The Pricing Effects of Securities Class Action Lawsuits and Litigation Insurance

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The price reactions to corrective disclosures often serve as a benchmark for settlements in securities class action lawsuits. When the firm bears litigation costs, this benchmark creates a feedback effect that exacerbates the price reaction to news that contradicts managers' earlier reports. Litigation insurance provides value in this setting by reducing the need for investors to price the effects of anticipated litigation. Insurance also affects how changes in the litigation environment impact the firm, with some changes having opposite effects on the frequency of lawsuits against uninsured and insured firms. The pricing behavior of rational investors eliminates the valuation impact of the portion of settlements paid to the investors, similar to dividends. The valuation impact of litigation arises from transaction costs, such as attorney fees, which the firm can mitigate by constraining misreporting and by purchasing insurance. (JEL G14, G30, K22, M41).

1. Introduction

Securities litigation plays an important role in the regulation of corporate disclosure in the United States and in other common law countries. In a class action lawsuit, a collective group of investors can seek compensation for losses caused by a disclosure that impacted the price they paid for the shares. Under the fraud-on-the-market theory, investors need not demonstrate that they personally relied on any disclosure—they only need to provide evidence that the disclosure distorted market prices (Cornell and Morgan 1990). Regardless of whether such suits name managers or the corporation, corporations fund nearly all securities class action
settlements either directly or indirectly via indemnities under insurance policies purchased with corporate funds (Klausner and Hegland 2010).\(^1\) As expressed in the *New York Times* DealBook column (Sorkin 2010), “While the public lusters for the hanging of a corporate executive, the shareholders . . . take the big hit in the wallet.” In other words, investors are both plaintiffs and defendants in securities class actions, creating what the legal literature refers to as the “circularity problem” (Coffee 2006). The beneficiaries of misreporting—investors who sold at inflated prices—are absent from the process.

I develop a model in which investors can sue for losses caused by misleading disclosures, and then examine the effects of the litigation environment and litigation insurance on the value of the firm and the frequency of litigation. In my model with risk-neutral investors, litigation reduces firm value via the portion of the firm’s litigation costs that does not flow to investors (hereafter “transaction costs”). Such costs might include attorney fees, opportunity costs of managers’ time, and bad publicity. In the absence of transaction costs, litigation resembles a randomly paid dividend and has no effect on the firm value, given risk-neutral investors—it merely transfers money from the investors’ firm holdings to their cash holdings. Since rational investors incorporate anticipated litigation costs into security prices, their ability to litigate does not directly add to firm value.\(^2\) With transaction costs, litigation reduces firm value because it is not a dollar-for-dollar transfer from the firm to the investors. The firm’s ex ante equilibrium price equals the value of the firm’s operations less the expected transaction costs of litigation.

This study formalizes how investors’ anticipation of litigation amplifies their reactions to news indicating that managers previously, and inappropriately, inflated the firm’s share prices. The amplification arises because investor reactions to such news incorporate both revisions to the firm’s fundamental value and, in the absence of insurance, the costs of potential lawsuits.\(^3\) In turn, settlements are highly correlated with estimates of losses based on the postcorrection drop in market price (Cox et al. 2005). As a result, investors’ anticipation of litigation costs creates a feedback loop that exacerbates the price declines used as an input for determining settlements. If the likelihood of successful litigation and investors’ expected recovery are sufficiently high, the feedback effects can lead to a collapse in the firm’s share price.

Litigation insurance alleviates the need to price litigation costs, to the extent they fall within the firm’s coverage level, and thus mitigates the

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\(^1\) While managers rarely personally pay damages in class actions, they often face forced turnover and damaged reputations (Romano 1991; Fich and Shivdasani 2007; Karpoff et al. 2008a).

\(^2\) The model abstracts from possible positive effects of lawsuits, such as improved corporate governance.

\(^3\) Gande and Lewis (2009) provide empirical evidence that investors price anticipated litigation costs.
feedback effect. Even though I assume that insurers fairly price their policies, insurance has value when transaction costs increase with overall litigation costs. By mitigating the feedback effect and its exacerbation of transaction costs, the act of insuring against a loss reduces the expected loss being insured.

I also show that insurance increases the likelihood of litigation. Although this may seem at odds with the prediction that insurance increases firm value, the higher likelihood of litigation actually results from the higher firm value. Specifically, insurance reduces the feedback effect of news that can trigger litigation, which reduces transaction costs and increases firm value. This increase in firm value leads to a higher ex ante price, but the higher ex ante price increases the likelihood of a subsequent price drop sufficient to trigger litigation. Since insurance reduces the feedback effect, the lawsuits that occur have smaller costs than if insurance coverage had not been purchased. These predictions are consistent with an environment where most firms carry insurance and face frequent, but small lawsuits.4

Finally, I show that constraints on securities litigation increase firm value, but may also increase the likelihood of suits against the firms that carry low insurance coverage.5 This prediction relates to the increase in share prices around the passage of the Private Securities Litigation Reform Act (PSLRA) of 1995, despite the higher subsequent frequency of litigation (Johnson et al. 2000; Perino 2003). My model suggests that the increase in post-PSLRA litigation may be concentrated among firms with low insurance coverage.

Prior analytical work on disclosure-related litigation has largely focused on settings in which the corporation’s assets are not at stake in litigation.6 Hughes and Thakor (1992) study how litigation against underwriters can lead to initial public offering (IPO) underpricing, and also explain how company-funded damages can lead to the apparent underperformance of the IPOs. Analytical work focused on settings in which the firm pays litigation costs includes Evans and Sridhar (2002), who examine the impact of litigation on the separation of good- and bad-type firms in a model, where firms weigh their reports’ effects on financing costs against their reports’ effects on attracting competitors. They allow for litigation in cases where misreporting leads to mispricing arising from pooling or

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4. In a separate analysis, available upon request, I show that high fixed, uninsurable litigation costs reduce and can possibly eliminate the value of insurance. This occurs because insurance increases the likelihood of litigation and thus increases the likelihood of incurring high uninsurable costs.

5. This takes the firm’s constraints on managerial behavior as given. A lower likelihood of facing litigation may diminish firms’ incentives to dissuade managerial misreporting. See Bernardo et al. (2000) for a discussion of how moral hazard can mitigate a firm’s benefit from constraints on litigation.

6. For example, in Kraakman et al. (1994), Hensler (1995), Trueman (1997), and Laux and Stocken (2012), managers or company founders fund litigation from their personal wealth.
mixed strategy equilibria, but they do not examine prices, per se. Spindler (2011) also predicts that litigation can cause the separation of good- and bad-type firms; however, his model allows for litigation even in the separating equilibrium where there is no mispricing. Dye (2011) examines how litigation affects voluntary disclosure.

This study covers several topics not addressed by prior studies. First, in a setting where litigation requires evidence of mispricing, I demonstrate an amplified price reaction to bad news when investors anticipate litigation directly against the firm. Second, I show that litigation insurance creates value by diminishing the amplified price reaction. This occurs even though I assume risk neutrality, and it may explain the pervasiveness of litigation insurance, despite the role such insurance plays in exacerbating moral hazard (e.g., Baker and Griffith 2007) with respect to manager’s reporting behavior. Third, I show how changes in the reporting environment affect firm value via the costs and the likelihood of litigation. In particular, I show that constraints on securities litigation can increase firm value while also increasing the likelihood of litigation.

A testable prediction of my model is that insurance coverage affects price reactions to a revelation that managers caused mispricing and to changes in the legal environment. Feedback effects arise when investors expect litigation that will outstrip the firm’s insurance coverage. Also, constraints on litigation and the magnitude of settlements can increase the likelihood of litigation for firms that have low insurance coverage. Based on the available data, I expect that the amplification of the shareholder reactions has the greatest impact on small firms with major corrective disclosures.7

For the remainder of the article, Section 2 provides background on class action lawsuits and litigation insurance. Section 3 illustrates the importance of transaction costs and how anticipated litigation amplifies the price reaction to bad news. Section 4 develops the main model and equilibrium. Section 5 examines constraints on misreporting, litigation insurance, and the litigation environment. Section 6 concludes. All proofs are included in Appendix A.

2. The Litigation Environment

Private litigation plays a significant role in the enforcement of US securities laws (Cox and Thomas 2009). La Porta et al. (2006) find that private

7. The amplification depends on the likelihood of successful litigation and the expected settlement amount relative to the price drop around the corrective disclosure. Plaintiffs successfully litigate against about half of the firms with restatements to “core accounts” (Palmrose and Scholz 2004). Settlements relative to price declines range from 82% for small dollar declines to 4% for large declines (Ryan and Simmons 2009). Combining the two figures and using the derivations from the model, the amplification ranges from $69\% = (50\% \times 82\%)/(1 - (50\% \times 82\%))$ for small dollar declines to $2\% = (50\% \times 4\%)/(1 - (50\% \times 4\%))$ for larger ones.
enforcement of disclosure laws benefits financial development, whereas public enforcement has little impact. Class actions allow shareholders to collectively sue for losses incurred due to buying at inflated prices. Total damages in a class action are estimated from prices and trading volume, and are paid on a \textit{pro rata} basis to shareholders who join the class (Alexander 1994). During 2000–02, the monetary settlements of securities class actions exceeded the combined total of all public monetary sanctions (Coffee 2006; Table 2).

The implications of this study rely on the relation between legal settlements and price reactions to news indicating that managers previously caused mispricing. Potential damages in a class action depend on the estimated damages per share multiplied by the estimated shares damaged (Thakor 2005). The per-share damages typically depend on the difference between the beginning- and the end-of-class period share prices, which reflects the correction to the allegedly misleading disclosure, with adjustments for market-wide changes and unrelated news.\footnote{Securities litigation typically uses the “value-line” approach to estimate damages. Denoting the beginning and end of class period prices by \( p_1 \) and \( p_2 \), respectively, the estimated damages roughly correspond to the price drop and equal \( \hat{p}p_2 \), where \( \hat{p} \) is an estimated discount factor from Times 1 to 2 based on, for example, the capital asset pricing model (CAPM) (Cornell and Morgan 1990). Lev and de Villiers (1994) discuss and critique the practice of an announcement period price drop as the starting point for computing damages. Dybvig et al. (2000) state that this critique played a role in modifying the maximum damages allowable under the PSLRA to the difference between the investors’ purchase price and the 90-day average price following the correction of the company’s misstatement. I expect that this cap is also correlated with the price drop at Time 2.}

Settlements can be large in absolute terms, but tend to be small when compared to alleged damages. Median settlements, scaled by the overstatement’s estimated impact on market value, depend on the estimation method and ranged from 2.3\% to 5.7\% in 2009 and from 2.9\% to 9.7\% during 2002–08 (Ryan and Simmons 2009).\footnote{These figures provide rough gages of settlements as a fraction of claimed damages because only a fraction of shares is purchased at inflated prices, whereas the market values used to compute the percentages are based on all outstanding shares. The higher percentages correspond to damages estimated from the price drop at the time the disclosure was corrected. The lower percentages, 2.3\% and 2.9\%, correspond to “plaintiff style” damages computed using the value line approach. Value line damages are larger because they include an estimate of expected returns in addition to the effect of the correction period price drop.} Griffin et al. (2004) find that the median settlement equals 2.1\% of end-of-class-period (i.e., postcorrection) market value and Palmrose and Scholz (2004) find that the median settlement in restatement-related litigation is 6\% of the assets.\footnote{The figures vary between the studies due to different samples and methodologies. Based on the median settlement-to-market-capitalization and end-of-class-period return data, the figures in Griffin et al. (2004) are consistent with settlements being about 13.2\%.}
The frequency and the results of securities litigation vary with firm characteristics. Settlements appear to have a greater impact on small firms, as evidenced by median settlements of 82% of the decline in value at the end-of-class period for declines of less than $10 million during 2002–08 versus 4.2% of the declines greater than $500 million (Ryan and Simmons 2009). However, large firms sometimes do pay very large settlements. For example, Cendant Corporation’s market value declined by more than $1 billion in 2 days around its April 15, 1998, announcement that it had overstated earnings by $100 million. It ultimately settled a class action lawsuit for $2.8 billion, about 25% of the market value decline, with more than $260 million of that amount going toward plaintiff attorney fees.11 Litigation is also more common for some types of firms, such as bio-pharmaceutical companies, which faced between 9% and 14% of the securities class action lawsuits initiated from 2006 to 2009 (Francis et al. 1994; Cohn and Swick 2010).

Securities litigation plays a role in shaping corporate reporting. Managers face severe reputational, financial, and even criminal consequences if the government charges them with misreporting (Karpoff et al. 2008a). Rogers and van Buskirk (2009) find that firms reduce disclosure activity after litigation, possibly due to a perception that attorneys use disclosure as a pretext for litigation. Frankel et al. (1995) find evidence consistent with litigation motivating firms, on average, to make unbiased forecasts around securities issuances. Firms also appear to disclose bad news early to reduce the likelihood of litigation and to reduce the settlements (Skinner 1994, 1997; Field et al. 2005).

The valuation effect of litigation in my model depends on the existence of transaction costs—costs to the firm that do not flow to investors. Empirically, transaction costs constitute a large portion of the firm’s litigation costs. Plaintiff attorneys receive about 32% of any settlements (Martin et al. 1999), and firms pay their attorneys an average of 30% of the settlements (Coffee 2006). Thus, for every dollar paid in settlement, the firm spends $1.30, including the costs of its attorney, and attorneys receive 48% (($0.30 + $0.32)/$1.30) of the direct costs paid by the firm. Firms also incur indirect costs from damaged reputations and diverted managerial attention. In Securities and Exchange Commission (SEC) enforcement actions—an extreme situation that often leads to class action lawsuits—indirect costs, adjustments to unmanipulated financial figures, and legal penalties and settlements comprise 66.6%, 24.5%, and 8.8%, respectively, of the 38% average loss in the market value associated with the enforcement action (Karpoff et al. 2008b). Griffin et al. (2004) analyze a broader sample of the price drop at the time of the corrective disclosure, as compared to 9.7% in Ryan and Simmons (2009). Palmrose and Scholz (2004) do not report sufficient data to estimate the ratio of settlement to the price drop.

11 Price decline data are from the Center for Research in Security Prices (CRSP) database, and the settlement data are from Stanford’s Securities Class Action Clearinghouse.
of 3000 suits from 1990 to 2002 and find that the mean (median) ratio of settlement to end-of-class-period market value is 8.1% (2.1%) versus the market value decline at the end-of-class period of 16.6% (13.7%).\footnote{12

Although civil lawsuits are subject to a more-likely-than-not standard once they arrive in court, the PSLRA introduced substantial hurdles for plaintiffs filing securities class action suits. In particular, the PSLRA increased the evidence required for plaintiffs to overcome the defendant’s inevitable motion to dismiss (Cox and Thomas 2009).\footnote{13} For example, in Re: Silicon Graphics Inc. Securities Litigation, Court of Appeals, 9th Circuit, 97-16204 and 97-16240 (1999), the court interpreted the PSLRA as requiring that plaintiffs “plead, at a minimum, particular facts giving rise to a strong inference of deliberate recklessness.” The model incorporates these constraints by allowing lawsuits only when investors both suffer a loss and show that new public, pre-discovery, information provides sufficient evidence that the manager overstated.

Most public companies carry director and officer (D&O) insurance policies that reimburse the costs of indemnifying managers, and cover the costs of managers in situations that prohibit the company from indemnifying (Towers Watson 2008).\footnote{14} Policy limits average $30 million for publicly traded companies, and $129 million for companies with assets greater than $10 billion (Towers Watson 2008). Annual insurance premiums average $482 thousand for publicly traded companies, and $2.2 million for companies with assets greater than $10 billion (Towers Watson 2008). Deductibles average 2\% of the coverage limit (Bhagat et al. 1998). The policies do not apply when there is a finding of fraud, but most class actions settle out of court (Cox and Thomas 2009). Insurers charge premiums that take into account the risk of settlements both when management is indeed at fault and when claims are nonmeritorious (Core 1997, 2000; Cao and Narayanamoorthy 2005). The sophistication of the insurers thus prevents companies from using D&O policies to escape the costs of misreporting—they can place only a fair ex ante bet on these costs. Accordingly, I assume that the premium for litigation insurance allows the insurer to breakeven, and that the insurer has sufficient knowledge about the firm to infer the manager’s reporting incentives.\footnote{15

12. The larger contribution of indirect costs in Karpoff et al. (2008b) may stem from the SEC enforcement actions’ focus on more egregious actions.

13. Attorneys make some attempts to bypass the PSLRA restrictions. For example, suing in state courts reduces the restrictions on pretrial discovery (Cox et al. 2008) while the theory of reckless, rather than fraudulent, behavior reduces the need to present a “strong inference” that managers deliberately made misleading disclosures (Cohn and Swick 2010).

14. Klausner and Hegland (2010) find that insurers pay the entire settlement in 53% of the cases. They pay a partial amount, largely due to retentions and policy limits, in another 35\% of the cases.

15. In practice, insurance premiums exceed the breakeven level used in my model. Incorporating an insurer profit would not affect the results so long as the insurer’s profit leaves some value to the firm. Insurance creates a surplus that the firm and its insurer can divide based on their relative bargaining power. If the insurer demands a premium that
3. Model and Timeline

This section describes the model’s timeline and illustrates how the rational anticipation of litigation affects prices. The model’s timeline has four steps, as shown in Figure 1. Table 1 summarizes the notation used in this study. Investors are risk neutral and perfectly competitive. From the perspective of a well-diversified investor, litigation likely represents an idiosyncratic risk so that risk neutrality seems particularly appropriate for examining behavior with respect to lawsuits. The firm begins at Time 0 and pays a terminal dividend $v$ at Time 3. At Time 1, the Time 0 shareholders sell to Time 1 shareholders, and the firm’s manager privately observes a signal $r$ of firm value. The results of the model still obtain if some fraction of Time 0 shareholders do not sell due to, for example, lockup periods or hedging benefits.\(^\text{16}\)

The manager has private information $b_m$ about his incentives to bias his public report $r_m$ to shareholders. Investors might know the manager’s compensation agreement, but they cannot see inside his mind to know precisely how much the manager cares about maximizing the share price for both nonpecuniary reasons, such as reputation, and the value he attaches to his compensation. Although investors rationally anticipate that the manager might opportunistically bias his report ($r_m \neq r$), the manager’s private knowledge of his incentives $b_m$ prevents shareholders from fully removing bias from the report, which creates scope for litigation. The Time 1 investors use the report $r_m$ in conjunction with other information $y$ to determine the share price $p_1$. The Time 2 investors observe an informative signal $s$ and determine the share price $p_2$ using all public information ($r_m, y, s$).

If the Time 2 information indicates that a misleading manager report caused Time 1 investors to overpay, then the Time 1 investors can sue in an attempt to recover their overpayment.\(^\text{17}\) For example, if the manager exceeds the value created by insurance, then the firm will be better off forgoing insurance. Core (2000) notes that the D&O insurance market is highly competitive, which suggests that firms are able to appropriate much of the value created by obtaining insurance.

16. Complete turnover arises endogenously if shareholders believe that litigation is possible, but a variety of forces outside of my model drive empirically observed trading volume (See, e.g., Lo and Wang 2010). The complete turnover occurs in my model because Time 1 investors value shares more than Time 0 investors do, on account of being able to sue if subsequent news indicates the manager overreported at Time 1. The frictionless, risk-neutral setting of my model leaves the Time 0 investors with no offsetting benefit from holding shares. In practice, the ex ante likelihood of litigation is low so that a small amount of friction could yield incomplete turnover. Derivations with incomplete turnover are available upon request.

17. Some turnover at Time 2 is needed for investors to have any incentive to sue. If investors had a binding commitment to hold shares indefinitely, litigation would be fruitless because it would merely transfer funds from the value of their share holdings into their cash holdings, less any transaction costs. If some fraction of Time 1 investors sells at Time 2 due to, for example, liquidity reasons, then they have an incentive to sue if Time 2 information indicates that the manager’s Time 1 report caused mispricing. Litigation requires a subset of investors to represent the class, and damages will be based on a statistical analysis with settlements allocated on a pro rata basis to investors who join the class (Alexander 1994).
Firm’s inception Time 0 Manager with bias parameter $b_m$ observes $r$ and reports $r_m$, investors observe $r_m$ and $y$, Time 1 investors buy at price $p_1$

Time 2 information $s$ released, Time 2 investors buy at price $p_2$

Terminal dividend $v$ realized, if Time 1 investors successfully sue, they receive $1 - \alpha$ times the firm’s costs

Figure 1. Timeline.

Table 1. Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$v, v_1, v_2$</td>
<td>Terminal value of the firm, gross of any litigation costs; $v_1 = \mathbb{E}[v</td>
</tr>
<tr>
<td>$r = v + e_r, r_m$</td>
<td>Manager’s Time 1 signal of firm value ($r$) and the manager’s report ($r_m$), where $e_r$ is a mean zero noise term</td>
</tr>
<tr>
<td>$y = v + e_y, s = v + e_s$</td>
<td>Public information available to investors at Times 1 and 2, respectively, in addition to the manager’s report $r_m$, where $e_y$ and $e_s$ are mean zero noise terms</td>
</tr>
<tr>
<td>$p_0, p_1, p_2$</td>
<td>Price of firm’s shares at Times 0, 1, and 2</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Product of the likelihood of the firm losing a lawsuit and the proportion of the price drop that will be paid in the event of losing a lawsuit</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Transaction costs, which are the fraction of firm’s litigation costs that do not flow to Time 1 investors</td>
</tr>
<tr>
<td>$X, x$</td>
<td>The maximum litigation costs paid by the firm’s insurer ($X$) and the premium charged by the insurer ($x$)</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>Standard deviation of the Time 2 expected firm value conditional on Time 1 information ($\text{std}(v_2</td>
</tr>
<tr>
<td>$P, v_2, M$</td>
<td>A constraint on filing lawsuits that requires at least a minimum probability, conditional on Time 2 information, that the manager’s report caused overvaluation of the firm at Time 1; Require $P(\mathbb{E}[v</td>
</tr>
<tr>
<td>$b_m, b, \mu_b, \tau_b, c$</td>
<td>Manager’s privately known sensitivity $b_m$ to price, where investors’ prior beliefs are given by $b$ normally distributed with mean $\mu_b$, precision $\tau_b$, and the manager faces misreporting costs $\frac{c^2}{2}(r_m - r)^2$</td>
</tr>
<tr>
<td>$\Phi(\cdot), \phi(\cdot)$</td>
<td>Standard normal distribution and density, respectively</td>
</tr>
<tr>
<td>$D, G$</td>
<td>Expected total damages and expected net gain from litigation, respectively</td>
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records false credit sales at Time 1, then the failure to collect by Time 2 can cause the auditors to question the validity of the manager’s Time 1 report and to force a restatement. Alternatively, the auditors might require a large write-down of accounts receivable at Time 2, which would also cause investors to question the validity of the Time 1 sales. Adverse Time 2 information does not always imply that the manager overreported at Time 1, however, and could instead be related to unexpected factors, such as changes in consumer preferences. Similarly, positive Time 2 information does not imply that the manager did not overreport at Time 1. Indeed, a manager might overreport at Time 1 in anticipation of positive Time 2 fundamentals that will more than cover the Time 2 reversals of any Time 1 accounting manipulations.

Claimed losses typically vary with the price drop $p_1 - p_2$ that occurs when new information suggests that the manager made a misleading disclosure (Cox et al. 2005). This can result from, for example, the difficulty of determining an estimate of mispricing independent of the price drop. Cases typically settle for less than the full price drop (Cox et al. 2005). In addition, any attempted suit might fail to satisfy the PSLRA requirements to proceed to trial and thus to represent a credible threat to the corporation. Even if a suit can proceed to trial, the outcome is uncertain at Time 2. Accordingly, I denote the company’s litigation costs by the random variable $\mathbb{C}_{18}$, the likelihood of a credible suit, the likelihood that a suit succeeds, and the company’s total litigation costs as a fraction of the price drop. The use of the price drop as an input for the company’s direct (e.g., settlements) and indirect (e.g., managerial time) litigation costs creates a feedback effect that plays a key role in the model.

Since Time 2 investors are risk neutral, competitive, and receive no settlement amounts in the event of successful litigation, they set the Time 2 price $p_2$ as the firm value, net of litigation costs, given all available information, where $v_2 = \mathbb{E}[v | r_m, y, s]$ denotes the expected value of the firm’s operations given Time 2 information:

$$p_2 = \mathbb{E}[v - \theta \max\{0, p_1 - p_2\}|r_m, y, s] = v_2 - \mathbb{E}[\theta \max\{0, p_1 - p_2\}|r_m, y, s]$$

$$\Rightarrow p_2 = v_2 - 1_{v_2 < p_1} \frac{\mathbb{E}[\theta | r_m, y, s]}{1 - \mathbb{E}[\theta | r_m, y, s]} (p_1 - v_2).$$

(1)

Expression (1) allows for general reactions to news. For example, a high likelihood of successful litigation will shift the Time 2 conditional
distribution of $\theta$ to higher values. The $1/1-E[\theta|r_m,y,s]$ multiplier in the second line of equation (1) reflects the feedback effect from price drops to litigation costs.

Time 1 investors observe the manager’s report $r_m$ and other public information $y$ when setting the Time 1 price $p_1$. Their expected payoff depends on the Time 2 price $p_2$, at which they expect to sell, and their share of any settlements. If litigation succeeds, not all of the costs to the company accrue to the suing shareholders. Fraction $\alpha$ of the loss in firm value represents transaction costs, such as attorney fees and deadweight costs. Similar to the company’s litigation costs, $\alpha$ may be unknown at Time 2 and viewed as a random variable conditional on Time 2 public information. The Time 1 price is then:

$$p_1 = E[p_2 + \theta(1 - \alpha)\max(0, p_1 - p_2)|r_m, y]. \tag{2}$$

On the surface, the ability to sue appears to increase the Time 1 price; however, the potential for litigation reduces the Time 2 price. Substituting the Time 2 price from equation (1) into equation (2) shows that transaction costs reduce the Time 1 price of the firm as shown below, where $v_1 = E[v|r_m, y]$ is the expected value of $v$ conditional on Time 1 public information:

$$p_1 = E[E[v - \theta\max(0, p_1 - p_2)|r_m, y, s] + \theta(1 - \alpha)\max(0, p_1 - p_2)|r_m, y]$$

$$= v_1 - E[\theta\alpha\max(0, p_1 - p_2)|r_m, y]. \tag{3}$$

The Time 1 investors’ expected net-of-transaction-costs payout from lawsuits has no impact on $p_1$ because the Time 2 investors rationally anticipate any litigation costs.

Figure 2 plots an example of the price function (1). The price function (1) reflects the expected litigation, which amplifies a price drop when news contradicts the manager’s report and therefore amplifies the Time 1 investors’ losses. The Time 2 price does not reflect the transaction costs $\alpha$ because new investors concern themselves with the firm’s total costs, regardless of what fraction of those costs flow to the Time 1 investors.

If there is 100% share turnover when the manager reports at Time 1, and litigation provides full recovery of the price drop with certainty ($E[\theta|r_m,y,s] = 1$), then the market price $p_2$ is undefined (approaches negative infinity) following any Time 2 information that indicates the manager’s Time 1 report overstated the firm’s value. For example, if the expected value $v_2$ dropped by $100, new investors would lower the price

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19. Plaintiff attorney fees account for about 32% of the settlements (Martin et al. 1999), whereas firms pay fees to their own attorneys that average 30% of the settlement amounts (Coffee 2006). Furthermore, firms incur indirect costs from litigation, such as diverted time and resources.
by $100 to reflect the lower value of the firm’s operations, but also by another $100 to reflect the certain litigation by Time 1 investors. The resulting price drop would allow the Time 1 investors to sue for $200, causing Time 2 investors to price a $100 drop in $v_2$ and a $200 lawsuit. This leads to a $300 suit, and so on because there would be no price at which investors would be willing to purchase shares. A well-defined price $p_2$ thus requires that $E[\theta|\text{rm}, y, s] < 1$ to reflect either partial or uncertain recovery in the event that Time 1 investors can sue for misleading Time 1 information. For example, if $E[\theta|\text{rm}, y, s] = 0.2$, then damages increase by $0.20 for every $1 decrease in $p_2$, and the feedback effect converges. The

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20. This is reflected in equation (1) by $1 - E[\theta|\text{rm}, y, s]$ in the denominator of the expected litigation costs. If $E[\theta|\text{rm}, y, s] < 1$, then the price is well-defined. A similar condition holds if we restrict attention to situations of limited liability. Limited liability requires carefully handling of the fact that $\theta$ incorporates the fraction of the price drop paid out in a successful lawsuit. Limited liability places restrictions on the amount of recovery because, for example, a firm with a fundamental value of $100 would be unable to fully compensate Time 1 investors for any price drop exceeding $100. Details of a limited liability setting are available upon request.

21. If only a fraction of shares traded at the time of the manager’s disclosure, the Time 2 price will be well-defined even if $E[\theta|\text{rm}, y, s] = 1$ because a dollar drop in price leads to a fractional drop in firm value—the fraction of shares that purchased based on the misleading disclosure. If transaction costs are zero ($\alpha = 0$), strict liability for price drops ($\theta = 1$) leads to a degenerate Time 1 price due to a “the more I pay now, the more I can sue for later” effect. Details of this setting are available upon request.
potential for undefined prices illustrates a possible bright side to the often criticized low recovery rates in securities class actions.

4. Model with linear equilibrium

While the generic structure in Section 3 allows for showing the existence of an equilibrium at Time 1 under mild conditions, it does not explicitly model the manager’s reporting choice and the need to credibly allege misreporting in order to file a securities class action suit. This section imposes additional structure on the model and derives a linear rational expectations equilibrium to facilitate analysis while preserving the features from Section 3, and introducing litigation insurance. I show the existence of a unique equilibrium in linear strategies, but not the uniqueness of the equilibrium across all possible sets of beliefs.

4.1 Model Setup

The first assumption is that all parties share the prior belief that the firm’s terminal dividend and the underlying signals (r, y, s) are jointly normally distributed. Specifically, I define the Time 1 signals as follows:

\[ r = v + e_r \]
\[ y = v + e_y \]
\[ s = v + e_s \]

where signal noise terms er, ey, and es are mean zero, normal random variables, independent of v. I assume that the covariance matrix is such that higher values of r, y, and s are interpreted as “good news.” In other words, E[v|r, y] is increasing in both r and y, and E[v|r, y, s] is increasing in r, y, and s. This assumption implies that signals are used primarily to interpret firm value rather than to filter noise from other signals and is satisfied, for example, if the errors er, ey, and es are uncorrelated. I also assume that the Time 2 signal s is at least weakly, positively associated with the manager’s signal r (cov(r, s|y) > 0), so that it is useful in determining the likelihood that the manager caused mispricing at Time 1.

22. For example, sufficient conditions for the existence of a Time 1 price \( p_1 < v_1 = E[v|r_m, y] \) are that v has constant support, the Times 1 and 2 expectations are well defined, and that there is a positive probability of successful litigation for some realizations of Time 2 information.

23. Models of prices with asymmetric information commonly use a linear rational expectations equilibrium, which imposes linear conjectures, but places no restrictions on strategies. In equilibrium, the linear conjectures yield linear best response functions. See Brunnermeier’s (2001) textbook treatment or, for example, Grossman and Stiglitz (1980), Kyle (1985), and Admati (1985).

24. If \( \text{cov}(r, s|y) < 0 \) and \( \text{cov}(v, s|r, y) > 0 \), then high values of s raise not only the Time 2 price but also increase the likelihood that the manager misreported at Time 1. I require both a loss \( (p_2 < p_1) \) and some likelihood that the manager caused mispricing in order to sue. Under reasonable restrictions, such as requiring that it be more-likely-than-not that the manager overreported, lawsuits never occur in equilibrium when \( \text{cov}(r, s|y) < 0 \) (details are available from the author upon request). The absence of litigation makes this case uninteresting in the context of this study and the “positive surprise today means you overstated results yesterday” phenomenon seems pathological, so I restrict the model to cases where \( \text{cov}(r, s|y) > 0 \) which holds, for example, when error terms are independent. The restriction primarily precludes...
As in Fischer and Verrecchia (2000), I assume that the manager reports \( r_m \) to maximize the following objective function:\(^\text{25}\)

\[
\max_{r_m} E [ b_m p_1 - \frac{c}{2} (r_m - r)^2 | r, b_m ].
\]

(4)

The cost function (4) can be viewed as a reduced form, where \( b_m p_1 \) represents the manager’s financial and reputational/intrinsic benefits from high valuations, and the term \( \frac{c}{2} (r_m - r)^2 \), \( c > 0 \) represents the manager’s misreporting costs (e.g., the effort required to deceive auditors and expected penalties in the event that others discover misreporting). The manager privately knows \( b_m \), and investors’ prior belief about \( b_m \) is represented by a normal distribution with mean \( \mu_b \) and precision \( \tau_b \). I use \( b \) to denote the manager’s bias from the investors’ perspective, which is a random variable, and \( b_m \) to denote the actual bias. The investors’ prior belief is such that \( b \) is independent of \( v, r, y, \) and \( s \). Uncertainty about \( b_m \) prevents investors from inferring the manager’s information, thereby providing scope for mispricing.

Following the linear rational expectations equilibrium approach, I assume that investors conjecture that the manager adopts a linear strategy, and vice versa. Given the linear conjectures, the equilibrium best responses are linear so that the conjectures are fulfilled. Time 1 investors observe the manager’s report \( r_m \) and the signal \( y \), but they must form a conjecture of the manager’s reporting strategy to extract information from his report. Similarly, the manager must form a conjecture of the price function when determining his report to maximize (4). I denote the respective conjectures by \( \hat{r}_m \), with coefficients \( (\hat{\rho}_0, \hat{\rho}_r > 0, \hat{\rho}_b > 0) \), and \( \hat{p}_1 \), with coefficients \( (\hat{\pi}_0, \hat{\pi}_r, \hat{\pi}_y) \):\(^\text{26}\)

\[
\hat{r}_m = \hat{\rho}_0 + \hat{\rho}_r r + \hat{\rho}_b b_m, \quad \hat{p}_1 = \mathbb{E}[v | r_m, y] + \hat{\pi}_0 + \hat{\pi}_r r_m + \hat{\pi}_y y.
\]

(5)

When modeling the possibility for securities litigation, I assume that the two criteria must be met for Time 1 investors to file a lawsuit. First, they must have suffered a loss \( (p_2 < p_1) \), and second, there must be sufficient evidence that mispricing occurred at Time 1. In particular, if

\(^{25}\) I provide further discussion of this objective function in Section 4.5. The equilibrium is qualitatively the same if the manager has an objective function \( E[p_1 - \frac{c}{2} (r_m - r - e)^2 | r, e] \) as in Dye and Sridhar (2004), where \( e \) denotes a privately observed cost of misreporting.

\(^{26}\) I verify that \( \hat{\rho}_r, \hat{\rho}_b > 0 \) in equilibrium, but the restrictions \( \hat{\rho}_r, \hat{\rho}_b > 0 \) are unnecessary for any of the results. I impose \( \hat{\rho}_r, \hat{\rho}_b > 0 \) at this stage to facilitate the discussion of the condition \( P(v_1 > \mathbb{E}[v | r, y] | r_m, y, s) > P \) at the beginning of the next section (Section 4.2). The expectation \( \mathbb{E}[v | r_m, y] \) represents investors’ beliefs computed using their conjecture \( \hat{r}_m \). Under the assumptions of normality and linear conjectures, \( \mathbb{E}[v | r_m, y] \) is linear so that the price conjecture \( \hat{p}_1 \) in equation (5) is equivalent to an arbitrary linear conjecture.
the manager’s report results in overvaluation of the firm, then \( v_1 = \mathbb{E}[v|\eta, y] > \mathbb{E}[v|\eta, y] \). I specify this second criterion with a threshold likelihood \( P \) that \( v_1 > \mathbb{E}[v|\eta, y] \). Time 1 investors can therefore sue if \( p_1 < p_2 \) and \( P(v_1 > \mathbb{E}[v|\eta, y]|\eta, y, s) > P. \)

Although the absence of noise trade and risk aversion implies that price drops are directly related to new information that contradicts the manager’s prior report, price drops need not be sufficient to trigger litigation due to factors such as the need for sufficient evidence under the PSLRA (Johnson et al. 2007). The threshold probability \( P \) represents these additional factors.

If investors file a suit at Time 2, the firm’s expected costs are \( \Theta(p_1 - p_2) \), where \( \Theta \in [0,1) \) represents the combined effects of the likelihood of the suit succeeding and the fraction of the price drop that the firm pays upon losing a suit. As noted in Section 3, \( \Theta \) must be less than one for an equilibrium to exist. If the Time 1 investors file a suit at Time 2, they expect to receive \( \Theta(1 - \alpha)(p_1 - p_2) \), where the parameter \( \alpha \) represents the proportional transaction costs. These costs incorporate any of the expected costs \( \Theta(p_1 - p_2) \) to the firm that do not flow to the investors. Modeling them as proportional costs reflects both the typical contingency fee arrangement for plaintiff attorneys and the notion that the amount of time and resources the managers divert to defending against the lawsuit are likely proportional to the damages sought. To facilitate the analysis of the model, I assume that the parameters \( \Theta \) and \( \alpha \) are constant. This assumption does not affect the existence of a linear equilibrium, but does affect the ability to conduct subsequent analysis. Indeed, as long as the Time 2 price is a piecewise linear function of the Time 2 signal, a linear equilibrium will result.

Finally, I assume that at Time 0, the firm can purchase litigation insurance from rational, perfectly competitive, risk-neutral insurers. The policy indemnifies the firm and its managers against litigation costs up to a maximum of \( X \), and the insurer charges a premium \( x \). I assume that there is no asymmetric information between the firm and the insurer at Time 0. As in actual settings, the insurer is perfectly aware that it might pay settlements

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27. The threshold \( P \) could arise from the legal system, as discussed in Section 2, or from a plaintiff attorney’s decision to file suit if the expected payoff exceeds the attorney’s fixed costs. A derivation of the case based on an attorney’s filing decision is available upon request.

28. The strict pleading requirement under the PSLRA suggests that the threshold \( P \) for a suit to proceed strictly exceeds the more-likely-than-not standard applied in the civil trial, itself. See Section 2.

29. Alexander (1996) discusses the prevalence of contingency fees and Coffee (2006) states that defense costs typically range from 25% to 35% of the settlement. These costs exclude losses from the diversion of managerial time and attention, and therefore underestimate the true extent of transaction costs.

30. This assumption is justified on the empirical ground that the providers of D&O insurance are highly sophisticated (Core 2000).
when the manager misreports. Given the lack of information asymmetry at Time 0, the insurer anticipates the manager’s reporting strategy and incorporates the likelihood of misreporting into the premium $x$. This level of sophistication has been used in the empirical literature to allow for the use of litigation insurance premiums as a proxy for corporate governance and litigation risk (Core 1997, 2000; Cao and Narayanamoorthy 2011).

The following subsections derive the equilibrium prices, manager’s report, and insurance premium, beginning with the Time 2 price. At Time 2, investors take as given the Time 1 price and the conjectured reporting strategy $\hat{r}_m$ given in equation (5). They also know the firm’s insurance coverage $X$, its premium $x$, and the threshold likelihood $P$. I first show the conditions under which litigation can occur and then derive the resulting Time 2 price. Working backward, I derive the Time 1 price, which depends on the Time 1 investors’ expected payoffs from both selling shares at Time 2 and from litigating. In the resulting unique linear equilibrium, the Time 1 price equals firm value less the expected transaction costs of litigation. Last, I discuss the assumptions of the model.

4.2 Time 2 price

Before deriving the Time 2 price, I first show that the necessary condition for litigation, $P(v_1 > E[v|r,y]|r_m,y,s) > P$, corresponds to a drop in the Time 2 expected value of the firm, $v_2 = E[v|r_m,y,s]$. Under the investors’ conjecture (5), the effect of the manager’s misreporting is:

$$v_1 - E[v|r,y] = \frac{\rho_h \text{cov}(v,r|y)}{\rho_r \text{var}(r|y)} (b_m - E[b|r_m,y]),$$

(6)

which is proportional to $b_m - E[b|r_m,y]$ under the assumption that $\text{cov}(v,r|y) > 0$ and the investors’ conjecture that $\hat{\rho}_b, \hat{\rho}_r > 0$. In other words, the investors’ Time 1 assessment of firm value, $v_1$, exceeds the true value, $E[v|r,y]$, if the manager has a greater than expected preference for biasing. This highlights the importance of the manager’s private knowledge of $b_m$. If investors can infer $b_m$ so that $E[b|r_m,y] = b_m$, then there is no scope for mispricing and therefore no scope for subsequent litigation.

The linear conjectures imply that $b_m - E[b|r_m,y]$ is normally distributed conditional on $(r_m,y,s)$, giving the following expression of the litigation threshold in terms of the Time 2 updated beliefs about the manager’s bias,

---

31. The proof of Proposition 1 provides computations of the expressions included in this discussion.
where $\Phi(\cdot)$ denotes the standard normal distribution and $\Phi^{-1}(\cdot)$ denotes its inverse:

$$P(v_1 > E[v|r,y]|r_m,y,s) > P \Leftrightarrow \frac{E[b|r_m,y,s] - E[b|r_m,y]}{\text{std}(b|r_m,y,s)} \Phi^{-1}(P).$$

Expression (7) states that the revised estimate of the manager’s bias must shift by $\Phi^{-1}(P)$ standard deviations for the probability of overreporting, $v_1 > E[v|r,y]$, to exceed $P$. Since the Time 2 signal $s$ is the source of the Time 2 revision in beliefs about firm value $v$ and manager bias $b$, the following Proposition shows that condition (7) can be rescaled in terms of the Time 2 revision of expected firm value, $v_2 - v_1$, where $v_2 = E[v|r_m,y,s]$ denotes the Time 2 conditional expected value of the firm and $\sigma_1 = \text{std}(v_2|r_m,y)$ denotes the Time 1 conditional standard deviation of $v_2$:

**Proposition 1.** There is a threshold $v_2$ such that the Time 2 conditional probability of Time 1 overpricing exceeds the threshold $P$ if and only if the Time 2 expected value $v_2 < v_1$:

$$P(v_1 > E[v|r,y]|r_m,y,s) > P \Leftrightarrow v_2 < v_1 - \sigma_1 M = v_2,$$

where:

$$M = \Phi^{-1}(P) \sqrt{\frac{1 - \text{corr}(b,s|r_m,y)^2}{\text{corr}(b,s|r_m,y)^2}}.$$ 

(8b)

Proposition 1 states that the Time 2 expected value of the firm provides sufficient evidence to trigger litigation if and only if it falls $M$ standard deviations below the Time 1 expected value. The threshold $P$ enters $M$ as a proportional factor $\Phi^{-1}(P)$. If filing a suit merely requires that it is more-likely-than-not that the manager caused Time 1 overpricing, then $P = 1/2$ so that $\Phi^{-1}(P) = \Phi^{-1}(1/2) = 0$ and $M = 0$. In this case, a downward revision in beliefs ($v_2 < v_1$) provides sufficient evidence of mispricing. Thus, a price drop combined with a downward revision in beliefs will suffice to file a lawsuit. The term $\Phi^{-1}(P)$, and therefore $M$, is increasing in $P$ so that stricter evidence requirements (e.g., the 9th Circuit’s requirement of a “strong inference of deliberate recklessness” in the post-PSLRA regime) necessitate larger downward revisions in beliefs to justify filing litigation.

Proposition 1 requires the Time 2 signal to provide information about the manager’s bias in order for lawsuits to occur, putting constraints on purely frivolous litigation. Any signal that is informative about the manager’s Time 1 bias will also be useful for updating the estimate of firm value, but signals that provide information about firm value are not necessarily useful for updating beliefs about manager bias. The term $M$,
given in (8b), reflects the requirement that Time 1 investors can sue only if there is a sufficiently high likelihood that the manager’s misreporting led to overpricing. The correlation $\text{corr}(b_s|r_{ms},y)$ represents the new information that the Time 2 signal $s$ provides about the manager’s unknown bias parameter $b_m$.\footnote{32}

**Corollary 1.1.** If litigation requires a higher standard than more-likely-than-not, then the Time 2 signal $s$ must be incrementally informative about the manager’s bias in order for litigation to occur. Formally, if $P > 1/2$ by any amount, then $M \rightarrow \infty$ as $\text{corr}(b_s|r_{ms},y)^2 \rightarrow 0$, so that $\nu_2 \rightarrow -\infty$.

Time 2 investors value the firm at its expected value less the insurance premium and any litigation costs not covered by insurance, giving the price:

$$p_2 = v_2 - x - 1_{\nu_2 < \nu_2^*} 1_{p_1 - p_2 > x} \theta (p_1 - p_2 - X),$$

where the first indicator $1_{\nu_2 < \nu_2^*}$ reflects the filing requirement for evidence that the manager overreported at Time 1, and the second indicator $1_{p_1 - p_2 > x}$ reflects a loss that exceeds the firm’s insurance coverage.\footnote{33} Solving equation (9) for $p_2$ gives:

$$p_2 = v_2 - x - 1_{\nu_2 < \nu_2^*} 1_{p_1 - p_2 > x} \frac{\theta}{1 - \theta} (p_1 - (v_2 - x) - X),$$

where the indicator functions depend on the event:\footnote{34}

$$\{v_2 < \nu_2, p_1 - p_2 > X\} \Leftrightarrow v_2 < v_1 - \sigma_1 \max \left\{ M, -\frac{1}{\sigma_1} (p_1 - (v_1 - x) - X) \right\}.$$  

\footnote{32} The proof of Corollary 1.1 follows immediately from equation (8b), since $\Phi^{-1}(P) > 0$ for any $P > 1/2$.

\footnote{33} Since the factor $\theta$ incorporates both the likelihood of Time 1 investors successfully suing and the proportion of loss they recover, the insurance cap $X$ should be interpreted as the gross loss against which the firm is insured. For example, if a successful suit recovers fraction $\gamma$ of the price drop $p_1 - p_2$, then the maximum payout under the policy is $\gamma X$. It is possible to recast the model to explicitly separate the likelihood of a successful suit $\theta$, and the fraction of losses paid $\gamma$ by replacing $\theta$ with $\theta \gamma$ and $X$ with $X/\gamma$. I do not take this approach because it has no effect on the model and introduces unnecessary notational complexity.

\footnote{34} From equation (10a), $p_1 - p_2 - X = (p_1 - (v_2 - x) - X)(1 + 1_{\nu_2 < \nu_2^*} 1_{p_1 - p_2 > x} \frac{\theta}{1 - \theta})$, which is positive if and only if $v_2 < p_1 + x - X = v_1 + (p_1 - (v_1 - x) - X)$. Combining this with the requirement that $v_2 < \nu_2 = v_1 - \sigma_1 M$ yields equation (10b).
to trigger litigation. In the extreme case of $P = 1$, $\Phi^{-1}(P)$, and therefore $M$, approaches infinity so that requiring certainty of Time 1 overpricing precludes litigation altogether.

4.3 Time 1 Price

Time 1 investors value shares based on their expected payoff from selling the shares at Time 2 and the possibility of suing the firm:

$$p_1 = E[p_2 + 1_{v_2 < v_1} 1_{p_2 < p_1}(1 - \alpha)(p_1 - p_2)|r_m, y],$$

(11a)

where the indicator functions depend on the event:$^{35}$

$$\{v_2 < v_1, p_2 < p_1\} \Leftrightarrow v_2 < v_1 - \sigma_1 \max \left\{ M, \frac{1}{\sigma_1} (p_1 - (v_1 - x)) \right\}.$$  

(11b)

The second term in equation (11a) reflects the expected value of suing, net of transaction costs $\alpha$. From equation (10a), the only portion of $p_2$ unknown at Time 1 is $v_2$. Given the linear conjectures (5), $v_2$ is normally distributed with mean $v_1$ and standard deviation $\sigma_1$.

$^{35}$ Similar to the explanation of equation (10b), Expression (10a) implies that $p_1 - p_2 = p_1 - (v_2 - x) + 1_{v_2 < v_1} 1_{p_2 < p_1} \frac{\alpha}{\sigma^2} (p_1 - (v_2 - x) - X)$, which is positive for $p_1 - (v_2 - x) > 0$ or, equivalently, $v_2 < p_1 + x = v_1 + p_1 - (v_1 - x)$. Combining this with the requirement that $v_2 < \frac{v_1}{\sigma_1} = v_1 - \sigma_1 M$ yields equation (11b).
After substituting from equation (9) for $p_2$ into equation (11a), the Time 1 price satisfies the following relation, where $G$ denotes the expected net gain from litigation scaled by $\sigma_1$:

$$p_1 = (v_1 - x)$$

Price less firm

value, net of

insurance premium

$$= \theta(1 - \alpha)E[p_1 - X | \bar{p}_2 > p_1] - \theta E[p_1 - X | \bar{p}_2 > p_1](p_1 - p_2)$$

Expected net payout from litigation

$$- \theta E[p_2 - X](p_1 - p_2)$$

Expected value of uninsured losses

$$= \theta(1 - \alpha)E[p_1 - X | \bar{p}_2 > p_1] - \theta E[p_1 - X | \bar{p}_2 > p_1](p_1 - p_2)$$

$$= \theta(1 - \alpha)E[p_1 - X | \bar{p}_2 > p_1] - \theta E[p_1 - X | \bar{p}_2 > p_1](p_1 - p_2)[r_m, v].$$

The following lemma states that the expected net gain from litigation, $G$, depends on the parameters known at Time 0 and the conjectured manager strategy $5$:

**Lemma 1.** Given the conjectured strategy (5), the expected net gain from litigation, $G$, is an implicit function of the parameters $(\theta, \alpha, \sigma_1, M, X)$, where $M$ and $\sigma_1$ depend on the conjectured reporting strategy.

Rearranging equation (12) and applying Lemma 1 yields the following Time 1 price:

$$p_1 = v_1 - x + \sigma_1 G(\theta, \alpha, \sigma_1, M, X).$$

Since $G$ does not depend on the Time 1 information, the Time 1 price is a linear function of Time 1 news, as reflected in $v_1$, despite the nonlinear Time 2 price. As I discuss in Section 4.5, this is a feature of the normal distributions used in the model. The value of the Time 1 investors’ ability to sue, a put option, does not vary with Time 1 information. The investors’ claimed damages depend on the extent to which the manager’s report causes overvaluation. Since normal random variables have constant variances, the overvaluation is constant.

4.4 Equilibrium Conjectures

The manager’s conjectures given by equation (5) and his objective function (4) yield the first-order condition:

$$(13)$$

$$b_m \left( \frac{\text{cov}(v, r_m|y)}{\text{var}(r_m|y)} + \hat{\pi}_r \right) - c(r_m = r) \Rightarrow r_m = r = \frac{1}{c} \left( \frac{\text{cov}(v, r_m|y)}{\text{var}(r_m|y)} + \hat{\pi}_r \right) b_m.$$

$$b_m \left( \frac{\text{cov}(v, r_m|y)}{\text{var}(r_m|y)} + \hat{\pi}_r \right) - c(r_m = r) \Rightarrow r_m = r = \frac{1}{c} \left( \frac{\text{cov}(v, r_m|y)}{\text{var}(r_m|y)} + \hat{\pi}_r \right) b_m.$$
Expressions (13) and (14) imply that the conjectures (5) satisfy \( \hat{\pi}_r = \hat{\pi}_v = \hat{\rho}_0 = 0, \hat{\rho}_r = 1, \) and \( \hat{\pi}_r = -x + \sigma_1 G \) and that \( \hat{\rho}_b \) equals the term multiplying \( b \) in equation (14).

Last, the insurers set the premium \( x \) at Time 0, based on the rational expectations of the Times 1 and 2 pricing and litigating behavior. They set the premium \( x \) to cover their expected payout under the policy, giving \( x = \theta E[1_{v_2 < \Sigma} 1_{p_2 < p_1} \min\{X, p_1 - p_2\}] \). Defining the total losses \( D = E[1_{v_2 < \Sigma} 1_{p_2 < p_1} (p_1 - p_2)] \), we can write the insurance premium as follows, where the second line follows from substituting this definition of \( D \) and a substitution from expression to the following cubic equation and depends on the manager’s solution to the following cubic equation and depends on the manager’s

\[
\begin{align*}
D &= \frac{\theta E[1_{v_2 < \Sigma} 1_{p_2 < p_1} (p_1 - p_2)] - \theta E[1_{v_2 < \Sigma} 1_{p_2 > p_1} (p_1 - p_2 - X)]}{\text{Expected value of uninsured losses}} \\
&= \frac{\theta D - (\theta(1 - \alpha)D - \sigma_1 G)}{\text{Expected value of uninsured losses}}.
\end{align*}
\]

Combining the preceding results and computing the insurance premium provides the following characterization of the equilibrium:

**Proposition 2**  In the unique linear equilibrium, the expected losses for which Time 1 investors can sue are a function of the Time 0 parameters \((\theta, \alpha, \sigma_1, M, X)\):

\[
D = E[1_{v_2 < \Sigma} 1_{p_2 < p_1} (p_1 - p_2)]
= \frac{\sigma_1}{1 - \theta(1 - \alpha)} \left( \phi(\max\{M, -G\}) - \Phi(\max\{M, -G\})G \right).
\]

The Time 1 price \( p_1 \) and the insurance premium \( x \) are:

\[
p_1 = v_1 - \theta \alpha D, \quad x = \frac{\theta D}{\text{Expected total damages}} - \frac{\theta E[1_{v_2 < \Sigma} 1_{p_2 > p_1} (p_1 - p_2 - X)]}{\text{Expected damages paid by firm}}.
\]

The manager reports \( r_m = r + \rho_b \hat{h}_m \), where \( \rho_b > 0 \) is the unique real solution to the following cubic equation and depends on the manager’s cost \( c \), \( \text{var}(r|y) \), \( \text{cov}(v, r|y) \), and \( \tau_b \):

\[
\rho_b^3 + \tau_b \text{var}(r|y) \rho_b - \frac{\tau_b}{c} \text{cov}(v, r|y) = 0.
\]

Expression (3) in Section 3 shows that the Time 1 price equals the expected firm value less the expected transaction costs from litigation. Expression (16b) shows that insurance does not change this fact. The insurance premium equals the total expected costs to be paid by the insurer. Investors expect to receive some of this amount back as their share of any settlements, but the premium also incorporates expected transaction costs, which do not flow back to the investors. The net effect is that the price is reduced by the expected transaction costs—both the portion covered by the insurer and the portion paid directly by the firm.
4.5 Discussion of Key Assumptions

The amplification effect depends on two conditions. First, there must be some share turnover at Time 2 in order for some Time 1 investors to have incentives to initiate litigation. Investors who own shares at Time 2 have incentives to join a class action lawsuit because the settlement depends on a hypothetical class rather than on who actually joins (Alexander 1994). Time 1 investors who sell no shares at Time 2 have no incentive to initiate a class action lawsuit because, on a net basis, litigation reduces their payoff by the amount of transaction costs. The second condition is that the ultimate settlement at least partly depends on the Time 1 investors’ realized losses, as measured by the price decline between Times 1 and 2. If the court could determine the amount of Time 1 mispricing independently of the price reaction to the Time 2 corrective information, then there would be no feedback effect from litigation.37

The linearity of the Time 1 price \( p_1 \) and the constant value of litigation result from the use of normal distributions. Normal random variables exhibit homoscedasticity—the uncertainty about the manager’s report and the informativeness of the signals remain constant, regardless of whether the report is high or low. On the one hand, this provides tractability and ensures that litigation depends on the extent to which the manager actually misreports, rather than on the strength of the underlying news about the company. On the other hand, as with the entire class of models that use normal distributions, the model abstracts from the fact that the empirical distribution of the earnings and the news is not normal.

Two features of the manager’s objective function (4) play a crucial role in yielding a linear rational expectations equilibrium. First, the manager’s payoff does not directly depend on the Time 2 price, which can be viewed as representing a short horizon due to, for example, liquidating equity incentives or impending retirement.38 A short horizon is consistent with

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37. For example, suppose that the firm has no insurance, the Time 2 disclosure indicates that there was a 100% chance that misreporting caused an overvaluation at Time 1, and investors expect complete compensation. The discussion in Section 3 showed that the Time 2 price is undefined if the court cannot measure overvaluation independently of the price drop. However, if, without relying on the price drop, the court can determine that the overvaluation was, say $100, then the Time 2 price will decline by $200—$100 for the misvaluation and $100 for the lawsuit. The price drop of $200 exceeds the $100 Time 1 misvaluation; however, this does not create the feedback effect because the $200 price drop does not increase the expected damages.

38. Manager sensitivity to the Time 2 price would likely diminish, but not eliminate, the incentives to bias. The broader issue of Time 2 investors impounding expected litigation costs and therefore exacerbating price drops from bad news would, of course, remain. This is similar to Dye’s (2011) model of litigation related to the withholding of information, where the manager maximizes the disclosure date price. Bhojraj and Libby (2005) provide experimental evidence that managers sacrifice long-term value for short-term performance if they anticipate stock issuance. Similarly, Graham et al.’s (2005) survey evidence indicates that myopic behavior is fairly widespread in the context of choosing higher reported earnings over higher value projects. Bolton et al. (2006) show that compensating managers on short-term
Arlen and Carney’s (1992) argument that securities fraud is more likely when the manager fears dismissal. In principle, payments based on Time 2 information could mitigate Time 1 misreporting due to, for example, clawback provisions or deferred compensation. A dependence on the Time 2 price, which is nonlinear with respect to the underlying information, introduces nonlinearities in the manager’s strategy that render the model intractable.

The second feature of the manager’s objective function that yields a linear strategy is the quadratic cost function, which implies that the manager faces similar costs for over- and underreporting. Although the prior mean \( \mu_b \) can be set sufficiently high relative to its precision, so that underreporting occurs with negligible probability (e.g., when the manager expects option grants), the symmetry is unrealistic in that managers face relatively severe penalties for overreporting (Karpoff et al. 2008a). The extent to which the manager’s report causes mispricing, \( v_1 - E[v|r,y] \), is approximately linear in the degree of misreporting \( r_m - r \), so that the cost function provides a reasonable approximation of the penalties of overreporting.\(^{39}\)

5. Applications

From the Time 1 price \( p_1 \) given by equation (16b), the ex ante value of the firm equals \( E[v] - \theta \alpha D \), where the second term \( \theta \alpha D \) reflects expected transaction costs.\(^{40}\) Holding the distribution of \( v \) constant, which precludes the analysis of how litigation impacts the effectiveness of the manager compensation contracts, litigation affects ex ante firm value via expected transaction costs that do not flow to investors (e.g., deadweight costs and attorney fees). This section analyzes how firm value—in particular, transaction costs—varies with the model’s parameters.

5.1 Effects of the Reporting Environment

Within the general context of Section 3 and the equilibrium of Section 4, misleading disclosures impact ex ante firm value via their effect on the

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\(^{39}\) Direct computations show that \( E[v|r_m,y] - E[v|r_m,y] \) depends on \( r \) and \( r_m \) through a term that is nearly linear in \( r_m - r \). Basing costs directly on the extent of overvaluation \( E[v|r_m,y] - E[v|r,y] \), rather than \( r_m - r \), results in an equilibrium where no reporting occurs. This also occurs with a cost function similar to Dye and Sridhar (2004). This is related to a technical issue noted in (Caskey et al. 2010, note 14). If the cost function is \( (E[v|r_m,y] - E[v|r,y])^2 \) and investors believe that the manager’s report \( r_m \) is informative with some specific sensitivity \( \hat{\rho} \) to his signal \( r \), the actual report will be more sensitive to \( r \) as the manager seeks to reduce his expected cost of misreporting. The only case in which this does not occur is if investors simply ignore the manager’s report.

\(^{40}\) The transaction costs in this setting play a role similar to a budget breaker in Holmström (1982). In that setting, the provision of incentives requires that the team of workers loses some of its output when it fails to work hard. Similarly, misreporting causes investors to lose value to transaction costs.
transaction costs of litigation. Analogous to dividends, any transfers between investors have no impact on the firm value in the risk-neutral setting of this model. Litigation becomes costly because it results in the transfer of value to parties other than investors or, in the case of diverted managerial time, because it results in value that is lost altogether.

The reduced-form manager’s objective function in equation (4) does not explicitly include the effects of litigation, which limits the insights that can be drawn regarding the effects of litigation on reporting behavior. The manager’s reporting strategy in the equilibrium given in Proposition 2 exactly matches that in Fischer and Verrecchia (2000). The strategy depends on the manager’s cost parameter $c$, the precision $t_b$ of the investors’ prior beliefs on the manager’s bias preference $b$, and, via the $\text{var}(r|y)$ and $\text{cov}(v,r|y)$ terms, the covariance matrix of $(v,r,y)$. In the context of the model, any effect of litigation on the manager arises from its impact on these parameters. The following proposition states that the cost and the precision parameters both increase firm value by reducing the expected litigation costs.

**Proposition 3.** Total expected litigation costs $D$ are decreasing in the manager’s misreporting cost and in the precision of investors’ knowledge of the manager’s objective function ($dD/dc < 0$, $dD/dt_b < 0$). Ex ante firm value is thus increasing in the misreporting cost and the precision.

The parameters $c$ and $t_b$ can reflect, among other things, the litigation environment and the firm’s control structures. For example, high-quality external and internal audit functions can constrain misreporting, and would be represented in the model by an increase in $c$. A reduction in the likelihood of litigation due to, for example, the PSLRA restrictions on proceeding to discovery, can be modeled as a reduction in $c$. Detailed reports of the manager’s compensation plans can increase investors’ knowledge of the manager’s incentives, represented by an increase in $t_b$, but maintaining $t_b < \infty$. Increases in the cost and the precision parameters reduce litigation costs by both reducing the likelihood of litigation and, because the manager is less able (from $t_b$) and less willing (from $c$) to trigger mispricing, the magnitude of litigation.\(^{41}\)

5.2 Effect of Insurance on Firm Value and Litigation Frequency

When the Time 2 news contradicts the manager’s Time 1 report, the anticipation of any uninsured litigation costs amplifies the price reaction. This section shows that avoiding this price reaction provides a motivation to purchase litigation insurance, which is a ubiquitous practice (Coffee 2006). Investors have no need to price insured losses, so that insurance tempers the price reaction to news that contradicts the manager’s report.

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\(^{41}\) See the proof of Proposition 3 in the Appendix A for the analytic characterization of these effects.
The following proposition establishes that the value of the firm is strictly increasing in the amount $X$ of insurance coverage it purchases at Time 0:

**Proposition 4.** The ex ante value of the firm is strictly increasing in the cap $X$ on insurance coverage. The likelihood of litigation is weakly increasing in the cap $X$. Formally, $dE[p_1]/dX > 0$ and $dP(v_2 < v_1, p_2 < p_1)/dX \geq 0$.

Figure 4 illustrates Proposition 4 by comparing prices of a firm with no insurance to one with unlimited insurance. In Panel A, litigation requires a high likelihood $P$ that the manager overreported at Time 1. In this case, any news sufficiently bad to surpass the threshold $P$ will have caused a price drop, regardless of whether the firm has insurance. When news is good, the thick curve for the no-insurance case lies above the thin curve for full insurance, which reflects the insurance premium that the full-insurance firm must pay. If news triggers litigation, the Time 2 price of a no-insurance firm drops more steeply than the price of a full-insurance firm. In the latter case, the insurer will bear the costs of litigation. The larger price drop in the no-insurance case amplifies the transaction costs of litigation. Time 1 price equals the expected firm value less the expected transaction costs of litigation, which results in a lower Time 1 price for the uninsured firm, as reflected by the horizontal dashed lines.

Figure 4, Panel B illustrates a similar comparison between a fully insured firm and an uninsured firm, but with a more lenient litigation threshold $P$. A price drop is a binding constraint for litigation against the no-insurance firm, but not for the full-insurance firm. This occurs because, as in the example of Panel A, the Time 1 price $p_1$ of the no-insurance firm includes a relatively large deduction for the expected transaction costs of the litigation, and the no-insurance firm’s good news price $p_2$ is higher because it incurs no insurance premium. The full-insurance firm’s lower good news Time 2 price and higher Time 1 price combine to make a price drop more likely. In the example of Panel B, a price drop is insufficient to allow litigation for the insured firm, and the relevant constraint on litigation becomes the threshold $v_2$ that indicates sufficient evidence that the manager’s report led to Time 1 overpricing. The insured firm will more likely face litigation because $v_2$ exceeds the value of $v_2$ that triggers a price drop and litigation for the uninsured firm.

The important point of Proposition 4 is that insurance can reduce the transaction costs of litigation. Firms do not take unlimited amounts of coverage, as evidenced by insurance contributing a relatively smaller fraction of larger settlements (Klausner and Hegland 2010). In a richer setting, where transaction costs level off for large suits, firm value need not continue increasing in the coverage $X$ as $X$ increases. For example, incremental attorney fees, a component of transaction costs, decline as suits grow larger (Eisenberg and Miller 2004). A leveling off of transaction costs...
reduces the benefit of insurance because the value of insurance derives from reducing the transaction costs associated with large suits. In such a setting, firms have incentives to buy *some* insurance but might not purchase unlimited coverage.\footnote{An analysis that proves this in a setting with varying transaction costs is available upon request. I thank an anonymous referee for raising this issue.} For example, if transaction costs are 30% of any settlement of $100 or lower, and are capped at $30, then the marginal benefits of insurance eventually vanish. In such cases, an interior coverage level ($X_0 < \infty$) maximizes firm value.\footnote{When $P$ is low, a unique coverage level maximizes firm value. When $P$ is high, there is an interior coverage level above which further increases in coverage have no effect on firm value. The former effect is the result of insurance increasing the likelihood of litigation when a price drop is a binding constraint on litigation, as shown in Proposition 4. When $P$ is high, price drops are not a binding constraint on litigation, so that there is no detrimental effect of insurance increasing the likelihood of litigation.} In addition to concerns over moral hazard, nonlinear transaction costs can play a role in companies’ choices to limit their insurance coverage levels.

Proposition 4 does not take into account how insurance can affect the manager’s reporting strategy. On the one hand, insurance makes litigation more likely, which could be interpreted as increasing the misreporting cost $c$ and decreasing misreporting (See Proposition 3). On the other hand,
insurance might reduce incentives to monitor the manager (e.g., reduce $c$ or $\tau_h$) and lead to greater misreporting. Empirical work suggests that insurers are sufficiently sophisticated to at least partially anticipate such moral hazard (Core 1997, 2000). If insurers anticipate that insurance reduces firms’ incentives to constrain misreporting, the model’s equilibrium and results remain. Although firms can increase value by restricting misreporting, moral hazard implies that the credibility of constraints depends on what the firm prefers after negotiating the insurance contract.

The proof of Proposition 4 shows that the Time 1 investors’ expected net gain from litigation (See Expression (13)) is increasing in the insurance coverage $X$. This implies the following corollary:

**Corollary 4.1.** Taking the model’s other exogenous parameters as given, there is a unique insurance coverage level $X_0$ such that Time 1 investors expect a net benefit from litigation if and only if $X > X_0$, and they expect a net loss for $X < X_0$.

Corollary 4.1 implies that, after the insurance premium has been set, investors perceive a net benefit of litigation for highly insured firms. This follows because the insurance premium $x$ is a sunk cost from the perspective of the Times 1 and 2 investors. If the firm is fully insured and the Time 1 investors sue, the suit does not affect the insurance premium, which was established at Time 0. A suit also has no effect on the Time 2 price because the insurance policy will cover any settlements. Thus, Time 1 investors expect a strict benefit from suing if the firm is fully insured. If the firm has no insurance, Time 2 investors reduce the price they are willing to pay, dollar-for-dollar, for the expected total payout from litigation. However, transaction costs prevent Time 1 investors from recovering the entire payout, yielding an expected net loss from litigation. Nonetheless, Time 1 investors will sue if possible because they cannot commit not to sue, and after selling their shares to Time 2 investors, they have no incentive not to sue. Corollary 4.1 states that a unique coverage level $X_0$ exists, between the extremes of zero and complete insurance, above (below) which Time 1 investors perceive an incremental increase (decrease) in firm value from their ability to sue.

### 5.3 Effects of Litigation Environment

The model represents the litigation environment by the required probability that the manager overreported ($P$), the combined effect of the likelihood of successful litigation and the fraction of damages paid ($\theta$), and the proportional transaction costs ($\alpha$). To determine the effects of the threshold $P$, first recall that litigation requires both realized losses ($p_2 < p_1$) and sufficient evidence that the manager overreported ($v_2 < v_1$). The following proposition establishes that litigation insurance plays a role in determining whether the threshold $P$ represents a binding constraint on litigation.
and, in particular, whether “strike suits” based merely on price drops would lead to credible litigation threats in the model.

**Proposition 5.** If the threshold \( P \) is a binding constraint on litigation, then the ex ante value of the firm is increasing in the threshold \( P \), and the likelihood of litigation, \( P(v_2 < v_2, p_2 < p_1) \), is decreasing in \( P \). The minimum \( P \) for which the threshold is a binding constraint is a decreasing function of insurance coverage \( X \), and any threshold \( P > 1/2 \) is binding if the firm has complete insurance.

The first part of Proposition 5 is straightforward. The threshold \( P \) is irrelevant if any news sufficient to trigger a price drop also indicates a higher probability than \( P \) that the manager overreported. However, if the threshold \( P \) is a binding constraint, further increases in the threshold reduce the likelihood that litigation will occur, which decreases the expected loss of firm value to transaction costs of litigation. However, note that this result takes as given the manager’s misreporting cost \( c \) and the precision \( \tau_b \) of beliefs about the manager’s objective function. A lower likelihood of litigation could be interpreted as reducing the manager’s misreporting cost \( c \). Furthermore, a lower likelihood of a suit reduces the firm’s ex ante incentives to constrain misreporting. Both of these factors mitigate the impact of increasing \( P \) on firm value and the probability of litigation.\(^{44}\)

The intuition behind the second part of Proposition 5 is related to the fact that Time 1 investors in an uninsured firm expect a net loss from litigation, whereas investors in a fully insured firm expect a net benefit as shown in Corollary 4.1. As a result, the Time 1 price \( p_1 \) of an uninsured firm is relatively low due to the expected net loss from litigation. A price drop therefore necessitates a very negative signal to cause \( p_2 \) to drop below the already low Time 1 price \( p_1 \). If the threshold likelihood of misreporting \( P \) is low, such as a more-likely-than-not standard (1/2), any Time 2 news that causes a price drop will have surpassed the low threshold \( P \). Only a high threshold \( P \) represents a binding constraint on litigation for uninsured firms.

For fully insured firms, any threshold \( P > 1/2 \) is a binding constraint on litigation. From expression (10), \( p_2 = v_2 - x \) for a fully insured firm \( (X \to \infty) \). From expression (13), \( p_1 = v_1 - x + \sigma_1 G \), where \( \sigma_1 G \) is the expected net gain from litigation. Corollary 4.1 shows that the expected net gain from litigation is positive for a fully insured firm \( (\sigma_1 G > 0) \). The change in price is then \( p_2 - p_1 = v_2 - v_1 - \sigma_1 G \), where the expected net gain from litigation \( \sigma_1 G \) embedded in \( p_1 \) does not depend on Time 2 information. The Time 2 expected value \( v_2 \) must exceed \( v_1 \) by at least \( \sigma_1 G \) to avoid a price drop; however, Proposition 1 shows that \( v_2 \) must be strictly less than \( v_1 \) to surpass the evidence threshold when \( P > 1/2 \). Thus, if \( P > 1/2 \),

\(^{44}\) See Bernardo et al. (2000) for a discussion of the interaction between moral hazard and legal presumptions in a principal–agent setting.
then the binding constraint on litigation against fully insured firms is the evidence threshold $v_2 < v_2$, rather than a price decline.

The following proposition establishes the effects of the settlement likelihood parameter $\theta$ and the transaction costs parameter $\alpha$ for the full- and no-insurance cases.\(^{45}\)

**Proposition 6.** In both the full- and no-insurance cases, the ex ante value of the firm $E[p_1]$ is decreasing in $\theta$ and $\alpha$. If a price drop ($p_2 < p_1$) is a binding constraint on litigation, then the probability of litigation is decreasing in $\alpha$ in both cases and is decreasing (increasing) in $\theta$ in the no-insurance (full-insurance) case. If a price drop is not a binding constraint, then the probability of litigation does not depend on $\theta$ and $\alpha$.

The effects of $\theta$ and $\alpha$ follow from Expression (16b), which gives $E[p_1] = E[v] - \theta\alpha D$. The direct effects of $\alpha$ and $\theta$ on expected transaction costs—$\theta D$ and $\alpha D$, respectively—both decrease $p_1$. The proof of Proposition 6 shows that $D$ is increasing in $\theta$ and decreasing in $\alpha$ in both the full- and no-insurance cases. Thus, the effect of $\theta$ on $D$ compounds its direct effect on $E[p_1]$. The effect of $\alpha$ on $D$ mitigates its direct effect on $E[p_1]$, but the proof of Proposition 6 shows that the direct effect dominates. The reduction in $p_1$ due to higher $\alpha$ explains why higher transaction costs reduce the likelihood of litigation when a price drop is a binding constraint to allow a lawsuit. If the threshold likelihood of overreporting $P$ is the binding constraint, as in the case when the firm has sufficiently high insurance (See Proposition 5), then $\alpha$ has no effect on the likelihood of litigation.\(^{46}\)

The effect of $\theta$ on the probability of litigation depends on whether the firm is insured. It seems plausible that a credible suit requires that it at least be more-likely-than-not that the manager overreported, especially given PSLRA’s pre-discovery pleading requirements. In such a case, $P > 1/2$, and Proposition 5 implies that a price drop is not a binding constraint on litigation for fully insured firms. In conjunction with Proposition 6, this implies that a fully insured firm’s probability of litigation does not depend on $\theta$ when $P > 1/2$.\(^{47}\)

45. The derivations become intractable for intermediate cases.

46. This does not take into account how plaintiff attorneys might react to changes in $\alpha$ that represents allowable contingency fees.

47. If $P$ is sufficiently less than $1/2$, so that a price drop is a binding constraint for fully insured firms, then the probability of litigation is increasing in $\theta$ due to an increase in Time 1 investors’ expected net gain from litigation $G$. Specifically, when a price drop is the binding constraint for litigation, the likelihood of litigation depends on the realization of $v_2$ where $p_2$ crosses $p_1$. An increase in $\theta$ increases the insurance premium $x$ which, by itself, lowers $p_1$ and $p_2$ by an equal amount, leaving the intersection unchanged. The higher $\theta$ increases Time 1 investors’ expected payout from litigation, which increases $p_1$ and moves the intersection of $p_2$ and $p_1$ to higher realizations of $v_2$. The realization of $v_2$ more likely falls short of this higher crossing point, making litigation more likely.
litigation to occur and an increase in \( \theta \) reduces the likelihood of a lawsuit. This occurs because a higher \( \theta \) reduces the Time 1 price \( p_1 \), making a price drop less likely. Stated differently, a reduction in \( \theta \) increases the likelihood that an uninsured firm will face securities litigation.

The differential impact of the litigation parameter \( \theta \) on insured and uninsured firms provides an avenue for testing Proposition 6. The post-PSLRA regime saw both an increase in the filing of lawsuits (Perino 2003; Palmrose and Scholz 2004) and positive price reactions to the legislation (Spiess and Tkac 1997; Johnson et al. 2000).\(^{48}\)

6. Conclusion

In securities class action lawsuits, the firm’s shareholders ultimately both receive and pay damages. This study develops a linear rational expectations model that incorporates shareholders’ anticipation of lawsuits when the release of news indicates that the firm’s managers misreported. The anticipation of litigation amplifies the price reaction to bad news as investors impound both the news about the firm’s operations and the firm’s costs of litigation. This, in turn, exacerbrates the magnitude of lawsuits when settlements depend on the price reaction to news that reveals previous misreporting.

Litigation insurance provides value in the model by eliminating the need for investors to price the expected costs of litigation. In practice, firms need to weigh this benefit against the concern that insurance can exacerbate corporate managers’ opportunistic reporting behavior. While insurance increases firm value by reducing the magnitude of price drops and lawsuits, it also increases the frequency of litigation.

Prices in the model take into account any expected wealth transfers between the firm and the investors, including those from litigation. Thus, in the absence of transaction costs (e.g., attorney fees), lawsuits have no impact on firm value. The transaction costs of litigation provide the firm with ex ante incentives to constrain misreporting to prevent the slippage of value away from investors. Of course, the benefits of reducing transaction costs of litigation must be weighed against the costs of implementing constraints on misreporting.

Despite the complexities involved with determining prices for which litigation can occur, the model has a tractable form. Thus, the model can be useful for future research on settings in which litigation affects share prices. The model involves a setting with risk-neutral investors, managers with a short-term focus, and the constant conditional uncertainty

\(^{48}\) See Ali and Kallapur (2001) for evidence contradicting the positive price reactions to the PSLRA.
that results from modeling with normal distributions. Relaxing these assumptions provides avenues for future research on securities litigation. Risk aversion potentially plays a role in the demand for both litigation and litigation insurance to the extent that either litigation risk has systematic components or investors do not hold diversified portfolios. Managers with long horizons will be more sensitive to the impact their reports have on litigation. In addition, stock prices exhibit substantial information-dependent uncertainty. All of these features would eliminate the tractable linear structure of the model presented here, but may provide additional insights into the effects of litigation on managerial and investor behavior.

Appendix A
A.1 Proof of Proposition 1 and related expressions
From the rules for updating normal random variables, and using the fact that $r_m$ is a noisy version of $r$ for the purpose of inferring $v$ gives $E[v|r, y] = E[v|r_m, r, y] = E[v|r_m, y] + \frac{\text{cov}(v, r|r_m, y)}{\text{var}(r|r_m, y)}(r - E[r|r_m, y])$. The conjecture for $r_m$ stated in equation (5) implies that:

$$
\text{cov}(v, r|r_m, y) = \text{cov}(v, r|y) - \frac{\text{cov}(v, r, y)\text{cov}(r, r_m|y)}{\text{var}(r_m|y)} = \text{cov}(v, r|y)\left(1 - \frac{\hat{\rho}_v \text{var}(r|y)}{\text{var}(r_m|y)}\right)
$$

$$
\text{var}(r|r_m, y) = \text{var}(r|y) - \frac{\text{cov}(v, r, y)^2}{\text{var}(r_m|y)} = \text{var}(r|y)\left(1 - \frac{\hat{\rho}_v^2 \text{var}(v|y)}{\text{var}(r_m|y)}\right).
$$

(A1)

Rearranging equation (5) gives $r = \frac{1}{\hat{\rho}_r}(r_m - \hat{\rho}_b b_m)$, so that $r - E[r|r_m, y] = -\frac{\hat{\rho}_b}{\hat{\rho}_r}(b_m - E[b|r_m, y])$. The preceding relations imply that:

$$
E[v|r, y] = \frac{\hat{\rho}_b}{\hat{\rho}_r} \text{var}(r|y)\left(b_m - E[b|r_m, y]\right). \quad (A2)
$$

Expression (7) follows from applying equation (6), the assumption that $\text{cov}(v, r|y) > 0$, the investors’ conjecture that $\hat{\rho}_b, \hat{\rho}_r > 0$, and the joint normality assumptions:

$$
P(v_1 > E[v|r, y]|r_m, y, s) = P(b_m > E[b|r_m, y]|r_m, y, s)
$$

$$
= \Phi\left(\frac{E[b|r_m, y, s] - E[b|r_m, y]}{\text{std}(b|r_m, y, s)}\right), \quad (A3)
$$

which gives the condition (7) after substituting into $P(v_1 > E[v|r, y]|r_m, y, s) > P$.

To prove the proposition, we first have the following from the rules for updating normal random variables:

$$
E[b|r_m, y, s] - E[b|r_m, y] = \frac{\text{cov}(b, s|r_m, y)}{\text{var}(s|r_m, y)}(s - E[s|r_m, y]),
$$

$$
v_2 - v_1 = E[v|r_m, y, s] - E[v|r_m, y] = \frac{\text{cov}(v, s|r_m, y)}{\text{var}(s|r_m, y)}(s - E[s|r_m, y]). \quad (A4)
$$
This proof also uses the following inequalities, which follow from direct computations and the assumptions that \( \text{cov}(r, s|y) > 0 \), \( \text{cov}(v, r|y) > 0 \) (\( E[v|r, y] \) increasing in \( r \)), \( \text{cov}(v, s|r, y) > 0 \) (\( E[v|r, s, y] \) increasing in \( s \)), and the conjectures \( \hat{\rho}_r, \hat{\rho}_b > 0 \):

\[
\begin{align*}
\text{cov}(r, s|r, m, y) &= \text{cov}(r, s|y) \frac{\hat{\rho}_v^2 \text{var}(b)}{\text{var}(r|m|y)}, \\
\text{cov}(v, s|r, m, y) &= \text{cov}(v, s|y) \frac{\text{var}(r|y) \text{cov}(r, s|y)}{\text{var}(r|y) + \hat{\rho}_v^2 \text{var}(b)}, \\
&> \text{cov}(v, s|y) \frac{\text{cov}(r, v|y) \text{cov}(r, s|y)}{\text{var}(r|y)} = \text{cov}(v, s|y) > 0.
\end{align*}
\]

(A5)

Substituting equation (A4) into equation (7) and rearranging gives the following inequality, where the sign of the inequality follows from equation (A5):

\[
E[b|r, m, y, s] - E[b|r, m, y] > \text{std}(b|r, m, y, s) \Phi^{-1}(P)
\]

\[
\Leftrightarrow v_2 < v_1 + \frac{\text{cov}(v, s|r, m, y)}{\text{cov}(b, s|r, m, y)} \text{std}(b|r, m, y, s) \Phi^{-1}(P).
\]

Rescale from coefficient of \( s \) in \( E[b|r, m, y] \)

\[
\text{to coefficient of } s \text{ in } E[v|r, m, y, s]
\]

(A6)

The term that multiplies \( \Phi^{-1}(P) \) in equation (A6) can be written as follows, which uses \( \text{cov}(b, s|r, m, y) < 0 \) as shown in equation (A5) and

\[
\sigma_1 = \sqrt{\text{var}(v_2|r, m, y)} = \sqrt{\frac{\text{cov}(v, s|r, m, y)^2}{\text{var}(s|r, m, y)}}.
\]

\[
\frac{\text{cov}(v, s|r, m, y)}{\text{cov}(b, s|r, m, y)} \text{std}(b|r, m, y, s) = \frac{\text{cov}(v, s|r, m, y)}{\text{std}(s|r, m, y)} \frac{\text{std}(s|r, m, y)}{\text{cov}(b, s|r, m, y)} \text{std}(b|r, m, y, s)
\]

\[
= -\sigma_1 \frac{\text{var}(b|r, m, y, s)/\text{var}(b|r, m, y)}{\frac{\text{cov}(b, s|r, m, y)^2}{\text{var}(b|r, m, y)\text{var}(s|r, m, y)}}
\]

\[
= -\sigma_1 \frac{1 - \frac{\text{cov}(b, s|r, m, y)^2}{\text{var}(b|r, m, y)\text{var}(s|r, m, y)}}{\frac{\text{cov}(b, s|r, m, y)^2}{\text{var}(b|r, m, y)\text{var}(s|r, m, y)}}
\]

(A7)

which yields Expression (8b).

A.2. Proof of Lemma 1

The Time 2 expected value \( v_2 \) is the only random variable in equation (12) conditional on Time 1 information. Given the linear conjectures (5), \( v_2 \) is normally distributed with mean \( v_1 \) and standard deviation \( \sigma_1 \). The expectations in equation (12) can therefore be computed using the
formulas $\mathbb{E}[\mathbb{E}[E]] = \mu_a(\Phi(\frac{a-\mu_a}{\sigma_a}) - \Phi(\frac{a-\mu_a}{\sigma_a}))$ and $\mathbb{E}[\mathbb{E}[E]] = \Phi(\frac{a-\mu_a}{\sigma_a}) - \Phi(\frac{a-\mu_a}{\sigma_a})$ for $a \sim \mathcal{N}(\mu_a, \sigma_a^2)$, where $\Phi$ and $\phi$ denote the distribution and density of a standard normal variable. The expectations in equation (12) can be computed using the definitions (10) and (11b). Rearranging equation (12) after computing expectations and substituting $\sigma_1 G = p_1 - (v_1 - x)$ gives the following equilibrium condition for $G$:

$$0 = \theta (1 - \alpha)(\Phi(-\max[M, -G])G + \phi(\max[M, -G])) - G$$

$$-\frac{\theta (1 - \theta(1 - \alpha))}{1 - \theta} \left(\Phi\left(-\max[M, -(G - \frac{1}{\sigma_1} X)]\right)(G - \frac{1}{\sigma_1} X)\right) + \phi\left(\max[M, -(G - \frac{1}{\sigma_1} X)]\right).$$

Define the right-hand side of equation (A8) as a function $g(G; \theta, \alpha, \sigma_1, M, X)$, which does not depend on the specific realization of any Time 1 information. To show that a solution exists, direct computations show that $g \to -\infty$ as $G \to \infty$ and $g \to \infty$ as $G \to -\infty$. The following shows that $g$ is strictly decreasing in $G$, which proves that a unique $G$ solves $g(G) = 0$:

$$\frac{\partial g}{\partial G} = -\left(1 - \theta(1 - \alpha)\Phi(-\max[M, -G])\right)$$

$$+\frac{\theta(1 - \theta(1 - \alpha))}{1 - \theta} \Phi\left(-\max[M, -(G - \frac{1}{\sigma_1} X)]\right) < 0,$$

where the inequality follows from $\alpha \in [0,1]$ and $\theta, \Phi(\cdot) \in (0,1)$. Expression (A8) thus implicitly defines the net gains from litigation $G$ as a function of $(\theta, \alpha, \sigma_1, M, X)$.

**A.3. Proof of Proposition 2**

**A.3.1 Manager’s Conjectures**

From equation (14), the conjectures (5) and Lemma 1, which implies that $\hat{\pi}_r = \hat{\pi}_y = 0$, the manager’s reporting strategy solves $\rho_b = \frac{1}{c} \frac{\text{cov}(r, r_{m|y})}{\text{var}(r_{m|y}) + \rho_b^2 \tau_b}$, which implies that $\rho_b$ must solve the following cubic equation:

$$\rho_b^3 + \tau_b \text{var}(r|y) \rho_b - \frac{\tau_b}{c} \text{cov}(v, r|y) = 0. \quad (A10)$$

Equation (A10) is strictly increasing in $\rho_b$, approaches infinity (negative infinity) as $\rho_b$ approaches infinity (negative infinity), and at $\rho_b = 0$ equals $-\frac{\tau_b}{c} \text{cov}(v, r|y) < 0$ under the assumption that $r$ is good news ($\text{cov}(v, r|y) > 0$). Thus, the equilibrium $\rho_b$ exists and is greater than zero. The linear
conjectures are satisfied so that the joint normality assumptions used in deriving prices are valid in equilibrium.

### A.3.2 Insurance Premium and Expected Losses

Define $D = E[1_{1_{X_2}<X_2, 1_{p_2<p_1}}(p_1 - p_2)|r_m, y]$. Expression (12) implies that:

$$
D = \frac{1}{\theta(1 - \alpha)} \left( \sigma_1 G + \theta E[1_{1_{X_2}<X_2, 1_{p_2<p_1}}(p_1 - p_2)X|r_m, y] \right)
= \frac{\sigma_1}{1 - \theta(1 - \alpha)} \left( \Phi(\max\{M, -G\}) - \Phi(\max\{M, -G\})G \right),
$$

(A11)

where the second line follows from a substitution from $g(G) = 0$. The insurance premium is the following, where the second equality follows from the definition of $D$ and from equation (A11):

$$
x = \theta E\left[ E[1_{1_{X_2}<X_2, 1_{p_2<p_1}}(p_1 - p_2)|r_m, y] \right] - \theta E\left[ E[1_{1_{X_2}<X_2, 1_{p_2<p_1}}(p_1 - p_2)X|r_m, y] \right]
= \sigma_1 G + \theta \alpha D.
$$

(A12)

Substituting equation (A12) into equation (13) gives equation (16b). ■

### A.4. Proof of Proposition 3

The parameters $c$ and $\tau_b$ affect $D$ via the amount of noise introduced into the manager’s report, which I denote by $w = \rho_b^2 \tau_b^{-1}$. The noise $w$ impacts $D$ via its effect on $\sigma_1$. In addition, expression (8b) indicates that $M$ depends on the conditional correlation $\text{corr}(b, s|r_m, y)$, which is also affected by the noise $w$. This gives $\frac{dD}{dw} = \frac{dD}{d\sigma_1} \frac{d\sigma_1}{dw} + \frac{dD}{dM} \frac{dM}{dw}$.

The proof proceeds in steps:

1. $dD/d\sigma_1 > 0$: direct computations give 
   $\frac{dD}{d\sigma_1} = \frac{1}{\sigma_1} \frac{dD}{dG} > 0$, and 
   $\frac{d\sigma_1}{dG} = -\frac{\sigma_1}{\theta(1 - \theta(1 - \alpha))} \Phi(\max\{M, -G\} - X)$, 
   which implies that $\frac{d\sigma_1}{dG} < 0$. Along with $\frac{dG}{d\sigma_1} < 0$, from the proof of Lemma 1, 
   these inequalities imply that $\frac{dD}{d\sigma_1} < 0$ and $\frac{dD}{dM} = \frac{dD}{d\sigma_1} \frac{d\sigma_1}{dG} \frac{dG}{dM} > 0$.

2. $d\sigma_1/dw > 0$: the assumption of positive conditional covariances 
   $(\text{cov}(v, r|y) > 0, \text{cov}(v, r|y, s) > 0)$ implies that $\sigma_1$ is increasing in 
   $\text{var}(r_m|y) = \text{var}(r|y) + w$ that is increasing in $w$.

3. $dD/dM \leq 0$: direct computations give:

$$
\frac{dD}{dM} = \begin{cases}
0 & \text{if } M \leq -G, \\
\frac{\sigma_1 \phi(M)}{\sigma_1} \left(1 + \frac{\theta}{1 - \theta} \Phi(M - \frac{1}{\sigma_1} X)\right) (M + G) < 0, & \text{if } -G < M < -(G - \frac{1}{\sigma_1} X), \\
\frac{1}{1 - \theta} \frac{\sigma_1 \phi(M)}{\sigma_1} \left(G + M - \theta \Phi(M - \frac{1}{\sigma_1} X)\right) < 0, & \text{if } M \geq -(G - \frac{1}{\sigma_1} X),
\end{cases}
$$

(A13)
where, in addition to \( \frac{\partial g}{\partial X} < 0 \), the first inequality follows from \( M > -G \) and the second follows from \( M > -(G - \frac{1}{\sigma_1} X) \).

4. \( dM/dw < 0 \): Direct computations give \( \text{corr}(b, s|r_m, y)^2 = \text{corr}(r, s|y)^2 \frac{w}{\text{var}(r|y, s) + w} \), which implies that \( d \text{corr}(b, s|r_m, y)/dM > 0 \), and equation (8b) implies that \( dM/dw < 0 \) in this interval.

5. \( dw/dt_b, dw/dc < 0 \): From equation (A10), \( \frac{d\rho_b}{dt_b} = -\frac{\rho_b}{t_b} \frac{\text{cov}(v, r|y)}{\text{var}(r|y)} < 0 \) and \( \frac{d\rho_c}{dc} = -\frac{\rho_c}{c} \frac{\text{cov}(v, r|y)}{\text{var}(r|y)} < 0 \), where the inequalities follow from the assumption \( \text{cov}(v, r|y) > 0 \), which implies that \( \rho_b < 0 \). This gives \( \frac{dw}{dt_b} = \frac{\rho_b}{t_b} (2 \frac{d\rho_b}{dt_b} - \frac{\rho_c}{t_c}) < 0 \) and \( \frac{dw}{dc} = 2 \frac{\rho_b}{t_b} \frac{d\rho_c}{dc} < 0 \).

The results from Steps 1 through 5 imply that \( \frac{dD}{dc}, \frac{dD}{dt_b} \) in the generation of the parameters, the second part of \( \frac{dD}{dc}, \frac{dD}{dt_b} \geq 0 \), the first part of \( \frac{dD}{dc}, \frac{dD}{dt_b} > 0 \) reflects the impact of the parameters on the likelihood of litigation, which only comes to bear when the filing hurdle \( (v_2 < v_1) \), rather than a price drop \( (p_2 < p_1) \), is the binding constraint on filing a suit.

A.5. Proof of Proposition 4

The expected value of the firm \( E[p_1] \) is increasing in \( X \) if and only if \( D \) is decreasing in \( X \). Expression (A11) defines \( D \), giving \( \frac{dD}{dx} = \frac{\partial D}{\partial g} \frac{dg}{dx} = -\frac{\partial D}{\partial g} \frac{\partial g}{\partial X} \), where \( g \) is the function given by equation (A8) and defines \( G \). The proof of Proposition 3, Step 1 gives \( \frac{dD}{dx} < 0 \) and the proof of Lemma 1 gives \( \frac{dg}{dx} < 0 \), so that \( \frac{dD}{dx} \) has the same sign as \( -\frac{dg}{dx} \). Direct computations give \( \frac{dg}{dx} = \frac{d(1 - (1 - \alpha) \Phi(-\max(M, -(G - \frac{1}{\sigma_1} X)))}{1 - \alpha} > 0 \), which implies that \( dD/dX < 0 \). The probability of litigation is \( P(v_2 < v_2, p_2 < p_1) = \Phi(-\max(M, -G)) \). Thus, \( \frac{dp(v_2 < v_2, p_2 < p_1)}{dx} = 1_{M < -G} \frac{dG}{dx} \geq 0 \), with strict inequality only if a price drop \( (p_2 < p_1) \), rather than the filing hurdle \( (v_2 < v_2) \), is the binding constraint on filing a suit.

A.6. Proof of Corollary 4.1

The expected net gain from litigation is \( \sigma_1 G \), where \( G \) is given by \( g(G) = 0 \) with \( \frac{dg}{dg} < 0 \), as given in Lemma 1. Direct computations give:

\[
\lim_{X \to 0} g(G; X) = -\frac{\partial g}{1 - \alpha} (\Phi(-\max(M, -G))G + \phi(\max(M, -G))) - G,
\]

\[
\lim_{X \to \infty} g(G; X) = \theta(1 - \alpha)(\Phi(-\max(M, -G))G + \phi(\max(M, -G))) - G,
\]

(A14)

so that \( g(0; X = 0) = -\frac{\partial g}{1 - \alpha} (\Phi(M) < 0 \) and \( g(0; X \to \infty) = \theta(1 - \alpha) \phi(M) > 0 \). Since \( \frac{dg}{dx} < 0 \), these inequalities imply that \( G < 0 \) for \( X = 0 \) and \( G > 0 \) for \( X \to \infty \). The proof of Proposition 4 shows that \( G \) is decreasing in \( X \), which implies that there is a unique \( X_0 \) such that \( G \), and therefore the
expected net loss from litigation $\sigma_1 G$, equals zero at $X = X_0$, is positive for $X > X_0$, and is negative for $X < X_0$.

A.7. Proof of Proposition 5

The expected value of the firm $E[p_1]$ is increasing in $P$ if and only if $D$ is decreasing in $P$, where $\frac{dD}{dP} = \frac{dD}{dM} \frac{dM}{dP}$. The proof of Proposition 3, Step 3 gives $\frac{dD}{dM} \leq 0$, and the probability of litigation is $P(v_2 < v_2, p_2 < p_1) = (-\max(M, -G))$ with $\frac{d\Phi(-\max(M, -G))}{dM} = -1_{M>-G}\phi(M) \leq 0$. Both inequalities are strict only if the filing hurdle $(v_2 < v_2)$, rather than a price drop $(p_2 < p_1)$, is the binding constraint on filing a suit ($M > -G$).

For the second part, the threshold $P$ is a binding constraint on litigation if and only if $M > -G$. Since $M > 0$ for any $P > 1/2$ (See Expression (8b) in Proposition 1) and $G > 0$ when the firm has complete insurance (See Corollary 4.1), the threshold is binding for any $P > 1/2$ when the firm has complete insurance. If $M = -G$, then direct computations show that $g(G = -M)$ is increasing in $M$, approaches $-\infty$ as $M \to -\infty$ and approaches $\infty$ as $M \to \infty$, which implies that a unique $M$ solves $0 = g(G = -M)$. Direct computations also show that $g(G = -M)$ is increasing in $X$, so that implicit function implies that the threshold $P$, such that $M = -G$ is decreasing in $X$.

A.8. Proof of Proposition 6

A.8.1 No-insurance case ($X = x = 0$)

Since the insurance premium $x = 0$, a comparison of equation (13) to equation (16b) implies that $\sigma_1 G = -\theta x D$. The trigger for litigation, $(v_2 < v_2, p_2 < p_1)$, is then $v_2 < v_1 - \sigma_1 \max(M, -G) = v_1 - \sigma_1 \max(M, \frac{1}{\sigma_1} \theta x D)$. Substituting $p_1 = v_1 - \theta x D$ and from equation (10a) for $p_2$, with $x = X = 0$, gives:

$$D = E[1_{v_2<v_2}, p_2<p_1](p_1 - p_2)] = \frac{1}{1 - \theta} E[1_{v_2<v_2}, p_2<p_1](v_1 - \theta x D - v_2)].$$

(A15)

Applying the implicit function theorem to equation (A15) gives:

$$\frac{dD}{d\theta} = \frac{1}{1 - \theta} \left(1 - \theta E[1_{v_2<v_2}, p_2<p_1]\right) D > 0, \quad \frac{dD}{d\alpha} = -\frac{\theta}{1 - \theta} E[1_{v_2<v_2}, p_2<p_1] D < 0.$$  

(A16)

49. When computing the derivatives in equation (A.16), expression (A.15) can be written as $D = \frac{1}{1 - \theta} E \left[ v_1 - \sigma_1 \max \left\{ M, \frac{1}{\sigma_1} \theta x D \right\} \right] (v_1 - \theta x D - v_2) F_1(v_2)$, where $F_1(v_2)$ denotes the distribution of $v_2$ conditional on Time 1 public information ($r_{nt}$).
Expression (A16) implies:
\[
\frac{d\theta_D}{d\theta} = \alpha D + \theta \frac{d\theta_D}{d\alpha} = \frac{1 + \frac{\theta}{\sigma_1}}{1 - (1 - \alpha) \theta E[1_{\nu_2 < \Sigma, \eta_2 < P_1}]} \alpha D > \alpha D > 0,
\]
\[
\frac{d\theta_D}{d\alpha} = \theta D + \theta \frac{d\theta_D}{d\alpha} = \frac{1}{1 - (1 - \alpha) \theta E[1_{\nu_2 < \Sigma, \eta_2 < P_1}]} \theta D \in (0, \theta D).
\]  

(A17)

The probability of litigation is \( E[1_{\nu_2 < \Sigma, \eta_2 < P_1}] = P(\nu_2 < \nu_1 - \sigma_1 \max\{M, \frac{1}{\sigma_1} \theta \alpha D\}) \), giving:
\[
\frac{d\Phi}{d\theta} \left( \max \left\{ M, \frac{1}{\sigma_1} \theta \alpha D \right\} \right) = -1 \frac{\theta}{\sigma_1} \Phi \left( \frac{1}{\sigma_1} \theta \alpha D \right) \frac{1}{\sigma_1} \frac{d\theta_D}{d\theta} \leq 0,
\]
\[
\frac{d\Phi}{d\alpha} \left( \max \left\{ M, \frac{1}{\sigma_1} \theta \alpha D \right\} \right) = -1 \frac{\theta}{\sigma_1} \Phi \left( \frac{1}{\sigma_1} \theta \alpha D \right) \frac{1}{\sigma_1} \frac{d\theta_D}{d\alpha} \leq 0.
\]  

(A18)

A.8.2 Full-insurance case \((X \to \infty, x > 0)\)

The fact that \(X \to \infty\) implies \(x = \theta D\) gives \(p_1 = \nu_1 - \theta D + \sigma_1 G\) for equation (13). Comparing to equation (16b) implies that \(\sigma_1 G = \theta (1 - \alpha) D\). The litigation trigger is then \(\nu_2 < \nu_1 - \sigma_1 \max\{M, \frac{1}{\sigma_1} \theta (1 - \alpha) D\}\). In the full-insurance case, \(\nu_2 = \nu_2 - x = \nu_2 - \theta D\), which, along with \(p_1 = \nu_1 - \theta \alpha D\), gives:
\[
D = E[1_{\nu_2 < \Sigma, \eta_2 < P_1} (p_1 - \nu_2)] = E[1_{\nu_2 < \Sigma, \eta_2 < P_1} (\nu_1 + \theta (1 - \alpha) D - \nu_2)].
\]  

(A19)

Applying the implicit function theorem to equation (A19) gives:
\[
\frac{dD}{d\theta} = \frac{(1 - \alpha) E[1_{\nu_2 < \Sigma, \eta_2 < P_1}]}{1 - (1 - \alpha) E[1_{\nu_2 < \Sigma, \eta_2 < P_1}]} > 0, \quad \frac{dD}{d\alpha} = -\frac{E[1_{\nu_2 < \Sigma, \eta_2 < P_1}]}{1 - (1 - \alpha) E[1_{\nu_2 < \Sigma, \eta_2 < P_1}]} \theta D < 0.
\]  

(A20)

Expression (A20) implies:
\[
\frac{d\theta_D}{d\theta} = \alpha D + \theta \frac{d\theta_D}{d\alpha} = \frac{1 - (1 - \alpha) E[1_{\nu_2 < \Sigma, \eta_2 < P_1}]}{1 - (1 - \alpha) E[1_{\nu_2 < \Sigma, \eta_2 < P_1}]} \alpha D > \alpha D > 0,
\]
\[
\frac{d\theta_D}{d\alpha} = \theta D + \theta \frac{d\theta_D}{d\alpha} = \frac{1 - (1 - \alpha) E[1_{\nu_2 < \Sigma, \eta_2 < P_1}]}{1 - (1 - \alpha) E[1_{\nu_2 < \Sigma, \eta_2 < P_1}]} \theta D \in (0, \theta D).
\]  

(A21)

The probability of litigation is \( E[1_{\nu_2 < \Sigma, \eta_2 < P_1}] = \Phi(\max\{M, \frac{1}{\sigma_1} \theta (1 - \alpha) D\}) \), giving:
\[
\frac{d\Phi}{d\theta} \left( \max \left\{ M, \frac{1}{\sigma_1} \theta (1 - \alpha) D \right\} \right) = -1 \frac{1}{\sigma_1} \Phi \left( \frac{1}{\sigma_1} \theta (1 - \alpha) D \right) \frac{1}{\sigma_1} \frac{d\theta_D}{d\theta} \geq 0,
\]
\[
\frac{d\Phi}{d\alpha} \left( \max \left\{ M, \frac{1}{\sigma_1} \theta (1 - \alpha) D \right\} \right) = -1 \frac{1}{\sigma_1} \Phi \left( \frac{1}{\sigma_1} \theta (1 - \alpha) D \right) \frac{1}{\sigma_1} \frac{d\theta_D}{d\alpha} \leq 0.
\]  

(A22)
References


