50, pp. 745-787.
- Brunnermeier, Markus, Gary Gorton, and Arvind Krishnamurthy 2012. "Risk Topography." *NBER Macro Annual 2011*.
- Chernenko, Sergey and Faulkender, Michael 2011. "The Two Sides of Derivatives Usage: Hedging and Speculating with Interest Rate Swaps." *Journal of Financial and Quantitative Analysis* (46), pp. 1727-1754.
- Duffie, Darrell 2012. "Systemic Risk Exposures: A 10-by-10-by-10 Approach." *Systemic Risk and Macro Modeling*, Markus Brunnermeier and Arvind Krishnamurthy, editors.
- Duffie, Darrell and Kenneth Singleton 1999. "Modeling Term Structures of Defaultable Bonds." *Review of Financial Studies* 12, pp. 687-720.
- English, William B., Skander J. Van den Heuvel, and Egon Zakrajsek 2012. "Interest Rate Risk and Bank Equity Valuations." Working Paper, Federal Reserve Board.
- Gorton, Gary and Richard Rosen 1995. "Banks and Derivatives". *NBER Macroannual 1995*, pp. 299-349.
- Flannery, Mark J. and Christopher M. James 1984a. "The Effect of Interest Rate Changes on the Common Stock Returns of Financial Institutions." *Journal of Finance* 39, 4, pp. 1141-1153.

- Flannery, Mark J. and Christopher M. James 1984b. "Market Evidence on the Effective Maturity of Bank Assets and Liabilities." *Journal of Money, Credit and Banking*, 16, pp. 435-445.
- Flannery, Mark J., Allaudeen Hamed and Richard H. Harnett 1997. "Asset Pricing, time-varying risk premia and interest rate risk." *Journal of Banking and Finance* 21, pp. 315-335.
- He, Zhiguo, In Gu Khang, and Arvind Krishnamurthy 2010. "Balance-Sheet Adjustments during the 2008 Crisis." *IMF Economic Review* 1, 118-156.
- Hentschel, Ludger and S.P. Kothari 2001. "Are Corporations Reducing or Taking Risks with Derivatives?" *Journal of Financial and Quantitative Analysis* 36, pp. 93-118.
- Hirtle, Beverly J. 1997. "Derivatives, Portfolio Composition, and Bank Holding Company Interest Rate Risk Exposure." *Journal of Financial Services Research* 12, pp. 243-266.
- Jarrow, Robert, Haitao Li, Sheen Liu, and Chunchi Wu 2010. "Reduced-form valuation of callable corporate bonds: Theory and Evidence." *Journal of Financial Economics* 95, pp. 227-248.
- Jermann, Urban and Vivian Y. Yue 2012. "Interest Rate Swaps and Corporate Default." Working Paper, Wharton.
- Kelly, Bryan, Hanno Lustig and Stijn van Nieuwerburgh 2011. "Too-Systemic-To-Fail: What Option Markets Imply About Sector-wide Government Guarantees." *NBER Working Paper* 17149.
- Landier, Augustin, David Sraer, and David Thesmar 2013. "Banks Exposure to Interest Rate Risk and the Transmission of Monetary Policy." *NBER Working Paper* 18857.
- Lustig, Hanno, Chris Sleet and Sevin Yeltekin 2008. "Fiscal hedging with nominal assets." *Journal of Monetary Economics* 55, pp. 710-727.
- Minton, Bernadette, Rene Stulz and Rohan Williamson 2009. "How Much Do Banks use Credit Derivatives to Hedge Loans?" *Journal of Financial Services Research* 2009, 35, pp. 1-31.
- Piazzesi, Monika and Martin Schneider 2010. "Interest Rate Risk in Credit Markets". *American Economic Review* 100 (2), pp. 579-584.
- Shaby, Benjamin and Martin T. Wells 2010. "Exploring an Adaptive Metropolis Algorithm", Technical Report.

Venkatachalam, Mohan 1996. "Value-relevance of banks' derivatives disclosures." *Journal of Accounting and Economics* 22, pp. 327-355.

Appendix

A Balance Sheet Details

This appendix provides more details about the balance sheet in Table 1. Table A.1 shows a bank balance sheet which is based on the Consolidated Financial Statements for Bank Holding Companies (FR-Y-9C) from December 31, 2011. These financial statements are required by law and are filed by Bank holding companies to the Board of Governors of the Federal Reserve System. The assets of banks include cash which can be interest bearing (IB) or noninterest bearing (NIB) in domestic offices (DO) and foreign offices (FO), securities, Flow of Funds sold (FFS), loans and leases, trading assets, premises and fixed assets, other investment, intangible assets and other assets. The liabilities include deposits, Federal Funds purchased (FFB), trading liabilities, other borrowed money, subordinated notes, and other liabilities. The difference between assets and liabilities is capital. The item numbers "BH" followed by more letters and numbers refer to the entry into the financial statements by each bank holding company.

TABLE A.1: BANK BALANCE SHEETS IN CALL REPORTS

Assets		Liabilities	
1. Cash		13. Deposits	
NIB balances, currency and coin	BHCK0081	a. In DO: (1) NIB	BHDM6631
IB balances in US offices	BHCK0395	(2) IB	BHDM6636
IB balances in FO	BHCK0397	b. In FO, Edges, IBFs: (1) NIB	BHFN6631
		(2) IB	BHFN6636
2. Securities		14. FFP	
a. Held-to-maturity securities	BHCK1754	a. FFP in DO	BHDMB993
b. Available-for-sale securities	BHCK1773	b. Securities Sold to Repurchase	BHCKB995
3. FFS		15. Trading Liabilities	
a. FFS in DO	BHDMB987		BHCK3548
b. Securities Purchased	BHCKB989		
4. Loans & Leases		16. Other Borrowed Money	
a. Loans & leases held for sale	BHCK5369	Includes mortgage, indebtedness,	BHCK3548
d. Loans & leases, net of unearned income and allowance for loan & lease losses	BHCKB529	and obligations under capitalized leases (items 17., 18. are not applicable)	
5. Trading Assets		19. Subordinated Notes	
	BHCK3545	Subordinated notes and debentures	BHCK4062
		Subordinated notes payable to trusts	BHCKC699
6. Premises and fixed Assets		20. Other Liabilities	
	BHCK2145		BHCK2750
Other Investment		21. Total Liabilities	
7. Other real estate owned	BHCK2150		BHCK2948
8. Investments in uncons. subsidiaries	BHCK2130	Equity	
9. Direct & indirect investments in real estate ventures	BHCK3656	Total Equity Capital	BHCKG105
10. Intangible assets			
a. Goodwill	BHCK3163		
b. Other intangible assets	BHCK0426		
11. Other Assets			
	BHCK2160		
12. Total Assets	BHCK2170		

Abbreviations: domestic office (DO), foreign office (FO), interest bearing (IB), noninterest bearing (NIB), Federal Funds sold (FFS), and Federal Funds purchased (FFB).

Schedule HC-B—Securities

Dollar Amounts in Thousands	Held-to-Maturity									Available-for-Sale								
	(Column A) Amortized Cost			(Column B) Fair Value			(Column C) Amortized Cost			(Column D) Fair Value								
	BHCK	BI	MI	Thou	BHCK	BI	MI	Thou	BHCK	BI	MI	Thou	BHCK	BI	MI	Thou		
1. U.S. Treasury securities.....	0211				0213				1286				1287				1.	
2. U.S. government agency obligations (exclude mortgage-backed securities):																		
a. Issued by U.S. government agencies ¹	1289				1290				1291				1293				2.a.	
b. Issued by U.S. government-sponsored agencies ²	1294				1295				1297				1298				2.b.	
3. Securities issued by states and political subdivisions in the U.S.....	8496				8497				8498				8499				3.	
4. Mortgage-backed securities (MBS):																		
a. Residential pass-through securities:																		
(1) Guaranteed by GNMA.....	G300				G301				G302				G303				4.a.(1)	
(2) Issued by FNMA and FHLMC.....	G304				G305				G306				G307				4.a.(2)	
(3) Other pass-through securities.....	G308				G309				G310				G311				4.a.(3)	
b. Other residential mortgage-backed securities (include CMOs, REMICs, and stripped MBS):																		
(1) Issued or guaranteed by FNMA, FHLMC, or GNMA.....	G312				G313				G314				G315				4.b.(1)	
(2) Collateralized by MBS issued or guaranteed by FNMA, FHLMC, or GNMA.....	G316				G317				G318				G319				4.b.(2)	
(3) All other residential mortgage-backed securities.....	G320				G321				G322				G323				4.b.(3)	
c. Commercial MBS:																		
(1) Commercial pass-through securities.....	G324				G325				G326				G327				4.c.(1)	
(2) Other commercial MBS.....	G329				G329				G330				G331				4.c.(2)	
5. Asset-backed securities and structured financial products:																		
a. Asset-backed Securities (ABS).....	C025				C989				C989				C027				5.a.	
b. Structured financial products:																		
(1) Cash.....	G336				G337				G338				G339				5.b.(1)	
(2) Synthetic.....	G340				G341				G342				G343				5.b.(2)	
(3) Hybrid.....	G344				G345				G346				G347				5.b.(3)	
6. Other debt securities:																		
a. Other domestic debt securities.....	1737				1738				1739				1741				6.a.	
b. Foreign debt securities.....	1742				1743				1744				1746				6.b.	
7. Investments in mutual funds and other equity securities with readily determinable fair values.....									A510				A511				7.	
8. Total (sum of 1 through 7) (total of column A must equal Schedule HC, item 2.a) (total of column D must equal	bhct												bhct					

Figure 7: HB-C Schedule for Securities.

Figure 7 shows the Schedule HC-B for Securities.