

September 2011

McCombs Research Paper Series No. IROM-09-10



Durable Products, Time Inconsistency, and Lock-in

Stephen M. Gilbert

McCombs School of Business

The University of Texas at Austin

steve.gilbert@mcombs.utexas.edu

Sree Jonnalagedda

Indian Institute of Management, Bangalore

sreelata@iimb.ernet.in

This paper can also be downloaded without charge from the
Social Science Research Network Electronic Paper Collection:
<http://ssrn.com/abstract=1695691>

Durable Products, Time Inconsistency, and Lock-in

Stephen M. Gilbert* and Sreelata Jonnalagedda†

23 May 2011

Abstract

Many durable products cannot be used without a contingent consumable product, e.g. printers require ink, iPods require songs, razors require blades, etc. For such products, manufacturers may be able to lock-in consumers by making their products incompatible with consumables that are produced by other firms. We examine the effectiveness of such a strategy in the presence of strategic consumers who anticipate the future prices of both the durable product and the contingent consumable. Under a lock-in strategy, the manufacturer has pricing power over the contingent consumable which she can use to extract additional rents from higher valuation consumers, but such pricing power may also reduce consumers' willingness to pay for the durable because it subjects them to being held-up with higher consumables prices in the future. Restricting our attention to linear pricing policies, we find that if the manufacturer can commit to shut down production of her durable after an initial one-time sale, then competition from another consumable of an appropriately degraded level of quality can benefit the manufacturer by mitigating consumers' fears of being held-up. On the other hand, when the manufacturer cannot commit to shutting down production of her durable, then her own output of additional durables gives her an incentive to keep consumables prices low, and competition in the consumables market is less beneficial.

1 Introduction

There are many durable products for which manufacturers are able to charge consumers based on the amount of use that they derive from the product. Typically, this is done by selling contingent products or services. For example, printers do not print without ink cartridges, commercial aircraft do not fly without replacement parts, etc. Even some sophisticated business application software is nearly impossible to use without expensive consulting and maintenance services.

There are many ways in which the strategy of locking consumers into contingent products and services can be implemented. Until very recently, Amazon completely locked-in its Kindle buyers to its own e-books. Similarly, Abbott Labs remains the sole source of test strips for its FreeStyle glucose monitors. On the other hand, there are also examples of firms that, while not completely

*The University of Texas at Austin, McCombs School of Business, steve.gilbert@mcombs.utexas.edu

†Indian Institute of Management, Bangalore, Sreelata@iimb.ernet.in

locking-in the buyers of their durable to their own consumables, control the relative quality of the competitively supplied consumable. For example, an iPod user may not have access to the same features with a song obtained from a source other than iTunes. However, in all of these examples, the durable product is sold and the contingent consumable is priced linearly.

A lock-in strategy is intuitively appealing as it gives a manufacturer an additional degree of freedom in extracting consumer surplus. When consumables are priced linearly, it allows the manufacturer to implement a two-part tariff in which the price of the durable extracts consumer surplus and the price of the consumable guides consumers' choices of the quantity of consumption. Indeed, for the case in which there is a single, one-shot, interaction between the manufacturer and a set of perfectly homogeneous consumers, it is relatively obvious that the firm could set the price of consumables to marginal cost and extract the full surplus through the price of the durable, i.e. relying entirely on the fixed fee portion of the two-part tariff. Interestingly, although heterogeneity among the consumers prevents the full surplus from being extracted through a two-part tariff, as recognized by Oi (1971), it is precisely those situations in which consumers are heterogeneous that a two-part tariff is most preferred over a fixed fee pricing scheme because of the additional degree of freedom that it provides.

The problem with using a lock-in strategy to implement a two-part tariff is that the fixed portion of the tariff is collected only once, when the consumer purchases the durable product, and consumers' willingness to pay for it depends upon their anticipation of how the manufacturer's future actions will affect the prices of both the durable and the consumable. It is well known that consumers' willingness to pay for a traditional durable product is eroded by their anticipation of declining prices as a consequence of how the manufacturer's willingness to sell it to low valuation consumers changes as durables are sold (see Coase (1972), Bulow (1982), etc.). When consumers are locked into the manufacturer's own consumable, their willingness to pay for the durable is additionally eroded by their anticipation of how the manufacturer's incentive to set the consumable price changes as she sells more units of the durable product. For example, consumers of Amazon's reading device Kindle are concerned about the availability of e-books at reasonable prices (WIRED (2009)). This concern is at least partly caused by the rational anticipation that, once the manufacturer sells some units of the durable, she may have decreased motivation to continue to provide contingent consumables at a low price. This is problematic for the manufacturer to the extent that consumers' willingness to pay for the durable is driven by anticipation of the future utility that they will be able to obtain from it.

To better understand the issue, imagine that, after a one time sale of her durable, a manufacturer can commit to shutting down production of it. Although such a commitment would be unequivocally beneficial for a traditional *durable goods manufacturer* (DGM), it can be a double edged sword for one who has locked-in her consumers to her own consumable: Without the ability to extract the full surplus from the marginal consumer through the price of the durable in future periods, such a DGM will have an incentive to raise the price of her consumable. Moreover, strategic consumers' anticipation of higher consumable prices erode the DGM's ability to extract

surplus through the price of the durable. Alternatively, by making her product compatible with a competitively supplied consumable, the competitive pressure increases consumers' willingness to pay for the durable by providing reassurance of a lower future consumable price, but it can also cannibalize the DGM's income from consumables sales. In the extreme where the competitively supplied consumable is a perfect substitute for her own, the DGM must rely entirely upon the price of the durable for her income, effectively giving up the additional degree of freedom that comes with a two-part tariff. However, because the amount of competitive pressure that is generated by the competitively supplied consumable depends upon its relative quality, it may be possible for a DGM to strike a balance between the pricing flexibility of a two-part tariff and consumer's anticipation of higher consumables prices in the future if the competitively supplied consumable is of an appropriately degraded level of quality. We show that there exists an *ideal* level of quality for a competitively supplied consumable that allows the DGM to earn the same profit that she could if she could pre-commit to the future price of her consumable under a policy of locking-in her consumers.

Now consider a DGM who cannot commit to shut down production of her durable after the initial sale. Because her ability to sell additional durables allows her to extract the full surplus of the marginal consumer in every period, it also provides an incentive for her to set a lower consumable price. Consequently, when the DGM can continue to produce durables, she may have less to gain from allowing her consumers to have access to competitively supplied consumables. Further, because of the role that the continued sale of durables plays in mitigating the hold-up problem with respect to consumers, we also show that, a DGM who can lock-in her consumers may be better off if she does not commit to shutting down production of her durable after the first period, which is in sharp contrast with results for traditional DGMs.

The rest of the paper is organized as follows. In section 2 we review the literature. In section 3 we develop a model of how consumers derive decreasing marginal utility for a durable product in each period in which they have access to it. After deriving the optimal non-linear pricing policy for the DGM as a benchmark, we focus on policies in which the contingent consumable is sold at a constant price-per-unit. We first consider the case in which the DGM sells the durable only in the first period, and show that there exists an ideal level of quality for a competitively supplied consumable that benefits the DGM by reassuring consumers about the future consumable price without cannibalizing the DGM's consumables sales. We then extend our analysis to the case in which the DGM may produce durables in each of two periods, so that consumers are concerned not only with the future price of consumables, but also with the DGM's incentive to continue to produce the durable. Finally, we discuss our results and their implications for practice.

2 Literature Review

The strategy that we refer to as *lock-in* is closely related to a practice known as *tying*, in which a monopolist in one product (A) market requires its consumers to purchase from it another product

(B) in which it does not hold monopolistic power. Whinston (1990) was the first to demonstrate specific conditions, i.e. economies of scale or imperfect competition, under which tying can be beneficial by allowing the firm to make a credible commitment to a higher level of output in the contested market. While Whinston's analysis does not make any assumptions about the relationship between the two products, later work has focused more specifically on tying two complementary or contingent products. Carlton and Waldman (2002) investigate how tying in the early stages of a product's life cycle can discourage potential rivals from incurring the fixed costs of entry, while Choi and Stefanadis (2001) obtain similar entry deterrence results when investments in innovation and development are risky.

In contrast to these studies, our examination of lock-in does not assume that there is a one-to-one relationship between the two products. Instead, we focus on a situation in which a consumer's use of a durable (product A) is linearly related to the amount of a contingent consumable (product B) that he / she is able to obtain. As a consequence, when a monopolist in the market for the durable uses lock-in, she effectively implements a two-part tariff in which the price of the durable is the fixed part and the price of the consumable is the variable part. As discussed in Oi (1971), although consumer heterogeneity interferes with the extent to which a two-part tariff can extract the full consumer surplus, a two part tariff is always more effective than simpler schemes that rely exclusively upon either a fixed fee or linear pricing. However, when the fixed fee portion of the two-part tariff is collected through the sale of a durable product that can be used over time, intertemporal dynamics arise, and consumers' willingness to pay for the durable is affected by their anticipation of future prices of both the durable and the consumable.

One of the features of our analysis is our result that shows that competition in the consumable market can benefit a DGM. Several other studies have demonstrated circumstances under which firms can benefit from competition. In studying single product markets, Conner (1995) finds that in the presence of direct network effects, competition from a low-end rival can be beneficial. Sun et al. (2004) extend this analysis and show that strength of network effects plays an important role in determining whether firms should adopt product line extensions, lump sum fee, royalty fee or a free licensing strategy. Another issue that is closely related to the benefits of competition is that of compatibility among the products of different manufacturers of systems of components. Matutes and Regibau (1988) and Economides (1989) recognize that competing firms that manufacture a system of complements choose to make their products compatible to take advantage of the indirect network effects generated from increasing product variety and choice. They show that compatibility reduces price competition since a reduction in the price of one component leads to increased demand for all systems using that component. Farrell and Katz (2000) study a setting that is a bit closer to ours, and show that a monopolist never loses from independent innovation in a complementary market, and when the effects of innovation can be drastic, the monopolist has a general incentive to cooperate with independent suppliers of the complement. In our model, the potential for the DGM to benefit from competition arises entirely from the way that the competition mitigates strategic consumers' anticipation of future exploitation.

Of course, there is a large literature that address the issue of how a firm's incentives to set prices change over time. This issue was first recognized in the context of firms that sell durable products to an anonymous group of consumers by Coase (1972), and was later formalized by Bulow (1982). A number of studies examine the conditions under which anonymous consumers' anticipation of decreasing durables prices over time can be mitigated by leasing instead of selling, e.g. Bucovetsky and Chilton (1986), Desai and Purohit (1999), Desai and Purohit (1998), Bhaskaran and Gilbert (2009), and Bhaskaran and Gilbert (2005). Others examine alternative mitigation strategies when leasing is not possible, such as: under-investing in durability and/or employing an inefficient production technology (Bulow (1986)), using intermediaries (Desai et al. (2004) and (Arya and Mittendorf (2006))). Several other studies study the dynamics between demand information and a firm's pricing policy. For example, Hart and Tirole (1988) show that when there is either a single consumer or consumers are non-anonymous, then a firm may be better off selling than leasing. Villas-Boas (2004) considers a related issue in which consumers enter and leave a market and the firm can charge different prices to new and existing consumers, and finds that such a setting may lead to price cycles. Taking a somewhat different perspective, Braden and Oren (1994) allow for the possibility that, even when consumers are anonymous, the firm may not know the distribution of their valuations, but obtains information about this distribution as it observes sales. They find that, in such a setting, the firm's preference between a penetration and skimming pricing strategy depends upon whether consumers purchase repeatedly.

3 Model Description

Consider a monopolist durable goods manufacturer (DGM) who produces a durable product from which consumers can obtain utility only by using it in combination with a contingent consumable. In each period for which a consumer has access to the durable, his marginal utility is decreasing in both the amount that he uses the durable and in the quality of the contingent consumable. To represent this, we define the marginal utility function for a consumer of type a as follows:

$$U'(a, s, z) = sa - z \quad (3.1)$$

where $s \in [0, 1]$ is the quality of the consumable, and $z \geq 0$ is the quantity of consumable. This assumption of linearly decreasing marginal utility is similar to the one made by Bhaskaran and Gilbert (2005) in their micro-model of the utility that consumers derive from multiple units of a product that is complementary to a durable. By integrating (3.1), we can derive the following utility for a consumer of type a who consumes quantity z of a consumable of quality s :

$$U(a, s, z) = \int_0^z U'(a, s, x) dx = \frac{z(2sa - z)}{2} \quad (3.2)$$

It is worth pointing out that a consumer's type, a , corresponds to her *maximum marginal utility* for a consumable of quality $s = 1$. We take this type to be uniformly distributed between $[1 -$

$\delta, 1 + \delta]$ across a market of mass equal to one, where $0 \leq \delta \leq 1$, so that the density function for consumer type a is $f(a) = \frac{1}{2\delta}$ for $a \in [1 - \delta, 1 + \delta]$. The parameter δ allows us to capture the extent of heterogeneity among consumers, where the limiting cases of $\delta \rightarrow 0$ and $\delta = 1$ represent perfectly homogeneous and highly heterogeneous consumer populations respectively.

The parameter, s , allows us to consider the possibility that consumers may have access to an alternative supply of consumables. We normalize the quality of the *primary* consumable that is produced by the DGM to $s = 1$, and denote by $\beta \in [0, 1]$ the relative quality of the alternative. For example, if $\beta = 1$, then the externally supplied consumable is identical to the DGM's consumable, whereas if $\beta = 0$, then the externally supplied consumable effectively does not exist as an alternative. Thus, $\beta = 0$ can be used to represent either the case in which no externally supplied consumable exists, or the case in which the DGM's durable product is incompatible with externally supplied consumables. Intermediate values of β provide varying amounts of competitive pressure on the price that the DGM can charge for her consumable. To avoid unnecessary complexity, we assume that the market for the externally supplied consumable is perfectly competitive so that it is available at marginal cost.

We assume that consumables are perishable in the sense that consumers cannot purchase units in one period and consume them in the future. This is most easily justified for situations in which the contingent consumables are intangible, e.g. the songs or e-books that a consumer may want to download in the future are yet to be created.

Throughout our analysis we normalize the marginal costs of both the durable and the consumable to zero for the purpose of simplifying exposition. Allowing for positive marginal costs for consumables is straight forward and does not change the fundamental nature of the insights. Allowing for positive marginal costs for durables is similarly straightforward in the case in which the DGM can commit to not producing additional durables after the first period. However, for the case in which she cannot make such a commitment, then positive marginal costs reduce her incentive to produce additional durables beyond the first period, eventually eliminating such incentive altogether. As is common in the durable goods literature, we assume that the performance of the durable does not deteriorate and that there are exactly two periods, denoting by $\rho \leq 1$ the discount factor that is applied to period 2. Note that the assumption of 2 periods is not critical to our results, it is only critical that the number of periods be finite.

Because the durable products that motivate our work are generally not leased to consumers, we do not consider leasing as an option for our DGM. Of course, if the DGM could lease her durable product, then the hold-up problem with respect to the price of consumables would not exist. In other words, the hold-up problem that is the focus of our research is specific to durable products that are sold, rather than leased, and that require the use of a contingent consumable. However, there are many reasons why DGMs cannot lease durable products, including the moral hazard issues that arise when the actions taken by the user of a product are not observable. This may help to explain why leasing is not commonly observed in consumer electronics. In addition, we note that the products that have motivated our research, e.g. iPhone, iPod, Kindle, personal

printers, etc., are overwhelmingly sold rather than leased.

In addition to assuming that the durables are sold, rather than leased, we assume that consumables are sold according to a simple linear pricing mechanism. In practice, there are many obstacles to the use of more sophisticated forms of non-linear pricing, including the difficulty of preventing the resale of “broken” bundles. Moreover, even in situations in which such re-selling might be prevented, e.g. when the consumables are distributed electronically, firms often avoid sophisticated non-linear pricing that is required to implement mechanism design in favor of policies that more closely resemble linear pricing. For example, Apple has steadfastly safeguarded its policy of charging the same price for every song on iTunes. Similarly, while Amazon may charge different prices for individual e-books, it does not attempt to implement any sort of pricing policy in which high volume consumers receive lower per-unit prices. Because such policies are so common in practice, it is of interest to understand the interactions between a firm’s durable product and its contingent consumable when the contingent consumable is sold according to a linear pricing policy, and that is the primary focus of our paper.

3.1 Non-linear pricing for the contingent consumable

As a useful benchmark, let us suppose that if the DGM locks-in her consumers to her own contingent consumable, she can sell the consumable through a continuous menu of quantity and price pairs. In each period, she will determine a quantity and a total (rather than per-unit) price, which we denote by $\{q(a), t(a)\}$, for each consumer type so that a consumer of type a self-selects the quantity and price pair that was designed for him (her). Let Q_t be the number of consumers who hold a durable in period t .¹ Conditional upon Q_t , the DGM can maximize her income from using non-linear pricing on consumables in period t as follows:

$$\pi_t^{NLcons}(\delta, Q_t) = \underset{t(a), q(a)}{\text{Max}} \int_{1+\delta(1-2Q_t)}^{1+\delta} t(a)f(a)da$$

subject to:

$$V'(a) = q(a) \quad (3.3)$$

$$V(a) \geq 0 \quad (3.4)$$

$$q'(a) \geq 0 \quad (3.5)$$

where $V(a) = U(a, 1, q(a)) - t(a)$ is the information rent received by a consumer of type $a \in [1 - \delta, 1 + \delta]$. Constraints (3.3) and (3.4) are the incentive compatibility and individual rationality constraints for a consumer of type a . Constraint (3.5) ensures that the second order conditions are satisfied when each consumer self-selects the quantity and price pair intended for his type. This is a standard non-linear pricing problem, similar to the one originally studied in Maskin and Riley (1984). The only difference is that, because the consumable is of no use to a consumer who does not hold a durable, the DGM can sell consumables only to consumers of type $a \geq 1 + \delta(1 - 2Q_t)$.

¹We assume throughout that an efficient market allocates the durables to the consumers who value them most.

Just as in standard non-linear pricing problems, the price paid by the marginal consumer is exactly equal to his utility, i.e. $t(a) = U(a, 1, q(a))$ for $a = 1 + \delta(1 - 2Q_t)$. Consequently, if M were to attempt to charge consumers an additional fee for access to the durable, then the maximum price at which Q_t units could clear the market in any given period would be zero. Moreover, by limiting the number of consumers who are available to purchase the consumable, she may interfere with her ability to generate consumables income. This can be observed from the fact that $\pi_t^{NLcons}(\delta, Q_t)$ is non-decreasing in Q_t . It follows that if the DGM can use an optimal non-linear pricing policy for her durable, then she should give away her durable, effectively setting $Q_t = 1$ for $t = 1, 2$, and using the same non-linear pricing policy in both periods, i.e. $q_1(a) = q_2(a) = q(a)$ and $t_1(a) = t_2(a) = t(a)$. Under such an approach, let $\pi^{NL}(\delta) = (1 + \rho) \pi_t^{NLcons}(\delta, 1)$ denote her total profit over both periods. Let $Q_t^{NL}(\delta)$ be the total fraction of consumers who receive positive quantities of the consumable in period t .

Lemma 3.1. *The optimal non-linear pricing policy for the consumable can be characterized as follows:*

i) If $\delta \leq \frac{1}{3}$: then $\pi^{NL}(\delta) = (1 + \rho) \frac{3 - \delta(6 - 7\delta)}{6}$, and all consumers get positive quantities of the consumable, i.e. $Q_t^{NL}(\delta) = 1$. For all $a \in [1 - \delta, 1 + \delta]$, we have: $q(a) = 2a - (1 + \delta)$ and $t(a) = \frac{1}{2}(4a(1 + \delta) - 2a^2 - 3(2 - \delta)\delta - 1)$.

ii) If $\delta \geq \frac{1}{3}$: then $\pi_t^{NL}(\delta) = (1 + \rho) \frac{(1 + \delta)^3}{24\delta}$, and some consumers do not get positive quantities. Specifically, $Q_t^{NL}(\delta) = \frac{1 + \delta}{4\delta}$, so that $q(a) = t(a) = 0$ for $a \leq \frac{1 + \delta}{2}$, while for $a \geq \frac{1 + \delta}{2}$, we have: $q(a) = 2a - (1 + \delta)$ and $t(a) = \frac{1}{4}(8a(1 + \delta) - 4a^2 - 3(1 + \delta)^2)$.

The above non-linear consumables pricing policy represents an upper bound on the profits that the DGM can earn from her durable and her consumable.

3.2 Linear pricing of consumables, preliminaries

Let us now turn our attention to situations in which the DGM is restricted to linear pricing policies for the contingent consumable. To see why a linear pricing policy for consumables may lead the DGM to charge consumers for access to her durable, it is useful to highlight why she does not benefit from doing this under a non-linear pricing policy. Recall that one of the characteristics of the optimal non-linear pricing policy for consumables is that it allows the DGM to extract the full surplus from the marginal consumer through the price of the consumable alone in each and every period. Consequently, the DGM can rely exclusively upon the income from consumables sales and need not charge anything for the durable. However, once consumables are available to consumers at a constant price per-unit, the DGM may not be able to extract the full surplus from the marginal consumer without charging consumers a positive price for the use of the durable.

Before examining the implications of linearly priced consumables, let us do some preliminary analysis of how consumers make choices. After accounting for the price (p) of the DGM's consumable, a consumer of type a has *marginal net utilities* of $U'(a, 1, z) - p = a - z - p$ for the DGM's consumable and $U'(a, \beta, z) = \beta a - z$ for the competitively supplied consumable. It follows that a consumer of type a has a larger marginal net utility for the DGM's consumable if and only if

$p < a(1 - \beta)$, which implies that each consumer purchases one type of consumable exclusively, either the one provided by the DGM, or the one provided by the competitive market. For whichever type of consumable a consumer chooses, his total net utility is equal to his total utility, $U(a, s, z)$, less the total amount that he pays to obtain quantity z of the consumable. Thus, he maximizes this net utility by choosing the quantity, z , for which $U'(a, 1, z) = p$, for the DGM's consumable, or $U'(a, \beta, z) = 0$ for the alternative consumable. Defining $z(a, p)$ as the quantity of consumable purchased by a consumer of type a if he holds a durable, we have that:

$$z(a, p) = \begin{cases} \beta a & \text{for } a \leq p/(1 - \beta) \\ a - p & \text{for } a \geq p/(1 - \beta) \end{cases} \quad (3.6)$$

To determine the quantity of consumables that the DGM sells, we need to integrate $z(a, p)$ over the consumer types who both hold the durable and prefer the DGM's consumable to the alternative one. Note that if Q consumers hold the durable, then the marginal consumer will be of type $a_m = 1 + \delta(1 - 2Q)$. The total quantity of consumables sold by the DGM is as follows:

$$y(Q, p) = \begin{cases} \int_{a_m}^{1+\delta} z(a, p) f(a) da = Q(1 - p + \delta(1 - Q)) & \text{for } p \leq (1 - \beta)a_m \\ \int_{\frac{p}{(1-\beta)}}^{1+\delta} z(a, p) f(a) da = \frac{(p - (1-\beta)(1+\delta))(p(1-2\beta) - (1-\beta)(1+\delta))}{4(1-\beta)^2\delta} & \text{otherwise} \end{cases} \quad (3.7)$$

In order to determine the price at which the DGM will be able to sell quantity, Q , of durables, we will use a concept called the *implicit rental price*, which represents the maximum rental fee at which a given quantity of durables could be *rented* to consumers for a single period of use. Let $r(\beta, Q, p)$ be the implicit rental price when Q consumers hold the durable, the price of the DGM's consumable is p , and an alternative consumable of quality β is available at price zero. In the context of our model, the implicit rental price is the total utility that the marginal consumer of the durable will obtain, net of the price of the consumables. Recall that the *type* of the marginal consumer is $a_m = 1 + \delta(1 - 2Q)$. Specifically:

$$r(\beta, Q, p) = \begin{cases} U(a_m, 1, z(a_m, p)) - pz(a_m, p) & \text{for } p \leq a_m/(1 - \beta) \\ U(a_m, \beta, z(a_m, p)) & \text{for } p \geq a_m/(1 - \beta) \end{cases}$$

After substituting the utility function defined in (3.2), and substituting (3.6) for $z(a, p)$, we have:

$$r(\beta, Q, p) = \begin{cases} (1 + \delta(1 - 2Q) - p)^2 / 2 & \text{for } p \leq a_m/(1 - \beta) \\ (1 + \delta(1 - 2Q))^2 \beta^2 / 2 & \text{for } p \geq a_m/(1 - \beta) \end{cases} \quad (3.8)$$

In period 2, the market clearing price for durables is $r(\beta, Q_2, p_2)$, where p_2 is the price of the DGM's consumable in period 2. In period 1, the market clearing price is $r(\beta, Q_1, p_1) + \rho r(\beta, Q_2, p_2)$, where Q_1 and p_1 are the DGM's durable quantity and consumable price in period 1 and Q_2 and

p_2 are the *anticipated* total durable quantity and consumable price in period 2.

With these preliminaries in place, we can now examine the implications of a linear pricing policy for contingent consumables. It is well known that a manufacturer of a traditional durable product can benefit from being able to credibly commit to not reducing her price over time. When the use of a durable product requires a consumable, consumers' willingness to pay for the durable depends upon not only their anticipation of the future price of the durable, but also their anticipation of the future price of the consumable. We will soon see that, with a linearly priced consumable, even if our DGM commits to maintain the price of her durable, consumers' anticipation of higher consumables prices in the future will create a hold-up problem that adversely affects their willingness to pay for the durable.

However, let us first consider the hypothetical case in which our DGM could commit not only to the future price of her durable, but also to the future price of her consumable. In this hypothetical situation, our DGM's profits can be expressed as follows:

$$\begin{aligned}\pi^{cc}(\mathbf{Q}, \mathbf{p}) &= Q_1 (r(\beta, Q_1, p_1) + \rho r(\beta, Q_2, p_2)) + \rho (Q_2 - Q_1) r(\beta, Q_2, p_2) + \sum_{t=1}^2 \rho^{t-1} p_t y(Q_t, p_t) \\ &= \sum_{t=1}^2 \rho^{t-1} (Q_t r(\beta, Q_t, p_t) + p_t y(Q_t, p_t))\end{aligned}\quad (3.9)$$

where, \mathbf{Q} and \mathbf{p} are the two-dimensional quantity and price vectors. In the first line of the above expression, the first term represents the income that M receives from sales of durables in period 1; the second term represents the present value of the income that our DGM receives from additional durable sales in period 2; and the third term represents the present value of income from consumables sales. The latter, simpler expression for $\pi^{cc}(\mathbf{Q}, \mathbf{p})$ is obtained by combining terms. For this situation, the competition in the consumable market only interferes with our DGM's implementing the profit maximizing durables quantities and consumables prices without providing any offsetting benefits. If at all possible, our DGM would prefer to operate without competition, i.e. $\beta = 0$. For this case, her optimal policy can be characterized as follows:

Lemma 3.2. *If the DGM can credibly commit to the quantity of durables and the consumables price, then for the case in which $\beta = 0$, her optimal policy is to set $p_1 = p_2 = p^{cc}$ and $Q_1 = Q_2 = Q^{cc}$ where $p^{cc} = \delta Q^{cc}$ and:*

$$Q^{cc} = \begin{cases} 1 & \text{for } \delta \leq \frac{1}{4} \\ \frac{1+\delta}{5\delta} & \text{for } \delta \geq \frac{1}{4} \end{cases} \quad \pi^{cc} = \begin{cases} (1+\rho) \left(\frac{1}{2} - \delta(1-\delta)\right) & \text{for } \delta \leq \frac{1}{4} \\ (1+\rho) \frac{(1+\delta)^3}{25\delta} & \text{for } \delta \geq \frac{1}{4} \end{cases}$$

When the DGM can pre-commit to both the durable quantity and the consumable price for period 2, she effectively implements an optimal two-part tariff in both periods. Because neither the quantity of durables nor the consumable price change, the implicit rental price is the same in both periods. Consequently, even though consumers do not pay the DGM a fixed fee in period 2, the DGM implicitly extracts this fee through the price of the durable in period 1.

3.3 Sales of durables and consumables with durable sales in period 1 only

Unfortunately, it may not be possible for the DGM to pre-commit to her prices for period 2, and this may adversely affect consumers' willingness to pay for the durable in period 1. Let us first relax only the assumption that the DGM can pre-commit to her consumables price, continuing for the time being to assume that she can commit to not producing any additional durables after period 1. This combination of assumptions could represent a situation at the end of a product's life cycle where consumers want to obtain consumables after the durable has gone out of production. In addition, it allows us to highlight the main trade-off between a lock-in policy versus one that permits consumers to access consumables produced by a competitive market. Subsequently, in Section 3.4, we extend our analysis to the case in which the DGM cannot commit to shutting down production of her durable product after the first period.

For this setting in which the durable can be sold only in period 1, the sequence of events is as follows: Prior to the first period, the DGM determines whether to lock consumers into her own contingent consumable or to make her durable compatible with a contingent consumable of quality β that is provided by a competitive market at marginal cost (zero). This decision is observed by consumers. In period 1, the DGM announces a per-unit price, denoted by p_1 , for her own consumable and simultaneously determines an output quantity, denoted by Q for the durable which she sells at the price at which the marginal consumer is indifferent to purchasing the durable. Equivalently, the manufacturer could announce a price for the durable, and the quantity would be the one at which the marginal consumer is indifferent to purchasing the durable. In period 2, the DGM announces a price for her own consumable, denoted p_2 , that maximizes her profit from selling consumables to the Q consumers who own the durable. Throughout all of our analysis, we assume that either the same set of consumers are present in both periods or that the distribution of consumer types remains the same and there are no transaction costs in the second-hand market for durables. Either of these two assumptions is sufficient to ensure that the durables are allocated to the consumers with the highest valuation for it.

We can now analyze the problem using backward induction. Let $\pi_2(Q, p_2)$ be the DGM's profits in period 2 given that Q consumers hold a durable. Under the assumption that the DGM sells no additional durables, then her profits can be expressed as:

$$\pi_2(Q, p_2) = p_2 y(Q, p_2) \tag{3.10}$$

The above profit function is unimodal in p_2 , and the conditionally optimal price of the consumable in period 2 can be characterized as follows:

Lemma 3.3. *There exists a threshold, $\bar{Q} = \frac{\beta(1+\delta)}{\delta(4\beta-1+\sqrt{1-2\beta+4\beta^2})}$, such that the conditionally optimal con-*

sumable price is:

$$p_2^* = \begin{cases} \text{Min} \left\{ (1 - \beta)(1 + \delta(1 - 2Q)), \frac{1 + \delta(1 - Q)}{2} \right\} & \text{for } Q \leq \bar{Q} \\ \frac{(1 - \beta)(1 + \delta)}{2(1 - \beta) + \sqrt{1 - 2\beta + 4\beta^2}} & \text{for } Q > \bar{Q} \end{cases}$$

such that, when $Q \leq \bar{Q}$, the DGM prices her consumable low enough to attract purchases from all consumers who hold the durable. Otherwise, the DGM's consumables price causes some consumers who hold the durable to prefer to purchase the alternative consumable that is of quality β .

Of course, in equilibrium, the manufacturer never sells more than \bar{Q} durables in the first period, and the marginal consumer at least weakly prefers to buy the DGM's consumable. However, showing the existence of this threshold is important for establishing that the equilibrium is subgame perfect. The existence of the threshold, \bar{Q} can be explained as follows: As the quantity of durables that are in use increases, the maximum marginal utility of the marginal consumer ($a_m = 1 + \delta(1 - 2Q)$) becomes lower and lower, making the DGM less and less willing to price her consumable sufficiently low to attract purchases from this marginal consumer.

Corollary 3.4. \bar{Q} is decreasing in β and in δ , while p_2^* is non-increasing in β . p_2^* is increasing in δ for both $Q < \text{Max} \left\{ \frac{1}{2}, \frac{(2\beta - 1)(1 + \delta)}{(4\beta - 1)\delta} \right\}$ and for $Q > \bar{Q}$. However, if $\text{Max} \left\{ \frac{1}{2}, \frac{(2\beta - 1)(1 + \delta)}{(4\beta - 1)\delta} \right\} < \bar{Q}$, then p_2^* is decreasing in δ for $Q \in \left(\text{Max} \left\{ \frac{1}{2}, \frac{(2\beta - 1)(1 + \delta)}{(4\beta - 1)\delta} \right\}, \bar{Q} \right)$.

As the quality, β , of the alternative consumable increases, it begins to put downward pressure on the price that the DGM offers, but at the same time, it makes the DGM increasingly willing to allow the lowest valuation consumers to purchase the alternative consumable. To understand how the heterogeneity parameter, δ , affects the DGM's pricing of her consumable, it is useful to consider the average value of a , i.e. the maximum marginal utility, among all those consumers who hold the durable. This average maximum marginal utility can be calculated as:

$$\frac{(1 + \delta) + (1 + \delta(1 - 2Q))}{2} = 1 + \delta(1 - Q)$$

This can be interpreted as the average intercept of the potential consumers' individual linear demand functions, and it is clearly increasing in δ for all $Q \in (0, 1)$. Thus, as δ increases, for any $Q < 1$, the average value of a among consumers holding the durable increases, and this pushes the DGM toward a higher consumable price. At the same time, an increase in δ also increases the gap between the maximum marginal utility for the highest type consumer, $(1 + \delta)$, and that of the marginal consumer, $a_m = 1 + \delta(1 - 2Q)$, and consequently the DGM becomes increasingly willing to allow these lowest type consumers to purchase the alternative consumable. The one situation in which p_2^* is increasing in δ is when both $p_2^* = (1 - \beta)a_m$ and $Q > \frac{1}{2}$. The first of these conditions, $p_2^* = (1 - \beta)a_m$, occurs when when the optimal price is just low enough to make the marginal consumer indifferent between the DGM's consumable and the substitute. Note that

it corresponds to the one point at which the DGM's profit is not continuously differentiable, so there can be a range of Q for which the p_2^* corresponds exactly to this point. The second condition, $Q > \frac{1}{2}$, implies that an increase in δ decreases $a_m = 1 + \delta(1 - 2Q)$ and subsequently decreases $p_2^* = (1 - \beta)a_m$. Intuitively, we can think of this in terms of the DGM's reluctance to give up the marginal consumer to the substitute consumable by keeping her price just low enough.

We can now turn our attention to period 1, when the DGM determines both the quantity, Q , of durables, and a price, p_1 , for the consumable. Recall that the market clearing price for the durable in period 1 is $r(\beta, Q_1, p_1) + \rho r(\beta, Q_2, p_2^*)$ where $r(\beta, Q, p)$ is as defined in (3.8), and p_2^* is the consumable price that consumers anticipate that the DGM will set in period 2. The DGM's profit function in period 1 can be expressed as follows:

$$\pi_1(\beta, Q, p_1) = Q(r(\beta, Q, p_1) + \rho r(\beta, Q, p_2^*)) + p_1 y(Q, p_1) + \rho \pi_2(Q, p_2^*) \quad (3.11)$$

Recall from Corollary 3.4 that p_2^* is non-increasing in β . Because the implicit rental price for the durable is decreasing in the price of the consumable, it follows that competition in the consumables market can increase the price at which the DGM can sell a given quantity of the durable in period 1. However, the availability of such an alternative consumable can also interfere with the DGM's ability to use the price of her own consumable as a means of extracting additional surplus from the high valuation consumers. However, as we show below, for any value of δ , there exists an intermediate level of quality for the alternative consumable for which the DGM can achieve the same level of profits as when she can commit to the second period consumable price.

Lemma 3.5. *There exists a level of quality for the alternatively supplied consumable, β^* , such that if $\beta = \beta^*$, then the DGM can achieve the same profits that she could if she could commit to the second period consumables price, where β^* can be characterized as follows:*

$$\beta^* = \begin{cases} (3 - 2\delta) / (3 + 3\delta) & \text{for } \delta \leq 1/4 \\ 2/3 & \text{otherwise} \end{cases}$$

In many cases, the DGM may be able to control the quality of a competitively supplied consumable by making the interface between it and her durable less than seamless. If the quality of the competitively supplied consumable is appropriately degraded, the DGM can always benefit from it when consumers will want access to consumables after the durable goes out of production.

Let us now consider and compare two extreme special cases of the problem, $\beta = 0$, which represents the case in which consumers do not have access to any consumable other than the one supplied by the DGM, and $\beta = 1$, which represents the case in which consumers have access to a perfect substitute for the DGM's consumable. For this special case, consumers effectively have *free access* to consumables of the same quality as those supplied by the DGM. Obviously, this eliminates the DGM's ability to obtain any income from consumables sales, so that she must rely entirely upon the income that she receives from durables. By substituting $\beta = 1$ and $p = 0$ into (3.6), we can see that $z(a, 0) = a$ when $\beta = 1$, so that each consumer who has a durable obtains

an *efficient* quantity of consumables. By substituting into (3.8), we can see that the implicit rental price in each period will be the following function of Q : $r(1, Q, 0) = \frac{1}{2} (1 + \delta(1 - 2Q))^2$. When the DGM sells durables in period 1 only, she earns no income in period 2 since $\beta = 1$ forecloses her sales of consumables, and her total profits are:

$$\pi_1(1, Q, 0) = Q(r(1, Q, 0) + \rho r(1, Q, 0)) = Q \frac{(1 + \rho)}{2} (1 + \delta(1 - 2Q))^2 \quad (3.12)$$

Lemma 3.6. *For the case in which the DGM sells durables in period 1 only and $\beta = 1$, her optimal quantity of durables and corresponding profit will be:*

$$Q^{\beta 1} = \begin{cases} 1 & \text{for } \delta \leq 1/5 \\ \frac{1+\delta}{6\delta} & \text{for } \delta \geq 1/5 \end{cases} \quad \pi_1^{\beta 1} = \begin{cases} \frac{(1+\rho)(1-\delta)^2}{2} & \text{for } \delta \leq 1/5 \\ \frac{(1+\rho)(1+\delta)^3}{27\delta} & \text{for } \delta \geq 1/5 \end{cases}$$

When $\delta \leq 1/5$, consumers are sufficiently homogeneous that the DGM sells the durable to all of them. Only as δ increases above this threshold does the DGM begin to ration the durable to only those consumers who are of sufficiently high types.

For the special case of $\beta = 0$, consumers are locked-in to the DGM's consumable, so that she has complete flexibility with respect to her consumable price. By substituting (3.7) into (3.10) and evaluating at $\beta = 0$, the DGM's second period profit function can be expressed as:

$$\pi_2(Q, p_2) = \begin{cases} p_2 Q (1 - p_2 + \delta(1 - Q)) & \text{for } p_2 \leq a_m \\ p_2 (1 - p_2 + \delta)^2 / 4\delta & \text{otherwise} \end{cases}$$

By applying the results from Lemma 3.3, we have that, when there is no alternative consumable, the conditionally optimal price at which the DGM sells her own consumable in period 2 is:

$$p_2^{\beta 0}(Q) = \begin{cases} (1 + \delta(1 - Q)) / 2 & \text{for } Q \leq (1 + \delta) / 3\delta \\ (1 + \delta) / 3 & \text{otherwise} \end{cases} \quad (3.13)$$

which corresponds to conditionally optimal second period profits of:

$$\pi_2^{\beta 0}(Q) = \begin{cases} Q(1 + \delta(1 - \delta)) / 4 & \text{for } Q \leq (1 + \delta) / 3\delta \\ (1 + \delta)^3 / 27\delta & \text{otherwise} \end{cases} \quad (3.14)$$

When $Q > (1 + \delta) / 3\delta$, the DGM sets the consumable price to be $p_2^* > a_m$, i.e. she prices the consumable above the maximum marginal utility of the marginal consumer, which implies that not all consumers who hold the durable will derive any value from it, and this will drive the implicit rental price for period 2 to zero. Obviously, this does not occur in equilibrium.

In the first period, the DGM determines both p_1 and Q to maximize $\pi_1(\beta, Q, p_1)$, as defined in (3.11), with $\beta = 0$ and $\pi_2(Q, p_2^*) = \pi_2^{\beta 0}(Q)$.

Lemma 3.7. *For the case in which the DGM sells durables in period 1 only and $\beta = 0$, her conditionally optimal consumables price for any Q is $p_1^{\beta 0} = Q\delta$. Her optimal quantity of durables and the corresponding profit can be characterized as follows:*

$$Q^{\beta 0} = \begin{cases} 1 & \text{for } \delta \leq \frac{4+3\rho}{8(2+\rho)} \\ \frac{(1+\delta)(4+3\rho)}{\delta(20+11\rho)} & \text{otherwise} \end{cases} \quad \pi_1^{\beta 0} = \begin{cases} \frac{4+3\rho-4(1-\delta)\delta(2+\rho)}{8} & \text{for } \delta \leq \frac{4+3\rho}{8(2+\rho)} \\ \frac{(1+\delta)^3(2+\rho)(4+3\rho)^2}{2\delta(20+11\rho)^2} & \text{otherwise} \end{cases}$$

Observe that, for $\delta > \frac{4+3\rho}{8(2+\rho)}$, the quantity of durables is larger than it would be in a single period problem, i.e. $\rho = 0$. This is because the DGM is willing to compromise her maximization of income in period 1 in order to increase the number of consumers who will be available to purchase the consumable in period 2. By comparing the results in Lemmas 3.6 and 3.7, we have:

Corollary 3.8. *For any values of the parameters δ and ρ , we have $Q^{\beta 0} \geq Q^{\beta 1}$, i.e. competition from a competitively supplied consumable that is of quality comparable to her own causes the DGM to produce fewer durables than when she faces no such competition.*

This corollary highlights the fact that, a lock-in policy ($\beta = 0$) allows the DGM to implement an unconstrained two-part tariff, whereas a free-access policy ($\beta = 1$) eliminates one of the degrees of freedom, i.e. the linear price of consumables, for the two-part tariff. Because the unconstrained two-part tariff allows the DGM to extract more than just the surplus of the marginal consumer from the high valuation consumers, it makes her more willing to sell to low valuation consumers. If there were only one period, the DGM would prefer the unconstrained two-part tariff that comes with a lock-in policy for all $\delta > 0$. However, when there is more than one period, and the DGM cannot commit to the future price of her consumable, then the lock-in policy also introduces a hold-up issue. We now characterize the conditions under which the DGM prefers to lock-in consumers to her own consumable over allowing them to have free access to a perfect substitute for her own consumable:

Proposition 3.9. *For the case in which the DGM sells durables in period 1 only, there exists a threshold, $\delta^{\beta 01}$, such that for $\delta < \delta^{\beta 01}$, the DGM prefers to allow her consumers to have access to competitively supplied consumables of quality $\beta = 1$ than to lock them in to her own consumable. Conversely, for $\delta > \delta^{\beta 01}$, the DGM prefers to lock consumers in to her own consumable, denying them access to a competitively supplied consumable of equivalent quality. For $\delta = \delta^{\beta 01}$, the DGM is indifferent. The threshold, $\delta^{\beta 01}$, can be expressed as:*

$$\delta^{\beta 01} = \begin{cases} \frac{1}{2} \left(-\rho + \sqrt{\rho(1+\rho)} \right) & \text{for } \rho \leq 4/5 \\ \frac{8(1+\rho)}{52+25\rho} & \text{otherwise} \end{cases} \quad (3.15)$$

In Figure 1, we compare the profits earned by the DGM with $\beta = 0$ to those with $\beta = \beta^*$ and with $\beta = 1$ for values of δ ranging between $(0, 1)$ for the case in which $\rho = 1$. For purposes of comparison, we have also included the total discounted profit that the DGM would earn under the optimal non-linear pricing policy, $\pi^{NL}(\delta)$. It can be seen that when $\delta \rightarrow 0$, and consumers are

homogeneous, the DGM's profits under both $\beta = \beta^*$ and with $\beta = 1$ converge to those under the optimal non-linear pricing policy. For this limiting case, the DGM can extract consumers' full surplus through the price of the durable, but only if she can convince them that they will have future access to consumables at marginal cost. Let us first consider the relationship between $\pi^{\beta 0}$ (lock-in) and $\pi^{\beta 1}$ (free access to consumables). Under lock-in, the DGM has two degrees of freedom in her pricing policy, the price of the durable and the price of the consumable, but she is subjected to the adverse effects of consumers' anticipation of higher future consumable prices. Alternatively, if she provides consumers with free access to a competitively supplied consumable, then she sacrifices a degree of freedom, i.e. the linear portion of the two-part tariff, and must rely entirely on the price of the durable to extract consumer surplus. When consumers are homogeneous, the DGM has little need for the second degree of freedom that comes with a two-part tariff, so it is more important for her to mitigate consumers fears of higher consumables prices by allowing them free access to competitively supplied consumables. However, as consumers become more heterogeneous, she prefers the more refined two-part tariff that comes with a lock-in strategy, even if it means suffering the consequences of rational consumers' anticipation of being subjected to higher future prices. It is worth noting that if, in the second period, we allowed our DGM an option similar to the ones studied in Essegai et al. (2002) and Sundararajan (2004), in which she could offer consumers unlimited consumption of consumables for a fixed fee, then she would prefer this to linear pricing for all $\delta < \frac{1}{2}$, and this provides her with total profit that is identical to $\pi^{\beta 1}$ without her having to make her product compatible with a competitively supplied consumable.²

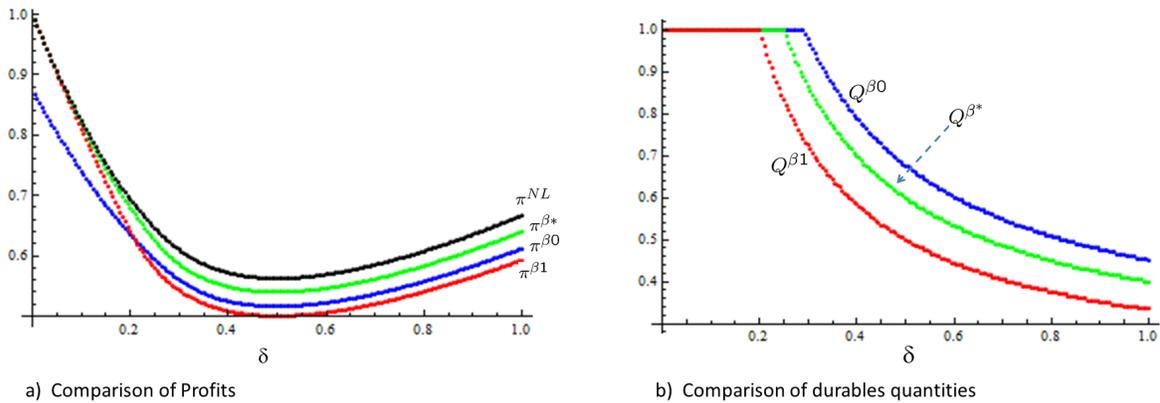


Figure 1: Comparison of profit and durable quantity with $\beta = 0, \beta = \beta^*$, and $\beta = 1$.

However, observe that, across the full range of δ , it is possible for the DGM to earn higher profits with a competitively supplied consumable of intermediate quality, i.e. $\beta = \beta^*$, than with

²We are grateful to an anonymous referee for pointing this out.

either lock-in ($\beta = 0$) or free-access ($\beta = 1$). This is because, the appropriate intermediate level of quality in the alternative consumable puts enough downward pressure on the price that the DGM can set for her own consumable in period 2 to eliminate the hold-up issue, without eliminating the DGM's ability to extract additional surplus from the higher valuation consumers.

Recall that the discount factor that is used in Figure 1 is $\rho = 1$. Because lower values of ρ imply more discounting of future cash flows, as we decrease the value of ρ , it is obvious that all four of the profit functions shown in Figure 1 would decrease. In addition, the point at which $\pi^{\beta 0} = \pi^{\beta 1}$ would shift to the left. This shifting of the indifference point can be verified mathematically by differentiating the upper and lower branches of (3.15) with respect to ρ , but it can also be explained intuitively: As we increase the rate at which future utility is discounted, the hold-up problem is less important relative to the flexibility that lock-in provides to extract different amounts of surplus from different consumer types. In the extreme of $\rho = 0$, i.e. a single period problem, we have $\delta^{\beta 0 1} = 0$ so that the DGM strictly prefers $\beta = 0$ for all $\delta > 0$.

It is also of interest to see how the DGM's output of durables is affected by the quality of the substitute consumable. Plot b) in Figure 1 corroborates Corollary 3.8: As β increases, the DGM has less flexibility to extract rent through the price of her consumable, and she compensates for this by reducing her output of durables.

It is worth pointing out that, although our assumption that the DGM can commit to shutting down production of her durable after period 1 is critical to this analysis, the assumption that there are only two periods is not. Once there are Q durables available to consumers, the DGM's pricing problem with respect to the consumable is the same in every period. Therefore, even if there are an arbitrary number, T , of periods, the DGM will set her consumables price as shown in Lemma 3.3 in each of periods 2, ..., T . Moreover, if we apply a discount factor of ρ_t to each of these periods $t = 2, \dots, T$, then by interpreting the discount factor, ρ , that appears in our analysis of period 1 as $\rho = \sum_{t=2}^T \rho_t$ then all of our results hold for an arbitrary number of periods. Of course, with this interpretation, we could well have $\rho > 1$. For values of $\rho > 1$, the hold-up issue becomes relatively more important, and we would expect that the DGM would prefer to allow her consumers access to a substitute consumable at higher values of δ . Indeed, as previously discussed, it is easy to confirm that the threshold, $\delta^{\beta 0 1}$, from Proposition 3.9 is increasing in ρ .

3.4 M cannot commit to shutdown production of the durable in period 2

So far, we have assumed that the DGM can commit to shut down her production of the durable after the first period. While this has allowed us to highlight the role that competition in the consumable market can play in mitigating the hold-up problem, it ignores the other, more famous, time inconsistency issue in which the DGM has an incentive to sell her durable to lower valuation consumers over time. Let us now relax the assumption that the DGM can commit to shut down production of her durable after the first period. As before, we will continue to assume that there are only two periods.

For this relaxed version of the problem, the sequence of events is exactly as described in Section

3.3, with one exception: In period 2, instead of just determining a price, p_2 , for the consumable, the DGM also determines an additional quantity of durables to make available. As in Sections 3.1 and 3.2, we denote by Q_t the total quantity of durables that are available in period $t = 1, 2$, so that the additional durables produced in period 2 is $Q_2 - Q_1$. Let $\Pi_2(Q_1, Q_2, p_2)$ be the DGM's second period profit given that Q_1 durables were sold in period 1, and that the DGM sells $Q_2 - Q_1$ durables and sets the consumables price to p_2 in period. We have:

$$\Pi_2(Q_1, Q_2, p_2) = (Q_2 - Q_1)r(\beta, Q_2, p_2) + p_2y(Q_2, p_2) \quad (3.16)$$

where we have used uppercase Π to distinguish these profits from the ones where the DGM can commit to shut down production of her durable after period 1. Denote the conditionally optimal solution to this second period problem by (Q_2^{**}, p_2^{**}) . As in Section 3.3, this solution is conditional upon the value of Q_1 . However, in contrast to the expression for $\pi_2(Q, p_2)$ that appears in (3.10), the DGM's second period profit function now includes a term for the income from additional durable sales. In period 1, the DGM's profit function is:

$$\Pi_1(\beta, Q_1, p_1) = Q_1(r(\beta, Q_1, p_1) + \rho r(\beta, Q_2^{**}, p_2^{**})) + p_1y(Q_1, p_1) + \rho\Pi_2(Q_2^{**}, p_2^{**}) \quad (3.17)$$

Because the DGM's problem in period 2 is not jointly concave in p_2 and Q_2 , we have been unable to obtain a closed form solution to this problem. However, we are able to characterize the optimal consumable price in each period conditional upon the durables quantities. In a slight variation in the notation used in Section 3.3, we will denote by $a_m(Q_t) = 1 + \delta(1 - 2Q_t)$ the type of the marginal consumer who holds a durable in period t .

Lemma 3.10. *In period 2, given Q_1 and Q_2 , the conditionally optimal price of the consumable can be characterized as follows: If $\Pi_2(Q_1, Q_2, p_h(Q_1, Q_2)) \geq \Pi_2(Q_1, Q_2, p_l(Q_1, Q_2))$, then $p_2^{co}(Q_1, Q_2) = p_h(Q_1, Q_2)$, and otherwise, $p_2^{co}(Q_1, Q_2) = p_l(Q_1, Q_2)$, where:*

$$p_h(Q_{t-1}, Q_t) = \text{Max} \left\{ \frac{(1 - \beta)(1 + \delta)}{2(1 - \beta) + \sqrt{1 - 2\beta + 4\beta^2}}, (1 - \beta)a_m(Q_t) \right\} \quad (3.18)$$

$$p_l(Q_{t-1}, Q_t) = \text{Min} \left\{ \frac{Q_{t-1} + (Q_{t-1} - 2Q_{t-1}Q_t + Q_t^2)\delta}{Q_{t-1} + Q_t}, (1 - \beta)a_m(Q_t) \right\} \quad (3.19)$$

Because the second period decisions and profit are independent of the consumable price in period 1, the conditionally optimal consumable price, $p_1^{co}(Q_1)$, can be characterized as follows: If $\Pi_2(0, Q_1, p_h(0, Q_1)) \geq \Pi_2(0, Q_1, p_l(0, Q_1))$, then $p_1^{co}(Q_1) = p_h(0, Q_1)$, and otherwise, $p_1^{co}(Q_1) = p_l(0, Q_1)$.

The above Lemma can be explained as follows: In period 2, given Q_1 and Q_2 , the DGM can either price the consumable low enough to attract purchases from the marginal consumer, i.e. $p_2 \leq (1 - \beta)a_m(Q_2)$, or she sets it above this threshold. When she sets the consumable price below this threshold, the marginal consumer purchases the DGM's consumable and the implicit rental price is $r(1, Q_2, p_2)$. Otherwise, if the DGM sets the consumable price above the threshold,

then the marginal consumer prefers to purchase the competitively supplied consumable, and the implicit rental price is $r(\beta, Q_2, 0)$. The first term inside the brackets in each of (3.18) and (3.19) represents the first order condition for each of these two constrained optimization problems, both of which are concave in p_2 . In period 1, we note that while p_1 affects the implicit rental price for period 1, it has no impact upon the implicit rental price or the profits in period 2. Thus, for any Q_1 , the DGM's pricing problem in period 1 is exactly the same as the one that she would face in period 2 if $Q'_1 = 0$ and $Q'_2 = Q_1$.

Although we are unable to provide a complete characterization of the DGM's optimal solution to the problem in which durables can be sold in both periods, we can provide a characterization of the special case in which $\delta = 1$ and either $\beta = 0$ or $\beta = 1$. Note that, when $\delta = 1$, consumers maximum marginal utilities for consumption are $U(0, 2)$, so this case is a re-scaled version of the one used in most of the durable goods literature in which consumers' valuations for the durable are $U(0, 1)$. By examining this special case, we will show how the DGM's ability to sell consumables affects her approach to selling durables. Let us begin with the case in which $\beta = 1$ where the DGM earns profits from the sales of durables only, and faces the problem traditionally associated with durable goods manufacturers.

Lemma 3.11. *For $\delta = 1$ and $\beta = 1$, if the DGM cannot commit to shut down durables output in period 2, then the optimal quantities of durables that she will make available in the first and second periods will be $Q_1^{\beta 1} = \frac{9}{27+8\rho}$ and $Q_2^{\beta 1} = \frac{1}{3} + \frac{6}{27+8\rho}$, and her total profit will be $\Pi_1^{\beta 1} = \frac{8(9+4\rho)^3}{27(27+8\rho)^2}$.*

Now let us turn our attention to the case in which $\beta = 0$ where the DGM is the sole supplier of consumables. For this case, the prices from Lemma 3.10 can be written as:

$$p_h(Q_1, Q_2) = \text{Max} \left\{ \frac{2}{3}, 2(1 - Q_2) \right\} \quad (3.20)$$

$$p_l(Q_1, Q_2) = \text{Min} \left\{ \frac{2Q_1 - 2Q_1Q_2 + Q_2^2}{Q_1 + Q_2}, 2(1 - Q_2) \right\} \quad (3.21)$$

For this special case, p_h represents the conditionally optimal consumable price, given that the marginal consumer holding a durable is indifferent between the DGM's consumable and an alternative of quality $\beta = 0$. Thus, when the DGM charges p_h the price of the durable is driven to zero. It follows that, if the DGM is going to price at p_h , she will sell consumables to exactly $\frac{2}{3}$ consumers, either by freely distributing durables so that $Q_2 = \frac{2}{3}$ if $Q_1 \leq \frac{2}{3}$, or by setting $Q_2 = Q_1$ and ignoring $[Q_1 - \frac{2}{3}]^+$ consumers. (Of course, in equilibrium, we will not have $Q_1 > \frac{2}{3}$.)

Lemma 3.12. *For $\delta = 1$ and $\beta = 0$, if the DGM cannot commit to shut down durables output in period 2:*
a) The conditionally optimal durables quantity and consumables price in period 2 are:

$$Q_2^{\beta 0}(Q_1) = \begin{cases} \frac{1-2Q_1+\sqrt{1+8Q_1(2+3Q_1)}}{5} & \text{for } Q_1 \leq \frac{1}{3} \\ \frac{2}{3} & \text{otherwise} \end{cases} \quad p_2^{\beta 0}(Q_1) = \begin{cases} \frac{2Q_1-2Q_1Q_2+Q_2^2}{Q_1+Q_2} & \text{for } Q_1 \leq \frac{1}{3} \\ \frac{2}{3} & \text{otherwise} \end{cases}$$

b) In period 1, the optimal quantity of durables is $Q_1^{\beta_0} = \frac{2}{5}$ and the corresponding consumables price is $\hat{p}_1^{\beta_0} = Q_1^{\beta_0} = \frac{2}{5}$. This leads to equilibrium second period durables quantity and consumables price of $Q_2^{\beta_0} = \hat{p}_2^{\beta_0} = \frac{2}{3}$. The total profit is: $\Pi_1^{\beta_0} = \frac{8}{25} + \frac{8\rho}{27}$.

Note that we have used the $\hat{\cdot}$ symbol to distinguish the optimal consumables prices in this case from those presented in Section 3.3 where the durable is sold only in period 1. Observe that, in contrast to the case where the DGM sells durables only in period 1, here the quantity that she produces in period 1 is exactly equal to what it would be in a single period problem.

Corollary 3.13. *For the special case of $\delta = 1$, the DGM makes more durables available in both periods and earns higher profits when her consumers do not have access to any competitively supplied consumables ($\beta = 0$) than when perfectly substitutable consumables ($\beta = 1$) are available at marginal cost. Specifically, $Q_1^{\beta_0} > Q_1^{\beta_1}$, $Q_2^{\beta_0} > Q_2^{\beta_1}$, and $\Pi_1^{\beta_0} > \Pi_1^{\beta_1}$.*

Thus, regardless of whether the DGM can commit to not selling durables in period 2, when consumers are very heterogeneous, she is better off locking them in to her own consumable than she would be allowing them to have access to perfectly substitutable consumables at marginal cost. At the other extreme, of perfectly homogeneous consumers ($\delta = 0$), the DGM would want to put durables in the hands of all consumers in period 1, regardless of the availability of alternative consumables, so her total profit would be unaffected by whether or not she can commit to not producing durables after the first period.

Proposition 3.14. *For the special case of $\delta = 1$, when the DGM is the sole supplier of consumables, then for all $\rho > 0$ the ability to produce additional durables in period 2 causes her to produce fewer durables in period 1, but more total durables, i.e., $Q_1^{\beta_0} \leq Q^{\beta_0} < Q_2^{\beta_0}$, and to charge a lower consumable price in both periods, i.e. $\hat{p}_2^{\beta_0} < p_2^{\beta_0}$ and $\hat{p}_1^{\beta_0} < p_1^{\beta_0}$. In addition, the DGM earns higher profits, i.e., $\Pi_1^{\beta_0} > \pi_1^{\beta_0}$.*

Recall that a traditional durable goods manufacturer who sells to a set of heterogeneous consumers would unequivocally benefit from being able to credibly commit to not producing durables after period 1. However, as shown above, this is not necessarily the case when she is the sole supplier of consumables. There are two main reasons for this: First, because her additional durable sales allow her to impose a two-part tariff upon the marginal consumer in period 2, this drives her toward a lower consumable price, and consumers' anticipation of this lower consumable price increases their willingness to pay for the durable in period 1. Second, because the DGM no longer needs to trade-off first period income against the number of consumers who will be available to purchase consumables in period 2, the quantity of durables that she produces in period 1 is now identical to what it would be in a single period problem. Consequently, when the DGM is the sole supplier of consumables, she may no longer benefit from an inability to produce durables after period 1. Of course, this also suggests that she may have less reason to benefit from a competitively supplied consumable of intermediate quality when she can produce durables in both periods, and we will explore this numerically.

Using the structure provided in Lemma 3.10 for the DGM's consumable pricing problem, we can use a search procedure to identify the optimal value of Q_2 conditional upon Q_1 , and then

subsequently search over the possible values for Q_1 . In Figure 2, we show two plots. In the plot a), we show how the DGM's profits vary with consumer heterogeneity (δ) with $\beta = 0$, $\beta = 1$, and $\beta = \beta^*$ (from Lemma 3.5), the same quality parameters that we used in Figure 1. For low values of δ , the DGM's order of preference is the same as when she could commit to shutting down her output of durables in period 2: As $\delta \rightarrow 0$, we have $\Pi^{\beta^*} > \Pi^{\beta^1} > \Pi^{\beta^0}$, while between about $\delta \approx 0.2$ and $\delta \approx 0.38$, the order of preference shifts to $\Pi^{\beta^*} > \Pi^{\beta^0} > \Pi^{\beta^1}$. This shift occurs for the same reason as in the absence of future output: As consumers become increasingly heterogeneous, the DGM benefits less from the commitment to low future consumables prices than from the ability to use a more refined pricing mechanism. However, in contrast to the case in which the DGM could commit to shutting down future durables output, we now see that, for sufficiently large δ (above about $\delta \approx 0.38$), we have $\Pi^{\beta^0} > \Pi^{\beta^*} > \Pi^{\beta^1}$. Moreover, for the larger values of δ , the gap between the DGM's profit with $\beta = 0$ versus with $\beta = 1$ is larger in Figure 2 than in Figure 1. Specifically, $\Pi^{\beta^0} - \Pi^{\beta^1} > \pi^{\beta^0} - \pi^{\beta^1}$ as $\delta \rightarrow 1$. These observations suggest that, as consumers become increasingly heterogeneous, the ability to continue to produce durables makes $\beta = 0$ increasingly attractive relative to either $\beta = 1$ or $\beta = \beta^*$.

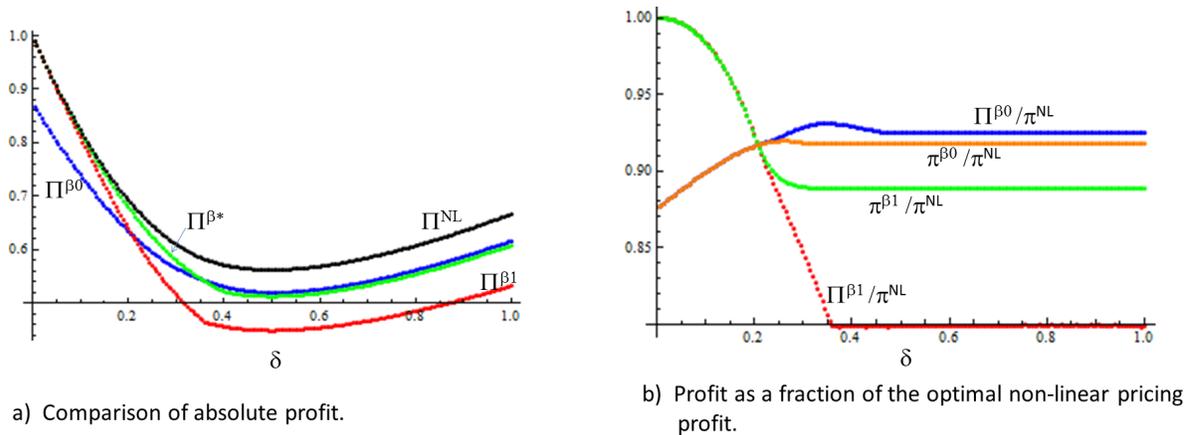


Figure 2: Comparison of profit when the DGM can produce durables in both periods.

This is even more evident from plot b) in Figure 2, where we show the DGM's profits for $\beta = 0$ and $\beta = 1$ as fractions of the profits that she could earn under the optimal non-linear pricing policy. Observe that for low values of δ , the ability to produce durables in period 2 has no consequence because the DGM sells durables to all consumers in period 1. However, for values of $\delta > 0.2$, we can see that the ability to produce durables in period 2 hurts the DGM a lot when $\beta = 1$. Indeed, when $\beta = 1$ and $\delta > 0.4$, the ability to continue to produce durables decreases her profits from about 88% of the non-linear pricing profits to only about 80%. Because $\beta = 1$ implies that consumers have access to consumables at marginal cost, this reduction in profit is due entirely

to the manufacturer's incentive to sell more durables that was recognized by Coase (1972), Bulow (1982), and others. However, when $\beta = 0$, the ability to continue to produce durables has a much different effect. For this case, where she faces no competition from an alternative consumable, she earns a slightly *higher* fraction of the non-linear pricing profits when she can continue durables production. Note that in plot b) of Figure 2, there are two reasons why $\frac{\Pi^{\beta 0}}{\pi^{NL}} < 1$: Not only is the linear policy a cruder mechanism than the non-linear policy, it also creates a hold-up issue related to the DGM's inability to pre-commit to p_2 . However, by comparing $\Pi^{\beta 0}$ to π^{cc} we can identify how much of the profit gap is a consequence of only the inability to pre-commit to p_2 . Indeed for $\rho = \delta = 1$, it can be shown that $\frac{\Pi^{\beta 0}}{\pi^{cc}} = 96\%$, i.e. by locking-in consumers and continuing to produce durables, the DGM can earn 96% of the profit that she could if she could pre-commit to the future quantities and prices.

To see why this is the case, recall from Proposition 3.14 that under a lock-in ($\beta = 0$) policy, for $\delta = 1$, the DGM's ability to sell durables in period 2 causes her to produce fewer durables in the first period, but more total durables as well as to set lower consumables prices. In Figure 3, we show that these results are robust over the full range of $\delta \in (0, 1)$. Recall that $Q^{\beta 0}$ is the optimal durable quantity when the DGM can commit to shut down production in period 2, while Q^{cc} and p^{cc} are the optimal durable quantity and consumable price that the DGM would set for both periods if she could pre-commit to both of them. In the figure, we show durables quantities and consumables prices under lock-in relative to Q^{cc} and p^{cc} .

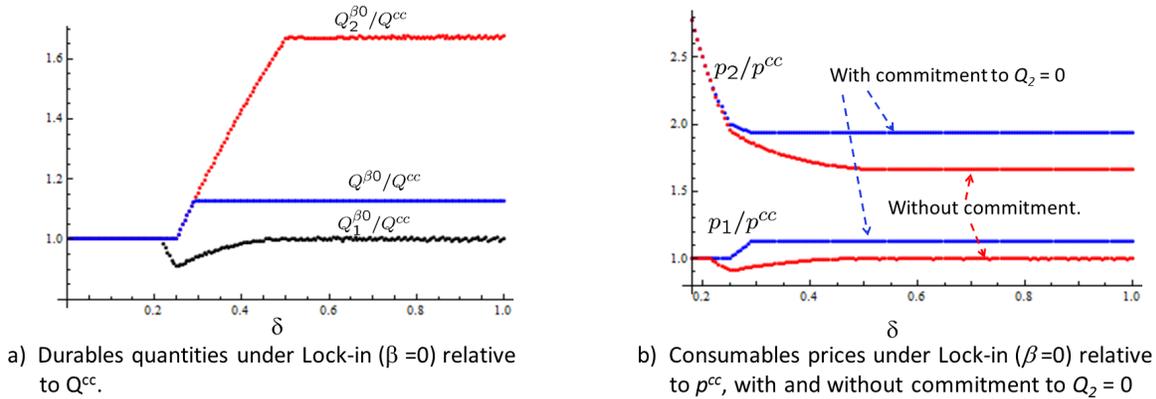


Figure 3: Impact of the ability to produce durables in period 2.

In part a) of Figure 3, we can see that under a lock-in policy, the DGM produces fewer durables in period 1, but more total durables when she cannot commit to shut down durables production, exactly what we would expect from results of traditional durables goods where consumers

implicitly have free access to consumables. However, as can be seen in part b) Figure 3, under a lock-in policy, the ability to produce durables in period 2 also causes her to offer lower consumables prices in both period 1 and in period 2. In period 1, because $Q_1^{\beta_0} \leq Q^{\beta_0}$, there is less heterogeneity among the consumers who hold the durable, and this shifts the DGM's priorities toward increasing the size of the surplus that she can extract from the marginal consumer. By offering a lower p_1 , she increases the size of the surplus that can be extracted through the price of the durable, but gives up some of the additional surplus that she could have extracted from the higher valuation consumers. However, the reduction in the amount of heterogeneity among the consumers holding the durable makes her willing to make this trade-off. In period 2, the DGM's ability to produce and sell more durables endows her with a mechanism for extracting the surplus from the marginal consumer in period 2 that she did not previously have. By lowering the price of her consumable, she can expand the size of this extractable surplus. As a consequence of this effect upon the DGM's incentives for pricing the consumable, the ability to continue to sell durables is less of a problem for her when she can lock her consumers in to a proprietary consumable. In fact, as shown in Figure 2, the DGM's ability to produce durables in period 2 actually increases her profit when she monopolizes the consumables market with $\beta = 0$, which contrasts sharply with existing results for durable goods manufacturers that are based on an implicit assumption that $\beta = 1$, i.e. consumers have free access to consumables.

4 Summary and Discussion

We have examined the choice that many manufacturers of durable products face when their products require a contingent consumable. Such a manufacturer can often lock-in her consumers so that they cannot use the durable without a proprietary consumable. Alternatively, she may be able to influence the extent to which her consumers can access competitively provided consumables. In order to examine the question of how and when such a firm should facilitate its consumers' use of competitively supplied consumables, we have developed a micro model consumer utility in which, in each period, consumers derive decreasing marginal utility from each use of the durable.

As we show, if the manufacturer could implement an optimal non-linear pricing policy for the consumable in each time period, then this would extract the maximum amount of surplus in each period from self-selecting consumers. Moreover, because such an optimal non-linear pricing policy would extract the full surplus from the marginal consumer in each period, the manufacturer have no reason to alter her menu of consumables prices over time, nor would she have any reason to use the durable as a means of extracting the consumer surplus. However, while such an optimal non-linear pricing policy provides a useful upper bound on the profits that our manufacturer can earn, there are many reasons why firms cannot or do not use non-linear pricing policies in practice, and there are many examples of firms that sell consumables according to a linear pricing policy (e.g. iTunes, Amazon e-books, etc.). For this reason, most of our attention is focused on the use of linear pricing policies for consumables.

To highlight how the manufacturer's incentives with respect to the price of the consumable change after she sells some durables, we first consider a situation in which the manufacturer can commit to shutting down production of her durable after the first period. Such a commitment would be unequivocally beneficial for a traditional DGM. However, when consumers are locked-in, their willingness to pay for the durable will be eroded by their anticipation of the DGM's incentive to raise the price of the consumable in the future. Alternatively, by making her product compatible with a competitively supplied consumable, the DGM subjects herself to competitive pressure that reassures consumers about the future consumable price and increases their willingness to pay for the durable. We show that there exists an ideal level of quality for a competitively supplied consumable that allows the DGM to earn the same profit that she could if she could pre-commit to the future price of her consumable under a policy of locking-in her consumers.

We then explore the situation in which the DGM cannot commit to shut down production of her durable after the first period. Because she is able to extract the full surplus of the marginal consumer in any period in which she sells a positive quantity of durables, these continued durable sales put downward pressure on the consumable price. Moreover, because strategic consumers anticipate these lower consumables prices, they have greater willingness to pay for the durable in the first period. Consequently, when the DGM can continue to produce durables, she may have less to gain from allowing her consumers to have access to competitively supplied consumables. Further, because of the role that the continued sale of durables plays in mitigating the hold-up problem with respect to consumers, we also show that, a DGM who can lock-in her consumers may be better off if she does not commit to shutting down production of her durable after the first period, which is in sharp contrast with results for traditional DGMs.

One of the clear implications of our analysis is that allowing consumers to access a competitively supplied consumable is most beneficial when the DGM either cannot or will not be producing additional durables in the future. This suggests that lock-in may be a dominant strategy early in a product's life-cycle, before the product has achieved a critical level of market penetration. In this early stage of the product-life-cycle, the effect of additional durable sales upon the DGM's incentives to offer low future consumables prices is strongest, and it alone may be sufficient to mitigate consumers fears of being held-up. However, as the market penetration increases, this effect weakens until it is eventually eliminated when the DGM shuts down production of the durable completely. Consequently, the benefit of allowing consumers to access a competitively supplied consumable should become greater as the product moves to more advanced stages of the product-life-cycle. This may help to explain why it is not uncommon for manufacturers to lock-in consumers early in the product-life-cycle, but then shift toward facilitating access to alternative consumables later on.

References

- Arya, A., B. Mittendorf. 2006. Benefits of channel discord in the sale of durable goods. *Marketing Science* **25** 91–96.
- Bhaskaran, S.R., S. Gilbert. 2005. Selling and leasing strategies for durable goods with complementary products. *Management Science* **51**(8) 1278–1290.
- Bhaskaran, S.R., S. Gilbert. 2009. Implications of channel structure for leasing or selling durable goods. *Marketing Science* **28**(5) 918–934.
- Braden, J. B., S. S. Oren. 1994. Nonlinear pricing to produce information. *Marketing Science* **13**(3) 310–326.
- Bucovetsky, S., J. Chilton. 1986. Concurrent renting and selling in a durable goods monopoly under threat of entry. *Rand Journal of Economics* **17** 261–278.
- Bulow, J. 1982. Durable goods monopolists. *Journal of Political Economy* **90**(2) 314–332.
- Bulow, J. 1986. An Economic Theory of Planned Obsolescence. *Quarterly Journal of Economics* **51** 729–750.
- Carlton, D., M. Waldman. 2002. The strategic use of tying to preserve and create market power in evolving industries. *The RAND Journal of Economics* **33** 194–220.
- Choi, J.P., C. Stefanadis. 2001. Tying, investment, and the dynamic leverage theory. *The RAND Journal of Economics* **32** 52–71.
- Coase, R. 1972. Durability and monopoly. *Journal of Law and Economics* **15** 143–149.
- Conner, K.R. 1995. Obtaining strategic advantage from being imitated: When can encouraging-clones pay? *Management Science* **41**(2) 209–225.
- Desai, P., O. Koenigsberg, D. Purohit. 2004. Strategic Decentralization and Channel Coordination. *Quantitative Marketing and Economics* **2**(1) 5–22.
- Desai, P., D. Purohit. 1998. Leasing and selling: Optimal marketing strategies for a durable goods firm. *Management Science* **44**(11) S19–S34.
- Desai, P., D. Purohit. 1999. Competition in durable goods markets: The strategic consequences of leasing and selling. *Marketing Science* **18**(1) 42–58.
- Economides, N. 1989. Desirability of compatibility in the absence of network externalities. *American Economic Review* **79**(5) 1165–1181.
- Essegaier, S., S. Gupta, Z. J. Zhang. 2002. Pricing access services. *Marketing Science* **21**(2) 139–159.

- Farrell, J., M. L. Katz. 2000. Innovation, rent extraction and integration in systems markets. *The Journal of Industrial Economics* **48** 413–432.
- Hart, O., J. Tirole. 1988. Contract renegotiation and Coasian dynamics. *Review of Economic Studies* **55** 509–540.
- Maskin, E., J. Riley. 1984. Monopoly with incomplete information. *RAND Journal of Economics* **15(2)**.
- Matutues, C., P. Regibau. 1988. "Mix and Match": Product compatibility without network externalities. *Rand Journal of Economics* **19(2)**.
- Oi, W. Y. 1971. A Disneyland dilemma: Two-part tariffs for a Mickey Mouse monopoly. *The Quarterly Journal of Economics* **85(1)** 77–96.
- Sun, B., J. Xie, H. Cao. 2004. Product strategy for innovators in markets with network effects. *Marketing Science* **23** 243–254.
- Sundararajan, A. 2004. Nonlinear pricing of information goods. *Management Science* **50(12)** 1660–1673.
- Villas-Boas, J. M. 2004. Price cycles in markets with consumer recognition. *RAND Journal of Economics* **35(3)** 486–501.
- Whinston, M. 1990. Tying, foreclosure, and exclusion. *The American Economic Review* **80(6)** 837–859.
- WIRED. 2009. Kindle readers ignite protest over e-book prices. URL <http://www.wired.com/gadgetlab/2009/04/kindle-readers/>.