Long-Run Risk Model in Detail: Technical Appendix

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This appendix provides a complete derivation of our modified long-run risk model in section A. It calibrates the model in Section B and discusses the results in Section C.

A The Model

A.1 Setup

Preferences The long-run risk literature works off the class of preferences due to Kreps and Porteus (1978), Epstein and Zin (1989), and Duffie and Epstein (1992). These preferences impute a concern for the timing of the resolution of uncertainty. A first parameter α governs risk aversion and a second parameter ρ governs the willingness to substitute consumption inter-temporally. In particular, ρ is the inverse of the inter-temporal elasticity of substitution (EIS). Bansal and Yaron (2004) show that the stochastic discount factor can be written as a function of consumption growth and the return to the wealth portfolio:

$$M_{t+1} = \beta^{\frac{1-\alpha}{1-\rho}} \left(\frac{C_{t+1}}{C_t} \right)^{-\rho \frac{1-\alpha}{1-\rho}} \left(R^c_{t+1} \right)^{\frac{\rho-\alpha}{1-\rho}}. \tag{1}$$

The return on a claim to aggregate consumption, the *total wealth return*, can be written as

$$R^c_{t+1} = \frac{W_{t+1}}{W_t - C_t} = \frac{C_{t+1}}{C_t} \frac{WC_{t+1}}{WC_t - 1}.$$

We start by using the Campbell (1991) approximation of the log total wealth return $r^c_t = \log(R^c_t)$ around the long-run average log wealth-consumption ratio $A^c_0 \equiv E[w_t - c_t]$.[1]

$$r^c_{t+1} = \kappa^c_0 + \Delta c_{t+1} + wc_{t+1} - \kappa^c_1 wc_t, \tag{2}$$

where we define the log *wealth-consumption* ratio wc as

$$wc_t \equiv \log \left(\frac{W_t}{C_t} \right) = w_t - c_t.$$

The linearization constants κ^c_0 and κ^c_1 are non-linear functions of the unconditional mean wealth-consumption ratio A^c_0:

$$\kappa^c_1 = \frac{e^{A^c_0}}{e^{A^c_0} - 1} > 1 \quad \text{and} \quad \kappa^c_0 = -\log \left(e^{A^c_0} - 1 \right) + \frac{e^{A^c_0}}{e^{A^c_0} - 1} A^c_0. \tag{3}$$

[1] Throughout, variables with a subscript zero denote unconditional averages.

Using the definition of the total wealth return, one can then show that the log SDF becomes:

$$m_{t+1} = \frac{1-\alpha}{1-\rho}\left[\log\beta + \kappa_0^c\right] - \kappa_0^c - \alpha\Delta c_{t+1} - \frac{\alpha-\rho}{1-\rho}\left(wc_{t+1} - \kappa_1^c wc_t\right) \tag{4}$$

Technology We mostly adopt the consumption growth specification of Bansal and Yaron (2004):

$$\begin{aligned}
\Delta c_{t+1} &= \mu_c + x_t + \sigma_t \eta_{t+1}, &(5)\\
x_{t+1} &= \rho_x x_t + \varphi_e \sigma_t e_{t+1}, &(6)\\
\sigma_{t+1}^2 &= \bar{\sigma}^2 + \nu_1(\sigma_t^2 - \bar{\sigma}^2) + \sigma_w \sigma_t w_{t+1}, &(7)
\end{aligned}$$

where (η_t, e_t, w_t) are i.i.d. standard normal innovations. Consumption growth contains a low-frequency component x_t and is heteroscedastic, with conditional variance σ_t^2. The two state variables x_t and $\sigma_t^2 - \bar{\sigma}^2$ capture time-varying growth rates and time-varying economic uncertainty.
The two changes we make relative to Bansal and Yaron (2004) are that (1) the innovation in equation (7) is not $\sigma_w w_{t+1}$ but $\sigma_w \sigma_t w_{t+1}$, and (2) we assume that x_t and to $\sigma_t^2 - \bar{\sigma}^2$ have a non-zero conditional correlation χ. We engineer this through a non-zero unconditional correlation χ between the contemporaneous shocks e_{t+1} and w_{t+1}:

$$Corr[e_{t+1}, w_{t+1}] = Cov[e_{t+1}, w_{t+1}] = E[e_{t+1} w_{t+1}] = \chi.$$

All other shocks have zero correlation with each other at all leads and lags. To find the correlation between x_t and to $\sigma_t^2 - \bar{\sigma}^2$, start by working out

$$\begin{aligned}
E_t[x_{t+1}(\sigma_{t+1}^2 - \bar{\sigma}^2)] &= E_t[(\rho_x x_t + \varphi_e \sigma_t e_{t+1})(\nu_1(\sigma_t^2 - \bar{\sigma}^2) + \sigma_w \sigma_t w_{t+1})],\\
&= \rho_x \nu_1 x_t (\sigma_t^2 - \bar{\sigma}^2) + \rho_x x_t \sigma_w \sigma_t E_t[w_{t+1}],\\
&\quad + \varphi_e \sigma_t \nu_1 (\sigma_t^2 - \bar{\sigma}^2) E_t[e_{t+1}] + \varphi_e \sigma_t^2 \sigma_w E_t[e_{t+1} w_{t+1}],\\
&\quad = \rho_x \nu_1 x_t (\sigma_t^2 - \bar{\sigma}^2) + \varphi_e \sigma_w \chi \sigma_t^2.
\end{aligned}$$

The conditional covariance between x_{t+1} and $\sigma_t^2 - \bar{\sigma}^2$ equals:

$$\begin{aligned}
Cov_t[x_{t+1}, \sigma_{t+1}^2 - \bar{\sigma}^2] &= E_t[x_{t+1}(\sigma_{t+1}^2 - \bar{\sigma}^2)] - E_t[x_{t+1}] E_t[(\sigma_{t+1}^2 - \bar{\sigma}^2)],\\
&= \rho_x \nu_1 x_t (\sigma_t^2 - \bar{\sigma}^2) + \varphi_e \sigma_w \chi \sigma_t^2 - \rho_x \nu_1 x_t (\sigma_t^2 - \bar{\sigma}^2),\\
&= \varphi_e \sigma_w \chi \sigma_t^2.
\end{aligned}$$

It follows that the conditional correlation between x_{t+1} and $\sigma_t^2 - \bar{\sigma}^2$ is also χ:

$$\begin{aligned}
Corr_t[x_{t+1}, \sigma_{t+1}^2 - \bar{\sigma}^2] &= \frac{Cov_t[x_{t+1}, \sigma_{t+1}^2 - \bar{\sigma}^2]}{Std_t[x_{t+1}] Std_t[\sigma_{t+1}^2 - \bar{\sigma}^2]},\\
&= \frac{\chi \varphi_e \sigma_w \sigma_t^2}{\varphi_e \sigma_w \sigma_t^2} = \chi.
\end{aligned}$$

The *unconditional* variance of x_{t+1} and $\sigma_t^2 - \bar{\sigma}^2$ is:

$$\begin{aligned}
V[x_{t+1}] &= E[x_{t+1}^2] = \frac{\varphi_e^2 \bar{\sigma}^2}{1 - \rho_x^2},\\
V[\sigma_{t+1}^2 - \bar{\sigma}^2] &= E[(\sigma_{t+1}^2 - \bar{\sigma}^2)^2] = \frac{\sigma_w^2 \bar{\sigma}^2}{1 - \nu_1^2}.
\end{aligned}$$

The *unconditional* covariance between x_{t+1} and $\sigma_t^2 - \bar{\sigma}^2$ equals:

$$Cov[x_{t+1}, \sigma_{t+1}^2 - \bar{\sigma}^2] = \frac{\chi \varphi_e \sigma_w \bar{\sigma}^2}{1 - \rho_x \nu_1}$$

The *unconditional* correlation between x_{t+1} and $\sigma_t^2 - \bar{\sigma}^2$ equals:

$$\begin{aligned} Corr[x_{t+1}, \sigma_{t+1}^2 - \bar{\sigma}^2] &= \frac{Cov[x_{t+1}, \sigma_{t+1}^2 - \bar{\sigma}^2]}{Std[x_{t+1}] Std[\sigma_{t+1}^2 - \bar{\sigma}^2]}, \\ &= \frac{\frac{\chi \varphi_e \sigma_w \bar{\sigma}^2}{1 - \rho_x \nu_1}}{\frac{\varphi_e \bar{\sigma}}{\sqrt{1-\rho_x^2}} \frac{\sigma_w \bar{\sigma}}{\sqrt{1-\nu_1^2}}} = \chi \frac{\sqrt{1-\rho_x^2}\sqrt{1-\nu_1^2}}{1 - \rho_x \nu_1}. \end{aligned}$$

The unconditional correlation between these two state variables is zero at all other leads and lags.

Dividend Growth We specify dividend growth processes for various assets or portfolios i, making two important modifications to the specification in Bansal and Yaron (2004):

$$\Delta d_{t+1}^i = \mu_d^i + \phi_{d,x}^i x_t + \phi_{d,\sigma}^i (\sigma_t^2 - \bar{\sigma}^2) + \varphi_d^i \sigma_t u_{t+1}^i \tag{8}$$

In Bansal and Yaron (2004), the shock u_t is assumed orthogonal to (η, e, w) and $\phi_{d,\sigma}^i = 0$. Then, correlation between consumption and dividend growth comes through the two state variables x_t and σ_t. Instead, we allow for a correlation ι^i between η_{t+1} and u_{t+1}^i. All other correlations are kept at zero. This opens up an additional channel of correlation and is a modification also made by (Bansal and Shaliastovich 2007). It turns out to be immaterial for our purposes. Much more important is the non-zero loading of dividend growth on economic uncertainty $\phi_{d,\sigma}^i \neq 0$. Its role will become clear below.
Defining returns ex-dividend and using the Campbell (1991) linearization, the log return on a claim to the dividend of portfolio i can be written as:

$$r_{t+1}^i = \Delta d_{t+1}^i + pd_{t+1}^i + \kappa_0^i - \kappa_1^i pd_t^i,$$

with coefficients

$$\kappa_1^i = \frac{e^{A_0^i}}{e^{A_0^i} - 1} > 1, \quad \text{and} \quad \kappa_0^i = -\log\left(e^{A_0^i} - 1\right) + \frac{e^{A_0^i}}{e^{A_0^i} - 1} A_0^i,$$

which depend on the long-run log price-dividend ratio A_0^i.

Inflation We follow Wachter (2006), Piazzesi and Schneider (2006), and Bansal and Shaliastovich (2007), and model inflation as

$$\pi_{t+1} = \bar{\pi}_t + \varphi_{\pi,\eta} \sigma_t \eta_{t+1} + \varphi_{\pi,e} \varphi_e \sigma_t e_{t+1} + \sigma_\pi \xi_{t+1}, \tag{9}$$

where expected inflation $\bar{\pi}_t = E_t[\pi_{t+1}]$ is given by

$$\bar{\pi}_{t+1} = \mu_\pi + \rho_\pi (\bar{\pi}_t - \mu_\pi) + \phi_\pi x_t + \zeta_{\pi,\eta} \sigma_t \eta_{t+1} + \zeta_{\pi,e} \varphi_e \sigma_t e_{t+1} + \sigma_z \xi_{t+1}, \tag{10}$$

The innovation ξ_t is orthogonal to all other innovations $(\eta_t, e_t, w_t, u_t^i)$. Expected inflation mean-reverts, it carries long-run risk with loading ϕ_π, and its innovations are correlated with unexpected inflation, and with the temporary and persistent component of consumption growth.
The nominal SDF is given by:

$$m_{t+1}^{\$} = m_{t+1} - \pi_{t+1}.$$

Expected inflation (demeaned), $\bar{\pi}_t - \mu_\pi$, is a third state variable that drives the nominal yield curve. We are interested in understanding its variance and covariance with the other two state variables x_{t+1} and $\sigma_t^2 - \bar{\sigma}^2$. First, we calculate

$$\begin{aligned} Cov[x_{t+1}, \bar{\pi}_{t+1} - \mu_\pi] &= \rho_x \rho_\pi Cov[x_t, \bar{\pi}_t - \mu_\pi] + \rho_x \phi_\pi E[x_t^2] + \zeta_{\pi,e} \varphi_e^2 \bar{\sigma}^2, \\ &= \frac{\frac{\rho_x \phi_\pi}{1-\rho_x^2} + \zeta_{\pi,e}}{1 - \rho_x \rho_\pi} \varphi_e^2 \bar{\sigma}^2. \end{aligned}$$

The covariance with economic uncertainty is

$$\begin{aligned} Cov[\sigma_{t+1}^2 - \bar{\sigma}^2, \bar{\pi}_{t+1} - \mu_\pi] &= \nu_1 \rho_\pi Cov[\sigma_t^2 - \bar{\sigma}^2, \bar{\pi}_t - \mu_\pi] + \nu_1 \phi_\pi Cov[x_t, \sigma_t^2 - \bar{\sigma}^2] + \chi \zeta_{\pi,e} \varphi_e \sigma_w \bar{\sigma}^2, \\ &= \chi \frac{\frac{\nu_1 \phi_\pi}{1-\rho_x \nu_1} + \zeta_{\pi,e}}{1 - \nu_1 \rho_\pi} \varphi_e \sigma_w \bar{\sigma}^2. \end{aligned}$$

We can now calculate the unconditional variance of expected inflation

$$\begin{aligned} E[(\bar{\pi}_{t+1} - \mu_\pi)^2] &= \rho_\pi^2 E[(\bar{\pi}_t - \mu_\pi)^2] + \phi_\pi^2 E[x_{t+1}^2] + \zeta_{\pi,\eta}^2 \bar{\sigma}^2 + \zeta_{\pi,e}^2 \varphi_e^2 \bar{\sigma}^2 + \sigma_z^2 + 2\phi_\pi \rho_\pi E[x_t, \bar{\pi}_t - \mu_\pi], \\ &= \frac{\zeta_{\pi,\eta}^2 \bar{\sigma}^2 + \left[\frac{\phi_\pi^2}{1-\rho_x^2} + \zeta_{\pi,e}^2 + 2\phi_\pi \rho_\pi \frac{\frac{\rho_x \phi_\pi}{1-\rho_x^2} + \zeta_{\pi,e}}{1-\rho_x \rho_\pi} \right] \varphi_e^2 \bar{\sigma}^2 + \sigma_z^2}{1 - \rho_\pi^2}. \end{aligned}$$

A.2 Equity pricing

Euler Equation The starting point of the analysis is the Euler equation $E_t[M_{t+1} R_{t+1}^i] = 1$, where R_{t+1}^i denotes a gross return between dates t and $t+1$ on some asset i and M_{t+1} is the SDF. In logs:

$$E_t[m_{t+1}] + E_t\left[r_{t+1}^i\right] + \frac{1}{2} Var_t[m_{t+1}] + \frac{1}{2} Var_t\left[r_{t+1}^i\right] + Cov_t\left[m_{t+1}, r_{t+1}^i\right] = 0. \tag{11}$$

The same equation holds for the real risk-free rate $y_t(1)$, so that

$$y_t(1) = -E_t[m_{t+1}] - \frac{1}{2} Var_t[m_{t+1}]. \tag{12}$$

The expected excess return becomes:

$$E_t\left[r_{t+1}^{e,i}\right] = E_t\left[r_{t+1}^i - y_t(1)\right] + \frac{1}{2} Var_t\left[r_{t+1}^i\right] = -Cov_t\left[m_{t+1}, r_{t+1}^i\right] = -Cov_t\left[m_{t+1}, r_{t+1}^{e,i}\right], \tag{13}$$

where $r_{t+1}^{e,i}$ denotes the excess return on asset i corrected for the Jensen term.

The Consumption Claim In what follows we focus on the return on a claim to aggregate consumption, denoted r^c, where

$$r^c_{t+1} = \kappa^c_0 + \Delta c_{t+1} + wc_{t+1} - \kappa^c_1 wc_t,$$

and derive the five terms in equation (11) for this asset.

Taking logs on both sides of the non-linear SDF expression in equation (1) delivers an expression of the log SDF as a function of log consumption changes and the log total wealth return

$$m_{t+1} = \frac{1-\alpha}{1-\rho}\log\beta - \frac{1-\alpha}{1-\rho}\rho\Delta c_{t+1} + \left(\frac{1-\alpha}{1-\rho} - 1\right) r^c_{t+1}. \tag{14}$$

We conjecture that the log wealth-consumption ratio is linear in the two states x_t and $\sigma^2_t - \bar{\sigma}^2$,

$$wc_t = A^c_0 + A^c_1 x_t + A^c_2\left(\sigma^2_t - \bar{\sigma}^2\right).$$

As BY, we assume joint conditional normality of consumption growth, x, and the variance of consumption growth. We verify this conjecture from the Euler equation (11).

Using the conjecture for the wealth-consumption ratio, we compute innovations in the total wealth return, and its conditional mean and variance:

$$\begin{aligned}
r^c_{t+1} - E_t\left[r^c_{t+1}\right] &= \sigma_t\eta_{t+1} + A^c_1\varphi_e\sigma_t e_{t+1} + A^c_2\sigma_w\sigma_t w_{t+1} \\
E_t\left[r^c_{t+1}\right] &= r_0 + \left(1 - (\kappa^c_1 - \rho_x)A^c_1\right)x_t - A^c_2(\kappa^c_1 - \nu_1)\left(\sigma^2_t - \bar{\sigma}^2\right) \\
V_t\left[r^c_{t+1}\right] &= \left[1 + (A^c_1\varphi_e)^2 + (A^c_2)^2\sigma^2_w + 2A^c_1\varphi_e A^c_2\sigma_w\chi\right]\sigma^2_t \\
r_0 &= \kappa^c_0 + A^c_0(1-\kappa^c_1) + \mu_c
\end{aligned}$$

Substituting in the expression for the log total wealth return r^c into the log SDF, we compute innovations, and the conditional mean and variance of the log SDF:

$$\begin{aligned}
m_{t+1} - E_t\left[m_{t+1}\right] &= -\alpha\sigma_t\eta_{t+1} - \frac{\alpha-\rho}{1-\rho}A^c_1\varphi_e\sigma_t e_{t+1} - \frac{\alpha-\rho}{1-\rho}A^c_2\sigma_w\sigma_t w_{t+1}, \\
E_t\left[m_{t+1}\right] &= m_0 + \left(-\alpha + \frac{\alpha-\rho}{1-\rho}A^c_1(\kappa^c_1 - \rho_x)\right)x_t + \frac{\alpha-\rho}{1-\rho}(\kappa^c_1 - \nu_1)A^c_2\left(\sigma^2_t - \bar{\sigma}^2\right) \\
V_t\left[m_{t+1}\right] &= \left[\alpha^2 + \left(\frac{\alpha-\rho}{1-\rho}\right)^2\left\{(A^c_1\varphi_e)^2 + (A^c_2\sigma_w)^2 + 2\chi A^c_1\varphi_e A^c_2\sigma_w\right\}\right]\sigma^2_t \\
m_0 &= \frac{1-\alpha}{1-\rho}\log\beta - \frac{\alpha-\rho}{1-\rho}\left[\kappa^c_0 + A^c_0(1-\kappa^c_1)\right] - \alpha\mu_c
\end{aligned} \tag{15}$$

The conditional covariance between the log consumption return and the log SDF is given by the conditional expectation of the product of their innovations

$$\begin{aligned}
Cov_t\left[r^c_{t+1}, m_{t+1}\right] &= E_t\left[r^c_{t+1} - E_t\left[r^c_{t+1}\right], m_{t+1} - E_t\left[m_{t+1}\right]\right] \\
&= \left[-\alpha - \frac{\alpha-\rho}{1-\rho}\left\{(A^c_1\varphi_e)^2 + (A^c_2\sigma_w)^2 + 2\chi A^c_1\varphi_e A^c_2\sigma_w\right\}\right]\sigma^2_t
\end{aligned}$$

Using the method of undetermined coefficients and the five components of equation (11), we can solve for the constants A_0^c, A_1^c, and A_2^c:

$$
\begin{aligned}
A_1^c &= \frac{1-\rho}{\kappa_1^c - \rho_x}, && (16)\\
A_2^c &= .5\frac{(1-\rho)(1-\alpha)}{\kappa_1^c - \nu_1}\left[1 + \frac{(A_1^c\varphi_e)^2 + (A_2^c\sigma_w)^2 + 2\chi A_1^c\varphi_e A_2^c\sigma_w}{(1-\rho)^2}\right], && (17)\\
0 &= \frac{1-\alpha}{1-\rho}\left[\log\beta + \kappa_0^c + (1-\kappa_1^c)A_0^c\right] + (1-\alpha)\mu_c + \\
&\quad \frac{1}{2}(1-\alpha)^2\left[1 + \frac{(A_1^c\varphi_e)^2 + (A_2^c\sigma_w)^2 + 2\chi A_1^c\varphi_e A_2^c\sigma_w}{(1-\rho)^2}\right]\bar{\sigma}^2 && (18)
\end{aligned}
$$

The first equation is the same as in the case of no correlation between the state variables. The second and third equations are implicit functions of A_2^c and A_0^c. Because κ_0^c and κ_1^c are non-linear functions of A_0^c, this system of three equations needs to be solved simultaneously and numerically. Our computations indicate that the system has a unique solution. This verifies the conjecture that the log wealth-consumption ratio is linear in the two state variables.

The Consumption Risk Premium According to (13), the risk premium (expected excess return corrected for a Jensen term) on the consumption claim is given by

$$
\begin{aligned}
E_t\left[r_{t+1}^{c,e}\right] &= -Cov_t\left[r_{t+1}^c, m_{t+1}\right]\\
&= \left[\alpha + \frac{\alpha-\rho}{1-\rho}\left\{(A_1^c\varphi_e)^2 + (A_2^c\sigma_w)^2 + 2\chi A_1^c\varphi_e A_2^c\sigma_w\right\}\right]\sigma_t^2\\
&= (\lambda_{m,\eta} + \lambda_{m,e}A_1^c\varphi_e + \lambda_{m,w}A_2^c\sigma_w + 2\chi\lambda_{m,e}\lambda_{m,w})\sigma_t^2, && (19)
\end{aligned}
$$

with the market price of risk vector $\Lambda = [\lambda_{m,\eta}, \lambda_{m,e}, \lambda_{m,w}]$ given by

$$
\begin{aligned}
\lambda_{m,\eta} &= \alpha,\\
\lambda_{m,e} &= \frac{\alpha-\rho}{1-\rho}A_1^c\varphi_e = \frac{\alpha-\rho}{\kappa_1^c-\rho_x}\varphi_e,\\
\lambda_{m,w} &= \frac{\alpha-\rho}{1-\rho}A_2^c\sigma_w.
\end{aligned}
$$

Note that the market price of temporary consumption growth shocks η_{t+1} is positive and equal to the risk aversion coefficient α, the market price of long-run consumption growth shocks e_{t+1} is positive, and the market price of economic uncertainty (innovations w_{t+1} in $\sigma^2 - \bar{\sigma}^2$) is *negative* because A_2^c will turn out to be negative.

Correlations expected consumption growth and expected total wealth return Expected total wealth returns and expected consumption growth rates are equal to:

$$
\begin{aligned}
E_t\left[r_{t+1}^c\right] &= r_0^c + \rho x_t - A_2^c(\kappa_1^c - \nu_1)(\sigma_t^2 - \bar{\sigma}^2), && (20)\\
E_t\left[\Delta c_{t+1}\right] &= \mu_c + x_t. && (21)
\end{aligned}
$$

The unconditional variance of these quantities is:

$$
\begin{aligned}
V\left[E_t\left[r_{t+1}^c\right]\right] &= \rho^2 V[x_t] + [A_2^c(\kappa_1^c-\nu_1)]^2 V[\sigma_t^2 - \bar{\sigma}^2] - 2\rho A_2^c(\kappa_1^c-\nu_1)Cov[x_t, \sigma_t^2 - \bar{\sigma}^2],\\
V\left[E_t\left[\Delta c_{t+1}\right]\right] &= V[x_t].
\end{aligned}
$$

The unconditional covariance between expected returns and expected consumption growth rates is

$$Cov\left(E_t\left[r^c_{t+1}\right], E_t\left[\Delta c_{t+1}\right]\right) \quad = \quad \rho V[x] - A^c_2(\kappa^c_1 - \nu_1)Cov[x_t, \sigma^2_t - \bar{\sigma}^2]$$

This then leads to the unconditional correlation, which is straightforward to compute.

The Dividend Claims We conjecture, as we did for the wealth-consumption ratio, that the log price dividend ratio on a stock i, where the notation i is suppressed, is linear in the two state variables:

$$pd^i_t = A^i_0 + A^i_1 x_t + A^i_2(\sigma^2_t - \bar{\sigma}^2).$$

As we did for the return on the consumption claim, we compute innovations in the dividend claim return, and its conditional mean and variance:

$$\begin{aligned}
r^i_{t+1} - E_t\left[r^i_{t+1}\right] &= \varphi^i_d \sigma_t u^i_{t+1} + A^i_1 \varphi_e \sigma_t e_{t+1} + A^i_2 \sigma_w \sigma_t w_{t+1} \\
E_t\left[r^i_{t+1}\right] &= r^i_0 + [\phi^i_{d,x} - A^i_1(\kappa^i_1 - \rho_x)]x_t + [\phi^i_{d,\sigma} - A^i_2(\kappa^i_1 - \nu_1)]\left(\sigma^2_t - \bar{\sigma}^2\right) \\
V_t\left[r^i_{t+1}\right] &= \left[(\varphi^i_d)^2 + (A^i_1\varphi_e)^2 + (A^i_2\sigma_w)^2 + 2\chi A^i_1 \varphi_e A^i_2 \sigma_w\right]\sigma^2_t, \\
r^i_0 &= \kappa^i_0 + A^i_0(1 - \kappa^i_1) + \mu^i_d
\end{aligned}$$

Finally, the conditional covariance between the log SDF and the log dividend claim return is

$$Cov_t\left[m_{t+1}, r^i_{t+1}\right] = -\left[\iota^i \alpha \varphi^i_d + \frac{\alpha - \rho}{1 - \rho}\left\{A^c_1 A^i_1 \varphi^2_e + \chi A^c_1 A^i_2 \varphi_e \sigma_w + \chi A^c_2 A^i_1 \varphi_e \sigma_w + A^c_2 A^i_2 \sigma^2_w\right\}\right]\sigma^2_t.$$

From the Euler equation for this return $E_t\left[m_{t+1}\right] + E_t\left[r^i_{t+1}\right] + \frac{1}{2}V_t\left[m_{t+1}\right] + \frac{1}{2}V_t\left[r^i_{t+1}\right] + Cov_t[m_{t+1}, r^i_{t+1}] = 0$ and the method of undetermined coefficients, we can use the same procedure as above, and solve for the constants A^i_0, A^i_1, and A^i_2:

$$\begin{aligned}
A^i_1 &= \frac{\phi^i_{d,x} - \rho}{\kappa^i_1 - \rho_x}, \\
A^i_2 &= \frac{\left[\frac{\alpha-\rho}{1-\rho} A^c_2(\kappa^c_1 - \nu_1) + \phi^i_{d,\sigma} + .5H^i\right]}{\kappa^i_1 - \nu_1}, \\
0 &= m_0 + \kappa^i_0 + A^i_0(1 - \kappa^i_1) + \mu^i_d + .5H^i\bar{\sigma}^2
\end{aligned}$$

where

$$\begin{aligned}
H^i = & \left\{\alpha^2 + (\varphi^i_d)^2 - 2\iota^i \alpha \varphi^i_d\right\} + \left\{(A^i_1\varphi_e)^2 + (A^i_2\sigma_w)^2 + 2\chi A^i_1 \varphi_e A^i_2 \sigma_w\right\} \\
& + \left(\frac{\alpha - \rho}{1 - \rho}\right)^2 \left\{(A^c_1\varphi_e)^2 + (A^c_2\sigma_w)^2 + 2\chi A^c_1 \varphi_e A^c_2 \sigma_w\right\} \\
& - 2\frac{\alpha - \rho}{1 - \rho}\left\{A^c_1 A^i_1 \varphi^2_e + \chi A^c_1 A^i_2 \varphi_e \sigma_w + \chi A^c_2 A^i_1 \varphi_e \sigma_w + A^c_2 A^i_2 \sigma^2_w\right\}
\end{aligned} \tag{22}$$

Again, this is a non-linear system in three equations and three unknowns, which we solve numerically.
The equity risk premium on dividend claim i is:

$$E_t\left[r^{e,i}_{t+1}\right] = \left[\iota^i \alpha \varphi^i_d + \frac{\alpha - \rho}{1 - \rho}\left\{A^c_1 A^i_1 \varphi^2_e + \chi A^c_1 A^i_2 \varphi_e \sigma_w + \chi A^c_2 A^i_1 \varphi_e \sigma_w + A^c_2 A^i_2 \sigma^2_w\right\}\right]\sigma^2_t. \tag{23}$$

Equation (23) shows that the equity risk premium consists of a compensation for bearing temporary shock risk (first term), long-run risk (second term) and uncertainty risk (third term). The fourth term is new and only arises when there is non-zero correlation χ between the two state variables. The market price of temporary cash-flow risk is $\alpha > 0$, the market price of long-run risk is $A_1^c \varphi_e > 0$, and the market price of uncertainty risk is $A_2^c \sigma_w < 0$. These signs make sense: when long-run risk is low (economic uncertainty is high), the marginal utility growth for the representative investor is high, and she is willing to pay a high price for a security that has high dividend growth in such states of the world. The new, fourth term in (23) lowers the equity risk premium when the state variables are positively correlated because $A_1^c A_2^i + A_2^c A_1^i < 0$. Indeed, when high uncertainty goes hand in hand with high future growth, the world is a less risky place. Finally, we note that the only source of variation in equity risk premia over time is economic uncertainty σ_t^2.

Correlations expected dividend growth and expected return Expected stock returns and expected dividend growth rates on any portfolio i are equal to:

$$\begin{aligned} E_t\left[r_{t+1}^i\right] &= r_0^i + \rho x_t + [\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1)](\sigma_t^2 - \bar{\sigma}^2), && (24) \\ E_t\left[\Delta d_{t+1}^i\right] &= \mu_d^i + \phi_{d,x}^i x_t + \phi_{d,\sigma}^i(\sigma_t^2 - \bar{\sigma}^2). && (25) \end{aligned}$$

The unconditional variance of these quantities is:

$$\begin{aligned} V\left[E_t\left[r_{t+1}^i\right]\right] &= \rho^2 V[x_t] + [\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1)]^2 V[\sigma_t^2 - \bar{\sigma}^2] + 2\rho[\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1)]Cov[x_t, \sigma_t^2 - \bar{\sigma}^2], \\ V\left[E_t\left[\Delta d_{t+1}^i\right]\right] &= (\phi_{d,x}^i)^2 V[x_t] + (\phi_{d,\sigma}^i)^2 V[\sigma_t^2 - \bar{\sigma}^2] + 2\phi_{d,\sigma}^i \phi_{d,x}^i Cov[x_t, \sigma_t^2 - \bar{\sigma}^2]. \end{aligned}$$

The unconditional covariance between expected stock returns and expected dividend growth rates is

$$\begin{aligned} Cov\left(E_t\left[r_{t+1}^i\right], E_t\left[\Delta d_{t+1}^i\right]\right) &= \phi_{d,x}^i \rho V[x_t] + \phi_{d,\sigma}^i[\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1)]V[\sigma_t^2 - \bar{\sigma}^2] \\ &\quad + \left(\rho\phi_{d,\sigma}^i + \phi_{d,x}^i[\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1)]\right) Cov[x_t, \sigma_t^2 - \bar{\sigma}^2] \end{aligned}$$

This then leads to the unconditional correlation, which is straightforward to compute.
The important thing to note is that this covariance is (1) asset-specific because it depends on both $\phi_{d,x}^i$ and $\phi_{d,\sigma}^i$, and (2) it can be positive or negative depending on $\phi_{d,x}^i$ and $\phi_{d,\sigma}^i$, holding fixed χ. In the standard LRR model, the covariance between the state variables is zero and $\phi_{d,\sigma}^i = 0$. In that case, the sign of the covariance between expected returns and expected dividend growth depends on the sign of $\phi_{d,x}^i \rho$, which is positive.
The unconditional correlation between expected and unexpected dividend growth is zero for all portfolios by the law of iterated expectations:

$$Corr\left(E_t\left[\Delta d_{t+1}^i\right], \Delta d_{t+1}^i - E_t\left[\Delta d_{t+1}^i\right]\right) = Corr(\phi_{d,x}^i x_t + \phi_{d,\sigma}^i(\sigma_t^2 - \bar{\sigma}^2), \varphi_d^i \sigma_t u_{t+1}) = 0.$$

Likewise, the correlation between expected and unexpected returns is zero. This is the same as in the standard BY model.

A.3 Bond Pricing

The Risk-free Rate According to equation (12), the expression for the risk-free rate is given by

$$\begin{aligned} y_t(1) &= -A(1) - B(1)x_t - C(1)(\sigma_t^2 - \bar{\sigma}^2) && (26)\\ A(1) &= m_0 + .5\Gamma(0)\bar{\sigma}^2, \\ B(1) &= -\rho, \\ C(1) &= \frac{\alpha-\rho}{1-\rho}(\kappa_1^c - \nu_1)A_2^c + .5\Gamma(0), \end{aligned}$$

where the notation $\Gamma(0)$ is defined as:

$$\Gamma(0) \equiv \alpha^2 + \left(\frac{\alpha-\rho}{1-\rho}\right)^2 \left\{(A_1^c\varphi_e)^2 + (A_2^c\sigma_w)^2 + 2\chi A_1^c\varphi_e A_2^c\sigma_w\right\} \qquad (27)$$

The Real Yield Curve The price of a τ-period real zero-coupon bond satisfies:

$$P_t(\tau) = E_t\left[e^{m_{t+1}+\log P_{t+1}(\tau-1)}\right].$$

This defines a recursion with $P_t(0) = 1$. The corresponding bond yield is $y_t(\tau) = -\log(P_t(\tau))/\tau$. It is easy to show that the LRR model gives rise to a affine real term structure (Bansal and Shaliastovich 2007):

$$y_t(\tau) = -\frac{A(\tau)}{\tau} - \frac{B(\tau)}{\tau}x_t - \frac{C(\tau)}{\tau}(\sigma_t^2 - \bar{\sigma}^2),$$

The coefficients $A(\tau)$, $B(\tau)$, and $C(\tau)$ satisfy the following recursions:

$$\begin{aligned} A(\tau+1) &= m_0 + A(\tau) + .5\Gamma(\tau)\bar{\sigma}^2, && (28)\\ B(\tau+1) &= \rho_x B(\tau) - \rho, && (29)\\ C(\tau+1) &= \nu_1 C(\tau) + \frac{\alpha-\rho}{1-\rho}(\kappa_1^c - \nu_1)A_2^c + .5\Gamma(\tau) && (30) \end{aligned}$$

where

$$\begin{aligned} \Gamma(\tau) \equiv\ & \alpha^2 + \left[B(\tau) - \frac{\alpha-\rho}{\kappa_1^c - \rho_x}\right]^2 \varphi_e^2 + \left[C(\tau) - \frac{\alpha-\rho}{1-\rho}A_2^c\right]^2 \sigma_w^2 \\ & +2\chi\left[B(\tau) - \frac{\alpha-\rho}{\kappa_1^c - \rho_x}\right]\varphi_e\left[C(\tau) - \frac{\alpha-\rho}{1-\rho}A_2^c\right]\sigma_w, && (31) \end{aligned}$$

and where the recursion is initialized at $A(0) = 0$, $B(0) = 0$, and $C(0) = 0$.

Proof. Guess and verify

$$\begin{aligned} P_t(\tau+1) &= E_t[\exp\{m_{t+1} + \log(P_{t+1}(\tau))\}] \\ &= E_t[\exp\{m_{t+1} + A(\tau) + B(\tau)x_{t+1} + C(\tau)(\sigma_{t+1}^2 - \bar{\sigma}^2)\}] \\ &= \exp\{m_0 + A(\tau) + (B(\tau)\rho_x - \rho)x_t + \left[\nu_1 C(\tau) + \frac{\alpha-\rho}{1-\rho}(\kappa_1^c - \nu_1)A_2^c\right](\sigma_t^2 - \bar{\sigma}^2)\} \times \\ & E_t\left[\exp\left\{-\alpha\sigma_t\eta_{t+1} + \left[B(\tau) - \frac{\alpha-\rho}{\kappa_1^c - \rho_x}\right]\varphi_e\sigma_t e_{t+1} + \left[C(\tau) - \frac{\alpha-\rho}{1-\rho}A_2^c\right]\sigma_w\sigma_t w_{t+1}\right\}\right] \end{aligned}$$

Note that the conditional expectation term is equal to $\exp\{.5\Gamma(\tau)\sigma_t^2\}$, where $\Gamma(\tau)$ is defined in (31). The rest follows from recognizing that

$$\log P_t(\tau+1) = A(\tau+1) + B(\tau+1)x_t + C(\tau+1)(\sigma_t^2 - \bar{\sigma}^2)$$

and matching up coefficients. $\square$

Real Bond Returns Define the 1-period log return on a real bond of maturity τ as:

$$r_{t+1}^b(\tau) = \tau y_t(\tau) - (\tau-1)y_{t+1}(\tau-1).$$

The one-period excess return subtracts the real short rate:

$$r_{t+1}^b(\tau) - y_t(1) = \tau y_t(\tau) - (\tau-1)y_{t+1}(\tau-1) - y_t(1).$$

In general, the excess log return on buying an τ-period real bond at time t and selling it at time $t+m$ as an $\tau - m$ period bond is:

$$\tau y_t(\tau) - (\tau-m)y_{t+m}(\tau-m) - my_t(1).$$

The same expression holds for nominal excess bond returns, which we denote with a \$ superscript. Innovations in one-period real bond returns are given by

$$\begin{aligned} r_{t+1}^b(\tau) - E_t\left[r_{t+1}^b(\tau)\right] &= (\tau-1)\left(-y_{t+1}(\tau-1) + E_t\left[y_{t+1}(\tau-1)\right]\right), \\ &= B(\tau-1)\varphi_e\sigma_t e_{t+1} + C(\tau-1)\sigma_w\sigma_t w_{t+1}. \end{aligned}$$

The conditional variance of the bond return therefore equals:

$$V_t\left[r_{t+1}^b(\tau)\right] = \left\{[B(\tau-1)\varphi_e]^2 + [C(\tau-1)\sigma_w]^2 + 2\chi B(\tau-1)\varphi_e C(\tau-1)\sigma_w\right\}\sigma_t^2.$$

The real bond risk premium (the expected excess return with Jensen adjustment) is:

$$\begin{aligned} E_t\left[r_{t+1}^{b,e}(\tau)\right] &= \tau y_t(\tau) - (\tau-1)E_t\left[y_{t+1}(\tau-1)\right] - y_t(1) + .5V_t\left[r_{t+1}^b(\tau)\right], \\ &= [.5\Gamma(0) - .5\Gamma(\tau-1) + .5\left\{[B(\tau-1)\varphi_e]^2 + [C(\tau-1)\sigma_w]^2 + 2\chi B(\tau-1)\varphi_e C(\tau-1)\sigma_w\right\}]\sigma_t^2 \\ &= \left\{B(\tau-1)\frac{\alpha-\rho}{\kappa_1^c-\rho_x}\varphi_e^2 + C(\tau-1)\frac{\alpha-\rho}{1-\rho}A_2^c\sigma_w^2\right. \\ &\quad \left. +\chi B(\tau-1)\varphi_e\frac{\alpha-\rho}{1-\rho}A_2^c\sigma_w + \chi C(\tau-1)\varphi_e\frac{\alpha-\rho}{\kappa_1^c-\rho_x}\sigma_w\right\}\sigma_t^2 \end{aligned}$$

We have verified that this risk premium equals $-Cov_t\left[m_{t+1}, r_{t+1}^b(\tau) - E_t\left[r_{t+1}^b(\tau)\right]\right]$.

The Nominal Yield Curve The price of a τ-period nominal zero-coupon bond satisfies:

$$P_t^{\$}(\tau) = E_t\left[e^{m_{t+1}^{\$} + \log P_{t+1}^{\$}(\tau-1)}\right].$$

This defines a recursion with $P_t^{\$}(0) = 1$. The corresponding bond yield is $y_t^{\$}(\tau) = -\log(P_t^{\$}(\tau))/\tau$. Nominal bond yields are also affine in the state vector (Bansal and Shaliastovich 2007):

$$y_t^{\$}(\tau) = -\frac{A^{\$}(\tau)}{\tau} - \frac{B^{\$}(\tau)}{\tau}x_t - \frac{C^{\$}(\tau)}{\tau}(\sigma_t^2 - \bar{\sigma}^2) - \frac{D^{\$}(\tau)}{\tau}(\bar{\pi}_t - \mu_\pi),$$

The coefficients $A(\tau)$, $B^{\$}(\tau)$, and $C^{\$}(\tau)$ satisfy the following recursions

$$A^{\$}(\tau+1) = m_0 - \mu_\pi + A^{\$}(\tau) + .5\left[-\sigma_\pi + D^{\$}(\tau)\sigma_z\right]^2 + .5\Gamma^{\$}(\tau)\bar{\sigma}^2, \tag{32}$$

$$B^{\$}(\tau+1) = B^{\$}(\tau)\rho_x - \rho + D^{\$}(\tau)\phi_\pi, \tag{33}$$

$$C^{\$}(\tau+1) = C^{\$}(\tau)\nu_1 + \frac{\alpha-\rho}{1-\rho}(\kappa_1^c - \nu_1)A_2^c + .5\Gamma^{\$}(\tau), \tag{34}$$

$$D^{\$}(\tau+1) = D^{\$}(\tau)\rho_\pi - 1, \tag{35}$$

initialized at $A^{\$}(0) = 0$, $B^{\$}(0) = 0$, $C^{\$}(0) = 0$, and $D^{\$}(0) = 0$, and where

$$\begin{aligned}\Gamma^{\$}(\tau) \equiv\ & \left[-\alpha - \varphi_{\pi,\eta} + D^{\$}(\tau)\zeta_{\pi,\eta}\right]^2 + \left[B^{\$}(\tau) - \frac{\alpha-\rho}{\kappa_1^c - \rho_x} - \varphi_{\pi,e} + D^{\$}(\tau)\zeta_{\pi,e}\right]^2\varphi_e^2 \\ & + \left[C^{\$}(\tau) - \frac{\alpha-\rho}{1-\rho}A_2^c\right]^2\sigma_w^2 + 2\chi\left[B^{\$}(\tau) - \frac{\alpha-\rho}{\kappa_1^c-\rho_x} - \varphi_{\pi,e} + D^{\$}(\tau)\zeta_{\pi,e}\right]\varphi_e\left[C^{\$}(\tau) - \frac{\alpha-\rho}{1-\rho}A_2^c\right]\sigma_w.\end{aligned}$$

Proof. We conjecture that the $t+1$-price of a τ-period bond is exponentially affine in the state and solve for the coefficients in the process of verifying this conjecture using the Euler equation:

$$\begin{aligned}P_t^{\$}(\tau+1) &= E_t[\exp\{m_{t+1}^{\$} + \log\left(P_{t+1}^{\$}(\tau)\right)\}] \\ &= E_t[\exp\{m_{t+1} - \pi_{t+1} + A^{\$}(\tau) + B^{\$}(\tau)x_{t+1} + C^{\$}(\tau)(\sigma_{t+1}^2 - \bar{\sigma}^2) + D^{\$}(\tau)(\bar{\pi}_{t+1} - \mu_\pi)\}] \\ &= \exp\{m_0 - \mu_\pi + A^{\$}(\tau) + \left[B^{\$}(\tau)\rho_x - \rho + D^{\$}(\tau)\phi_\pi\right]x_t \\ &\quad + \left[C^{\$}(\tau)\nu_1 + \frac{\alpha-\rho}{1-\rho}(\kappa_1^c - \nu_1)A_2^c\right]\left(\sigma_t^2 - \bar{\sigma}^2\right) + \left[D^{\$}(\tau)\rho_\pi - 1\right](\bar{\pi}_t - \mu_\pi)\} \times \\ &\quad E_t\left[\exp\{\left[-\alpha - \varphi_{\pi,\eta} + D^{\$}(\tau)\zeta_{\pi,\eta}\right]\sigma_t\eta_{t+1}\}\times\right. \\ &\quad \{+\left[-\frac{\alpha-\rho}{\kappa_1^c-\rho_x} - \varphi_{\pi,e} + B^{\$}(\tau) + D^{\$}(\tau)\zeta_{\pi,e}\right]\varphi_e\sigma_t e_{t+1}\}\times \\ &\quad \left.\{+\left[-\frac{\alpha-\rho}{1-\rho}A_2^c + C^{\$}(\tau)\right]\sigma_w\sigma_t w_{t+1} + \left[-\sigma_\pi + D^{\$}(\tau)\sigma_z\right]\xi_{t+1}\}\right]\end{aligned}$$

We use the joint standard normality of the four mutually-orthogonal shocks. Taking logs and collecting terms, we obtain a linear equation for $\log(P_t^{\$}(\tau+1))$ where the coefficients $A^{\$}(\tau+1)$ through $D^{\$}(\tau+1)$ satisfy (32)- (35). $\square$

We have verified that the one-period nominal short rate satisfies:

$$y_t^{\$}(1) = -E_t[m_{t+1}^{\$}] - .5V_t[m_{t+1}^{\$}] = -E_t[m_{t+1} - \pi_{t+1}] - .5V_t[m_{t+1} - \pi_{t+1}]$$

for the coefficients $A^{\$}(1)$ through $D^{\$}(1)$ in (32)- (35).

Inflation Risk Premium The expected excess return on a one-period nominal bond over a one-period real bond measures the inflation risk premium at the short end of the term structure. It is given by $-cov_t[m_{t+1}, \pi_{t+1}]$, where

$$-cov_t[m_{t+1}, \pi_{t+1}] = \left(\alpha\varphi_{\pi,\eta} + \frac{\alpha-\rho}{\kappa_1^c - \rho_x}\varphi_{\pi,e}\varphi_e^2 + \chi\frac{\alpha-\rho}{1-\rho}A_2^c\sigma_w\varphi_{\pi,e}\varphi_e\right)\sigma_t^2.$$

In principle, the inflation risk premium can be either negative or positive depending on the sign of χ. However, it takes the same sign for all periods. When $\varphi_{\pi,\eta} = 0$ and $\chi = 0$, which is the case in (Bansal and Shaliastovich 2007), the inflation risk premium equals the second term.

Nominal Bond Returns Define the 1-period log return on a nominal bond of maturity τ as:

$$\begin{aligned} r_{t+1}^{b,\$}(\tau) &= \tau y_t^{\$}(\tau) - (\tau-1)y_{t+1}^{\$}(\tau-1), \\ &= -A^{\$}(\tau) - B^{\$}(\tau)x_t - C^{\$}(\tau)(\sigma_t^2 - \bar{\sigma}^2) - D^{\$}(\tau)(\bar{\pi}_t - \mu_\pi) \\ &\quad + A^{\$}(\tau-1) + B^{\$}(\tau-1)x_{t+1} + C^{\$}(\tau-1)(\sigma_{t+1}^2 - \bar{\sigma}^2) + D^{\$}(\tau-1)(\bar{\pi}_{t+1} - \mu_\pi) \end{aligned}$$

Expected nominal bond returns are:

$$\begin{aligned} E_t\left[r_{t+1}^{b,\$}(\tau)\right] &= A^{\$}(\tau-1) - A^{\$}(\tau) + \left[B^{\$}(\tau-1)\rho_x + D^{\$}(\tau-1)\phi_\pi - B^{\$}(\tau)\right]x_t \\ &\quad + \left[C^{\$}(\tau-1)\nu_1 - C^{\$}(\tau)\right](\sigma_t^2 - \bar{\sigma}^2) + \left[D^{\$}(\tau-1)\rho_\pi - D^{\$}(\tau)\right](\bar{\pi}_t - \mu_\pi). \end{aligned}$$

Innovation in nominal bond returns are:

$$\begin{aligned} r_{t+1}^{b,\$}(\tau) - E_t\left[r_{t+1}^{b,\$}(\tau)\right] &= (\tau-1)\left(-y_{t+1}^{\$}(\tau-1) + E_t\left[y_{t+1}^{\$}(\tau-1)\right]\right), \\ &= D^{\$}(\tau-1)\zeta_{\pi,\eta}\sigma_t\eta_{t+1} + \left[B^{\$}(\tau-1) + D^{\$}(\tau-1)\zeta_{\pi,e}\right]\varphi_e\sigma_t e_{t+1} \\ &\quad + C^{\$}(\tau-1)\sigma_w\sigma_t w_{t+1} + D^{\$}(\tau-1)\sigma_z\xi_{t+1}. \end{aligned}$$

Expected excess bond returns adjusted for a Jensen term, or nominal bond risk premia, are given by:

$$\begin{aligned} E_t\left[r_{t+1}^{b,e,\$}(\tau)\right] &\equiv -Cov_t\left[m_{t+1}^{\$}, r_{t+1}^{b,e,\$}(\tau)\right] \\ &= Cov_t\left[\pi_{t+1}, r_{t+1}^{b,e,\$}(\tau)\right] - Cov_t\left[m_{t+1}, r_{t+1}^{b,e,\$}(\tau)\right] \\ &= r_0^{b,e,\$}(\tau) + F^{\$}(\tau)(\sigma_t^2 - \bar{\sigma}^2) \end{aligned}$$

where

$$\begin{aligned} r_0^{b,e,\$}(\tau) &= D^{\$}(\tau-1)\sigma_z\sigma_\pi + F^{\$}(\tau)\bar{\sigma}^2 \\ F^{\$}(\tau) &= (\varphi_{\pi,\eta} + \alpha)\,D^{\$}(\tau-1)\zeta_{\pi,\eta} + \\ &\quad \left(\varphi_{\pi,e} + \frac{\alpha-\rho}{\kappa_1^c - \rho_x}\right)\left[B^{\$}(\tau-1) + D^{\$}(\tau-1)\zeta_{\pi,e}\right]\varphi_e^2 + \frac{\alpha-\rho}{1-\rho}A_2^c C^{\$}(\tau-1)\sigma_w^2 + \\ &\quad \chi\frac{\alpha-\rho}{1-\rho}A_2^c\sigma_w\left[B^{\$}(\tau-1) + D^{\$}(\tau-1)\zeta_{\pi,e}\right]\varphi_e + \chi\frac{\alpha-\rho}{\kappa_1^c - \rho_x}\varphi_e C^{\$}(\tau-1)\sigma_w \end{aligned}$$

We have used the fact that:

$$
\begin{aligned}
Cov_t\left[\pi_{t+1}, rx_{t+1}^{\$}(\tau)\right] &= \Big\{\varphi_{\pi,\eta} D^{\$}(\tau-1)\zeta_{\pi,\eta} + \varphi_{\pi,e}\left[B^{\$}(\tau-1) + D^{\$}(\tau-1)\zeta_{\pi,e}\right]\varphi_e^2 \\
&\quad + \chi\varphi_{\pi,e}\varphi_e C^{\$}(\tau-1)\sigma_w\Big\}\sigma_t^2 + D^{\$}(\tau-1)\sigma_z\sigma_\pi \\
Cov_t\left[m_{t+1}, rx_{t+1}^{\$}(\tau)\right] &= \Big\{-\alpha D^{\$}(\tau-1)\zeta_{\pi,\eta} - \frac{\alpha-\rho}{\kappa_1^c - \rho_x}\left[B^{\$}(\tau-1) + D^{\$}(\tau-1)\zeta_{\pi,e}\right]\varphi_e^2 \\
&\quad - \frac{\alpha-\rho}{1-\rho}A_2^c C^{\$}(\tau-1)\sigma_w^2 - \chi\frac{\alpha-\rho}{1-\rho}A_2^c\sigma_w\left[B^{\$}(\tau-1) + D^{\$}(\tau-1)\zeta_{\pi,e}\right]\varphi_e \\
&\quad - \chi\frac{\alpha-\rho}{\kappa_1^c-\rho_x}\varphi_e C^{\$}(\tau-1)\sigma_w\Big\}\sigma_t^2.
\end{aligned}
$$

In sum, nominal expected excess bond returns (nominal bond risk premia) only vary over time because economic uncertainty $(\sigma_t^2 - \bar{\sigma}^2)$ varies. The loading $F(\tau)$ measures their sensitivity to this factor; it depends on the maturity of the bond.

The Cochrane-Piazzesi Factor Define the nominal forward rate for a loan between period $t+\tau$ and $t+\tau+1$ as usual from nominal bond prices:

$$
f_t^{\$}(\tau) = \log P_t^{\$}(\tau-1) - \log P_t^{\$}(\tau).
$$

In the model, the forward rate is

$$
\begin{aligned}
f_t^{\$}(\tau) &= \left(A^{\$}(\tau-1) - A^{\$}(\tau)\right) + \left(B^{\$}(\tau-1) - B^{\$}(\tau)\right)x_t + \\
&\quad \left(C^{\$}(\tau-1) - C^{\$}(\tau)\right)(\sigma_t^2 - \bar{\sigma}^2) + \left(D^{\$}(\tau-1) - D^{\$}(\tau)\right)(\bar{\pi}_t - \mu_\pi)
\end{aligned}
$$

Given that the nominal model has three state variables, we can invert the state variables from any three forward rates, say $f_t^{\$}(1)$, $f_t^{\$}(3)$, and $f_t^{\$}(5)$. Put differently, one linear combination of forward rates is perfectly correlated with $(\sigma_t^2 - \bar{\sigma}^2)$. This is the linear combination of interest, because this state variable is the only source of time variation in bond and stock risk premia. Hence, this linear combination is the best predictor of future bond returns. It is perfectly correlated with expected bond returns. We call in the Cochrane-Piazzesi factor because the weights (c_1, c_3, c_5) display a tent-shaped function as in Cochrane and Piazzesi (2005):

$$
CP_t = c_1 f_t^{\$}(1) + c_3 f_t^{\$}(3) + c_5 f_t^{\$}(5) = b_{CP}(\sigma_t^2 - \bar{\sigma}^2). \tag{36}
$$

In order to pin down the ratio of the standard deviation of CP_t relative to that of σ_t^2, b_{CP}, we form annual expected excess bond returns on nominal bonds of horizons 2-, 3-, 4-, and 5-years in the model. They only depend on σ_t^2. We set the CP factor equal to the equally-weighted average of these four returns, as we did in the data. Hence b_{CP} is not a free parameter but rather a function of the structural parameters of the model.

A.4 CP Betas

The question we want to ask of the long-run risk model is whether it is able to generate the cross-sectional variation in the exposures of excess returns of book-to-market-sorted stock portfolios to the CP factor. We showed in the main text that there is a clearly increasing pattern from growth to value. Since this is an equilibrium model, stock returns are endogenous, and only cash-flow growth can be specified exogenously. This leads us to impose the additional discipline that the model must match also the cross-sectional variation in the exposures of dividend growth rates of book-to-market-sorted stock portfolios to the CP factor. We also showed dividend growth betas that were decreasing from growth (mildly positive) to value (strongly negative). Third, the CP factor is a strong predictor of bond returns in the data; we want to capture this predictability quantitatively. We now provide closed-form expressions for the contemporaneous return and dividend growth betas and for lagged-CP betas for excess bond returns. We start from the expressions for expected stock returns, expected excess stock returns, and expected dividend growth rates on stock portfolios in equation (24)-(25).

Stock Returns and Excess Returns The unconditional, contemporaneous, CP beta for *excess* stock returns is equal to:

$$\beta^{res} \equiv \beta\left(r^{e,i}_{t+1}, CP_{t+1}\right) = \nu_1 \beta^{res}_{Lagged} + \beta^{res}_{Innov}, \tag{37}$$

$$\beta^{res}_{Lagged} = \frac{\phi^i_{d,\sigma} - A^i_2(\kappa^i_1 - \nu_1) + \frac{\alpha-\rho}{1-\rho} A^c_2(\kappa^c_1 - \nu_1) + .5\Gamma(0)}{b_{CP}}, \tag{38}$$

$$\beta^{res}_{Innov} = \left(\chi A^i_1 \frac{\varphi_e}{\sigma_w} + A^i_2\right) \frac{1-\nu_1^2}{b_{CP}}, \tag{39}$$

where the expression for $\Gamma(0)$ was given earlier in this appendix. The contemporaneous CP-beta of excess returns is the sum of the lagged beta , multiplied by the persistence of CP, ν_1, plus the innovation beta. The latter arises from the fact that pd innovations and CP innovations are both driven by the shock w_{t+1} and from the fact that innovations to w_{t+1} and e_{t+1} may be correlated ($\chi \neq 0$).
Likewise, the unconditional, contemporaneous, CP beta for stock returns is equal to:

$$\beta^{rs} \equiv \beta\left(r^i_{t+1}, CP_{t+1}\right) = \nu_1 \beta^{rs}_{Lagged} + \beta^{rs}_{Innov}, \tag{40}$$

$$\beta^{rs}_{Lagged} = \frac{\rho}{b_{CP}} \frac{Cov[x_t, \sigma^2_t - \bar{\sigma}^2]}{V[\sigma^2_t - \bar{\sigma}^2]} + \frac{\phi^i_{d,\sigma} - A^i_2(\kappa^i_1 - \nu_1)}{b_{CP}}, \tag{41}$$

$$\beta^{rs}_{Innov} = \left(\chi A^i_1 \frac{\varphi_e}{\sigma_w} + A^i_2\right) \frac{1-\nu_1^2}{b_{CP}}. \tag{42}$$

The unconditional lagged CP-beta for *realized* returns is the same as for expected returns: $\beta\left(r^i_{t+1}, CP_t\right) = \beta\left(E_t\left[r^i_{t+1}\right], CP_t\right)$. The reason is that unexpected returns have a zero unconditional correlation with CP_t by the law of iterated expectations.

Dividend Growth Rates The contemporaneous CP beta for dividend growth rates, denoted with a superscript Δd, can be shown to equal:

$$\beta^{\Delta d} \equiv \beta\left(\Delta d^i_{t+1}, CP_{t+1}\right) = \nu_1 \beta^{\Delta d}_{Lagged}, \tag{43}$$

$$\beta^{\Delta d}_{Lagged} = = \left(\chi \frac{\phi^i_{d,x}}{1-\rho_x \nu_1} \frac{\varphi_e}{\sigma_w} \frac{1-\nu_1^2}{b_{CP}} + \frac{\phi^i_{d,\sigma}}{b_{CP}}\right). \tag{44}$$

The contemporaneous CP-beta for dividend growth is proportional to the lagged CP-beta, with a constant of proportionality of ν_1. The reason is that dividend growth innovations are orthogonal to CP innovations because u_{t+1} and w_{t+1} are uncorrelated. The unconditional covariance between *realized* dividend growth rates Δd_{t+1} and CP_t is the same as the covariance with expected growth rates because unexpected dividend growth rates have a zero correlation with CP_t because $Cov\left(\varphi_d^2 \sigma_t u_{t+1}^i, \sigma_t^2 - \bar{\sigma}^2\right) = 0$, again because of the law of iterated expectations.

Bond Returns and Excess Returns For comparison, we also work out the betas of nominal *excess* bond returns with respect to CP:

$$\beta^{reb}(\tau) \equiv \beta\left(r_{t+1}^{b,e,\$}(\tau), CP_{t+1}\right) = \nu_1 \beta_{Lagged}^{reb}(\tau) + \beta_{Innov}^{reb}(\tau), \tag{45}$$

$$\beta_{Lagged}^{reb}(\tau) = \frac{.5\Gamma^{\$}(0) - .5\Gamma^{\$}(\tau-1)}{b_{CP}}, \tag{46}$$

$$\beta_{Innov}^{reb}(\tau) = \left(\chi\left[B^{\$}(\tau-1) + D^{\$}(\tau-1)\zeta_{\pi,e}\right]\frac{\varphi_e}{\sigma_w} + C^{\$}(\tau-1)\right)\frac{(1-\nu_1^2)}{b_{CP}}, \tag{47}$$

where the expression for $\Gamma^{\$}(\tau)$, for $\tau \geq 0$, is given above. We will focus on the excess bond return predictability by CP, by matching lagged excess return bond betas (equation 46) in model and data.
Finally, the contemporaneous CP beta of nominal bond returns is given by:

$$\beta^{rb}(\tau) \equiv \beta\left(r_{t+1}^{b,\$}(\tau), CP_{t+1}\right) = \nu_1 \beta_{Lagged}^{rb}(\tau) + \beta_{Innov}^{rb}(\tau), \tag{48}$$

$$\begin{aligned}\beta_{Lagged}^{rb}(\tau) = {} & \frac{\rho}{b_{CP}}\frac{Cov[x_t, \sigma_t^2 - \bar{\sigma}^2]}{V[\sigma_t^2 - \bar{\sigma}^2]} + \frac{1}{b_{CP}}\frac{Cov[\bar{\pi}_t - \mu_\pi, \sigma_t^2 - \bar{\sigma}^2]}{V[\sigma_t^2 - \bar{\sigma}^2]} \\ & + \frac{-\frac{\alpha-\rho}{1-\rho}(\kappa_1^c - \nu_1)A_2^c - .5\Gamma^{\$}(\tau-1)}{b_{CP}},\end{aligned} \tag{49}$$

$$\beta_{Innov}^{rb}(\tau) = \left(\chi\left[B^{\$}(\tau-1) + D^{\$}(\tau-1)\zeta_{\pi,e}\right]\frac{\varphi_e}{\sigma_w} + C^{\$}(\tau-1)\right)\frac{(1-\nu_1^2)}{b_{CP}}. \tag{50}$$

$$\tag{51}$$

Note that the first term of $\beta_{Lagged}^{rb}(\tau)$ is common with the beta of stock returns; it is a yield curve effect. The second term is also a yield curve effect. Both disappear if we look at excess returns.

Beta decomposition Recall the definition of a return

$$r_{t+1}^i = \kappa_0^i + \Delta d_{t+1}^i + pd_{t+1}^i - \kappa_1^i pd_t^i$$

and the definition of the pd ratio:

$$pd_t^i = A_0^i + A_1^i x_t + A_2^i(\sigma_t^2 - \bar{\sigma}^2)$$

Therefore, the contemporaneous return beta is:

$$\beta^{rs} - \beta^{\Delta d} = -\frac{A_1^i(\kappa_1^i\nu_1 - 1)}{b_{CP}}\frac{Cov[x_t, \sigma_t^2 - \bar{\sigma}^2]}{V[\sigma_t^2 - \bar{\sigma}^2]} - \frac{A_2^i(\kappa_1^i\nu_1 - 1)}{b_{CP}}$$

If $\kappa_1^i \nu_1 - 1 > 0$, which is a statement about the relative persistence of x and $\sigma_t^2 - \bar{\sigma}^2$, and $\chi < 0$, then both terms on the RHS are positive because $A_1^i > 0$ and $A_2^i < 0$. Since value stocks have higher A_1^i and more negative A_2^i, this explains the increasing pattern in the LHS from growth to value.
Another way to understand this is by using

$$pd_t^i = \frac{\kappa_0^i}{\kappa_1^i - 1} + \Delta d_t^{i,H} - r_t^{i,H},$$

Therefore, the contemporaneous return beta is:

$$\beta_{CP}^{rs} - \beta^{\Delta d} = \beta\left[CP_{t+1}, \Delta d_{t+1}^{i,H} - \kappa_1^i \Delta d_t^{i,H}\right] - \beta\left[CP_{t+1}, r_{t+1}^{i,H} - \kappa_1^i r_t^{i,H}\right] \tag{52}$$

The first beta on the RHS measures the covariance of innovations of CP_{t+1} with *news* about future dividend growth rates. It equals:

$$\beta\left[CP_{t+1}, \Delta d_{t+1}^{i,H} - \kappa_1^i \Delta d_t^{i,H}\right] = -\frac{\phi_{d,x}^i \nu_1}{b_{CP}} \frac{Cov[x_t, \sigma_t^2 - \bar{\sigma}^2]}{V[\sigma_t^2 - \bar{\sigma}^2]} - \frac{\phi_{d,\sigma}^i \nu_1}{b_{CP}} \tag{53}$$

$$+ \left\{\chi \frac{\phi_{d,x}^i}{(\kappa_1^i - \rho_x)} \frac{\varphi_e}{\sigma_w} + \frac{\phi_{d,\sigma}^i}{(\kappa_1^i - \nu_1)}\right\} \frac{(1 - \nu_1^2)}{b_{CP}} \tag{54}$$

The second beta on the RHS measures the covariance of innovations of CP_{t+1} with *news* about future stock returns. It equals:

$$\beta\left[CP_{t+1}, r_{t+1}^{i,H} - \kappa_1^i r_t^{i,H}\right] = \frac{-\rho\nu_1}{b_{CP}} \frac{Cov[x_t, \sigma_t^2 - \bar{\sigma}^2]}{V[\sigma_t^2 - \bar{\sigma}^2]} - \frac{\left[\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1)\right]\nu_1}{b_{CP}} \tag{55}$$

$$+ \left\{\chi \frac{\rho}{(\kappa_1^i - \rho_x)} \frac{\varphi_e}{\sigma_w} + \frac{\left[\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1)\right]}{\kappa_1^i - \nu_1}\right\} \frac{(1 - \nu_1^2)}{b_{CP}} \tag{56}$$

It is easy to verify that the difference between these two betas delivers the first expression we derived for $\hat{\beta}_{CP}^r - \hat{\beta}_{CP}^{\Delta d}$.

A.5 The Relationship Between Stocks and Bonds

The Equity Risk Premium and the Campbell-Shiller Decomposition Expected discounted future equity returns and dividend growth rates are given by:

$$r_t^{i,H} \equiv E_t\left[\sum_{j=1}^{\infty} \left(\kappa_1^i\right)^{-j} r_{t+j}^i\right] = \frac{r_0^i}{\kappa_1^i - 1} + \frac{\rho}{\kappa_1^i - \rho_x} x_t + \frac{\left[\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1)\right]}{\kappa_1^i - \nu_1}(\sigma_t^2 - \bar{\sigma}^2) \tag{57}$$

$$\Delta d_t^{i,H} \equiv E_t\left[\sum_{j=1}^{\infty} \left(\kappa_1^i\right)^{-j} \Delta d_{t+j}^i\right] = \frac{\mu_d^i}{\kappa_1^i - 1} + \frac{\phi_{d,x}^i}{\kappa_1^i - \rho_x} x_t + \frac{\phi_{d,\sigma}^i}{\kappa_1^i - \nu_1}(\sigma_t^2 - \bar{\sigma}^2) \tag{58}$$

From these expressions, it is easy to see that

$$pd_t^i = \frac{\kappa_0^i}{\kappa_1^i - 1} + \Delta d_t^{i,H} - r_t^{i,H},$$

and to compute the elements of the variance-decomposition:

$$V[pd_t^i] = Cov[pd_t^i, \Delta d_t^{i,H}] + Cov[pd_t^i, -r_t^{i,H}] = V[\Delta d_t^{i,H}] + V[r_t^{i,H}] - 2Cov[\Delta d_t^{i,H}, r_t^{i,H}].$$

We can also derive an expression for the expected future real bond yield (the risk-free rate):

$$y_t^H(1) \equiv E_t\left[\sum_{j=1}^{\infty}\left(\kappa_1^i\right)^{-j} y_{t+j-1}(1)\right] = \frac{-A(1)}{\kappa_1^i - 1} + \frac{\rho}{\kappa_1^i - \rho_x} x_t + \frac{-C(1)}{\kappa_1^i - \nu_1}(\sigma_t^2 - \bar{\sigma}^2) \tag{59}$$

$$\tag{60}$$

This makes it clear that the first term in the expected future stock return expression is a risk-free rate effect. It drops out once we look at expected excess returns:

$$r_t^{i,e,H} = r_t^{i,H} - y_t^H(1) = \frac{r_0^i + A(1)}{\kappa_1^i - 1} + \frac{\left[\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1) + C(1)\right]}{\kappa_1^i - \nu_1}(\sigma_t^2 - \bar{\sigma}^2)$$

The covariance between expected future dividend growth and expected future returns:

$$\begin{aligned} Cov\left(r_t^{i,H}, \Delta d_t^{i,H}\right) &= \frac{\phi_{d,x}^i \rho}{(\kappa_1^i - \rho_x)^2} V[x_t] + \frac{\phi_{d,\sigma}^i [\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1)]}{(\kappa_1^i - \nu_1)^2} V[\sigma_t^2 - \bar{\sigma}^2] \\ &\quad + \frac{\rho\phi_{d,\sigma}^i + \phi_{d,x}^i[\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1)]}{(\kappa_1^i - \rho_x)(\kappa_1^i - \nu_1)} Cov[x_t, \sigma_t^2 - \bar{\sigma}^2]. \end{aligned}$$

It is similar to that of expected one-period ahead dividend growth and expected returns in equation (57). Indeed, if $\rho_x = \nu_1$, then it equals the latter covariance divided by $(\kappa_1^i - \rho_x)^2 = (\kappa_1^i - \nu_1)^2$. Note that when $\chi = 0$ and $\phi_{d,\sigma}^i = 0$, only the first term survives in the covariance between expected returns and expected growth rates. Because $\rho > 0$ and $\phi_{d,x}^i > 0$, it is always positive. Furthermore, this is a pure risk-free rate effect: the correlation between the expected *excess* return and expected dividend growth rate is exactly zero when $\chi = 0$ and $\phi_{d,\sigma}^i = 0$. In our model, this is no longer necessarily the case because of the additional cash-flow effect coming through $\phi_{d,\sigma}^i \neq 0$:

$$\begin{aligned} Cov\left(r_t^{i,e,H}, \Delta d_t^{i,H}\right) &= \frac{\phi_{d,\sigma}^i [\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1) + C(1)]}{(\kappa_1^i - \nu_1)^2} V[\sigma_t^2 - \bar{\sigma}^2] \\ &\quad + \frac{\rho\phi_{d,\sigma}^i + \phi_{d,x}^i[\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1) + C(1)]}{(\kappa_1^i - \rho_x)(\kappa_1^i - \nu_1)} Cov[x_t, \sigma_t^2 - \bar{\sigma}^2]. \end{aligned}$$

Finally, for the consumption claim, we get the following expected future return, excess return, and growth expressions:

$$r_t^{c,H} = \frac{r_0^c}{\kappa_1^c - 1} + \frac{\rho}{\kappa_1^c - \rho_x} x_t - A_2^c(\sigma_t^2 - \bar{\sigma}^2) \tag{61}$$

$$\Delta c_t^H = \frac{\mu_c}{\kappa_1^c - 1} + \frac{1}{\kappa_1^c - \rho_x} x_t \tag{62}$$

$$r_t^{c,e,H} = \frac{r_0^c - A(1)}{\kappa_1^c - 1} + \frac{\frac{\alpha-1}{1-\rho} A_2^c(\kappa_1^c - \nu - 1) + .5\Gamma(0)}{(\kappa_1^c - \nu - 1)}(\sigma_t^2 - \bar{\sigma}^2) \tag{63}$$

The Term Structure of Equity We derive the price of dividend strips (or zero-coupon equity), the price of a claim to unit of dividends of firm i at some future date t. See also Lettau and Wachter (2007). Recall the log-linearized stock return expression for security or portfolio i:

$$r^i_{t+1} = \kappa^i_0 + \Delta d^i_{t+1} + pd^i_{t+1} - \kappa^i_1 pd^i_t. \tag{64}$$

Log price-dividend ratios on dividend strips of horizon τ, $\widetilde{pd}^i_t(\tau)$, are affine in the state vector:

$$\widetilde{pd}^i_t(\tau) = A^m(\tau) + B^m(\tau)x_t + C^m(\tau)(\sigma^2_t - \bar{\sigma}^2),$$

where the coefficients $A^m(\tau)$, $B^m(\tau)$, and $C^m(\tau)$ follow recursions

$$\begin{aligned}
A^m(\tau+1) &= A^m(\tau) + m_0 + \mu^i_d + \frac{1}{2}\Gamma^m(\tau)\bar{\sigma}^2, \\
B^m(\tau+1)' &= \rho_x B^m(\tau) + \phi^i_{d,x} - \rho, \\
C^m(\tau+1)' &= \nu_1 C^m(\tau) + \phi^i_{d,\sigma} + \frac{\alpha-\rho}{1-\rho}(\kappa^c_1 - \nu_1)A^c_2 + \frac{1}{2}\Gamma^m(\tau), \\
\Gamma^m(\tau) &= \alpha^2 + \varphi^2_d - 2\alpha\varphi_d\iota + \left[B^m(\tau) - \frac{\alpha-\rho}{1-\rho}A^c_1\right]^2\varphi^2_e + \left[C^m(\tau) - \frac{\alpha-\rho}{1-\rho}A^c_2\right]^2\sigma^2_w \\
&\quad +2\chi\left[B^m(\tau) - \frac{\alpha-\rho}{1-\rho}A^c_1\right]\left[C^m(\tau) - \frac{\alpha-\rho}{1-\rho}A^c_2\right]\varphi_e\sigma_w
\end{aligned}$$

initialized at $A^m(0) = 0$, $B^m(0) = 0$, and $C^m(0) = 0$.

Proof. We conjecture that the log $t+1$-price of a τ-period strip, scaled by the dividend in period $t+1$, is affine in the state

$$\widetilde{pd}^i_{t+1}(\tau) = A^m(\tau) + B^m(\tau)'x_{t+1} + C^m(\tau)(\sigma^2_{t+1} - \bar{\sigma}^2)$$

and solve for the coefficients $A^m(\tau+1)$, $B^m(\tau+1)$, and $B^m(\tau+1)$ in the process of verifying this conjecture using the Euler equation:

$$\begin{aligned}
\exp\widetilde{pd}^i_t(\tau+1) &= E_t\left[\exp\{m_{t+1} + \Delta d^i_{t+1} + \widetilde{pd}^i_{t+1}(\tau)\}\right] \\
&= \exp\{m_0 + \mu^i_d + A^m(\tau) + (\phi^i_{d,x} - \rho + B^m(\tau)\rho_x)x_t + \left[\phi^i_{d,\sigma} + \frac{\alpha-\rho}{1-\rho}(\kappa^c_1 - \nu_1)A^c_2 + C^m(\tau)\nu_1\right](\sigma^2_t \\
&\quad E_t\left[\exp\{-\alpha\sigma_t\eta_{t+1} + \left[B^m(\tau) - \frac{\alpha-\rho}{1-\rho}A^c_1\right]\varphi_e\sigma_t e_{t+1} + \left[C^m(\tau) - \frac{\alpha-\rho}{1-\rho}A^c_2\right]\sigma_w\sigma_t w_{t+1} + \varphi_d\sigma_t u_{t+}\right.
\end{aligned}$$

We use the log-normality of the shocks to work out the expectation. Taking logs and collecting terms, we obtain the conjectured linear expression for $pd^i_t(\tau+1)$ where the recursions are as given above. □

B Calibration

Our goal is to ask whether the model can quantitatively account for the patterns in CP betas and returns we document in the main text. Therefore, we refrain from a wholesale re-calibration or estimation of the long-run risk model and make minimal deviations from the literature. Table 1 compares our calibration in the first column to the calibration in Bansal and Shaliastovich (2007) (Column 2) and that of the original Bansal and Yaron (2004) (Column 3). All parameters are quarterly since we solve the model at quarterly frequency and compare it to quarterly data. This avoids complications arising from time aggregation. Our parameters are identical to those in Bansal and Shaliastovich (2007), with the following exceptions. First, we introduce heteroscedasticity in $\sigma_t^2 - \bar{\sigma}^2$. That is, we replace $\sigma_w w_{t+1}$ by $\sigma_w \sigma_t w_{t+1}$ in equation (7), and multiply our value of σ_w by $\bar{\sigma}$ to keep the unconditional variance of economic uncertainty the same. This change is inconsequential. Second, we estimate ν_1, the autocorrelation coefficient of economic uncertainty $\sigma_t^2 - \bar{\sigma}^2$. Third, we estimate the parameter χ, which governs correlation between shocks to x_t and σ_t^2. Fourth, we estimate the loadings of dividend growth $\phi^i_{d,\sigma}$ on $\sigma_t^2 - \bar{\sigma}^2$ and $\phi^i_{d,x}$ on x_t for each portfolio. We use 10 "book-to-market" portfolios and 1 "market" portfolio. Prior to the estimation, we set the unconditional mean log real dividend growth rate, μ^i_d, equal to its observed value in the data for each of the 11 portfolios. We then estimate $2+2\times 11 = 24$ parameters in order to minimize the distance between model and data along the following 38 dimensions: 11 contemporaneous log excess stock return CP betas, 11 contemporaneous log real dividend growth CP betas, 11 equity risk premia, and 5 lagged bond return CP betas for maturities 1-, 2-, 5-, 7-, and 10-years. The estimation is by non-linear least squares.

B.1 Mapping from Monthly to Quarterly Parameters

The Bansal-Yaron model is calibrated and parameterized to monthly data. Since we want to use data on quarterly consumption and dividend growth, and a quarterly series for the wealth-consumption ratio, we recast the model at quarterly frequencies. We assume that the quarterly process for consumption growth, dividend growth, the low frequency component and the variance has the exact same structure than at the monthly frequency, with mean zero, standard deviation 1 innovations, but with different parameters. This appendix explains how the monthly parameters map into quarterly parameters. We denote all variables, shocks, and all parameters of the quarterly system with a tilde superscript. Our parameter values are listed at the end of this section, together with details on the simulation approach.

Obviously, the preference parameters do not depend on the horizon ($\tilde{\alpha} = \alpha$ and $\tilde{\rho} = \rho$), except for the time discount factor $\tilde{\beta} = \beta^3$. Also, the long-run average log wealth-consumption ratio at the quarterly frequency is lower than at the monthly frequency by approximately log(3), because log of quarterly consumption is the log of three times monthly consumption.

We accomplish this by matching the conditional and unconditional mean and variance of log consumption and dividend growth. Log quarterly consumption growth is the sum of log consumption growth of three consecutive months. We obtain $\Delta\tilde{c}_{t+1} \equiv \Delta c_{t+3} + \Delta c_{t+2} + \Delta c_{t+1}$

$$\Delta\tilde{c}_{t+1} = 3\mu_c + (1+\rho_x+\rho_x^2)x_t + \sigma_t\eta_{t+1} + \sigma_{t+1}\eta_{t+2} + \sigma_{t+2}\eta_{t+3} + (1+\rho_x)\varphi_e\sigma_t e_{t+1} + \varphi_e\sigma_{t+1}e_{t+2} \quad (65)$$

Log quarterly dividend growth looks similar:

$$\Delta\tilde{d}_{t+1} = 3\mu_d + \phi(1+\rho_x+\rho_x^2)x_t + \varphi_d\sigma_t u_{t+1} + \varphi_d\sigma_{t+1}u_{t+2} + \varphi_d\sigma_{t+2}u_{t+3} + \phi(1+\rho_x)\varphi_e\sigma_t e_{t+1} + \phi\varphi_e\sigma_{t+1}e_{t+2} \quad (66)$$

First, we rescale the long-run component in the quarterly system, so that the coefficient on it in the consumption growth equation is still 1:

$$\tilde{x}_t = (1+\rho_x+\rho_x^2)x_t.$$

Table 1: Quarterly Calibration

Parameter	*Our model*	*BS 2008*	*BY 2004*
	Preferences		
α	8	8	10
ρ	2/3	2/3	2/3
β	.9961	.9961	.9970
	Consumption Process		
μ	.0048	.0048	.0045
$\bar{\sigma}$	.0056	.0056	.0135
ρ_x	.9732	.9732	.9383
φ_e	.1617	.1617	.1264
ν_1	EST	.9880	.9615
σ_w	.1984e-5/$\bar{\sigma}$	.1984e-5	.3932e-5
χ	EST	NA	NA
	Dividend Process		
μ_d^m	.0045	.0048	.0045
φ_d^m	5.9860	5.9860	4.4960
$\phi_{d,x}^m$	EST	1.5	3
ι^m	.10	.10	NA
$\phi_{d,\sigma}^m$	EST	NA	NA
	Inflation Process		
μ_π	.0096	.0096	NA
$\varphi_{\pi,\eta}$	0	0	NA
$\varphi_{\pi,e}$	-2	-2	NA
σ_π	.0061	.0061	NA
ρ_π	.5718	.5718	NA
ϕ_π	-.35	-.35	NA
$\zeta_{\pi,\eta}$	0	0	NA
$\zeta_{\pi,e}$	-1	-1	NA
σ_z	6.9282e-6	6.9282e-6	NA

Notes: This Table lists our benchmark parameter choices. All parameters are expressed in quarterly units. See Appendix B.1 on how we go from the monthly parameters of Bansal and Shaliastovich (2007) to the quarterly values in the second column and from the monthly values of Bansal and Yaron (2004) to the quarterly values in the third column.

Second, we equate the unconditional mean of consumption and dividend growth :

$$\tilde{\mu} = 3\mu,\ \tilde{\mu}_d = 3\mu_d.$$

These imply that we also match the the conditional mean of consumption growth:

$$E_t[\Delta c_{t+3} + \Delta c_{t+2} + \Delta c_{t+1}] = 3\mu + (1 + \rho_x + \rho_x^2)x_t = \tilde{\mu} + \tilde{x}_t = E_t[\Delta \tilde{c}_{t+1}]$$

Third, we also match the conditional mean of dividend growth by setting the quarterly leverage parameter $\tilde{\phi} = \phi$.

Fourth, we match the unconditional variance of quarterly consumption growth:

$$\begin{aligned}
V[\Delta\tilde{c}_{t+1}] &= (1+\rho_x+\rho_x^2)^2 V[x_t] + \sigma^2\left[3+(1+\rho_x)^2\varphi_e^2+\varphi_e^2\right] \\
&= (1+\rho_x+\rho_x^2)^2 \frac{\varphi_e^2\sigma^2}{1-\rho_x^2} + \sigma^2\left[3+(1+\rho_x)^2\varphi_e^2+\varphi_e^2\right] \\
&= \frac{\tilde{\varphi}_e^2\tilde{\sigma}^2}{1-\tilde{\rho}_x^2} + \tilde{\sigma}^2
\end{aligned}$$

The first and second equalities use the law of iterated expectations to show that

$$V[\sigma_{t+j}\eta_{t+j+1}] \equiv E\left[E_{t+j}\left\{\sigma_{t+j}^2\eta_{t+j+1}^2\right\}\right] - \left(E\left[E_{t+j}\left\{\sigma_{t+j}\eta_{t+j+1}\right\}\right]\right)^2 = E\left[\sigma_{t+j}^2\right] - 0 = \sigma^2$$

and the same argument applies to terms of type $V[\sigma_{t+j}e_{t+j+1}]$. Coefficient matching on the variance of consumption expression delivers expressions for $\tilde{\sigma}^2$ and $\tilde{\varphi}_e$:

$$\tilde{\sigma}^2 = \sigma^2\left[3+(1+\rho_x)^2\varphi_e^2+\varphi_e^2\right]$$

$$\begin{aligned}
\tilde{\varphi}_e^2 &= \varphi_e^2 \frac{(1-\tilde{\rho}_x^2)(1+\rho_x+\rho_x^2)^2}{1-\rho_x^2}\frac{\sigma^2}{\tilde{\sigma}^2} \\
&= \frac{(1-\rho_x^6)(1+\rho_x+\rho_x^2)^2}{1-\rho_x^2}\frac{\varphi_e^2}{3+(1+\rho_x)^2\varphi_e^2+\varphi_e^2},
\end{aligned}$$

where the third equality uses the first equality. Note that we imposed $\tilde{\rho}_x = \rho_x^3$, which follows from a desire to match the persistence of the long-run cash-flow component. Recursively substituting, we find that the three-month ahead x process has the following relationship to the current value:

$$x_{t+3} = \rho_x^3 x_t + \varphi_e\sigma_{t+2}e_{t+3} + \rho_x\varphi_e\sigma_{t+1}e_{t+2} + \rho_x^2\varphi_e\sigma_t e_{t+1}$$

which compares to the quarterly equation

$$\tilde{x}_{t+1} = \tilde{\rho}_x\tilde{x}_t + \tilde{\varphi}_e\tilde{\sigma}_t\tilde{e}_{t+1}$$

The two processes now have the same auto-correlation and unconditional variance.
Fifth, we match the unconditional variance of dividend growth. Given the assumptions we have made sofar, this pins down φ_d:

$$\tilde{\varphi}_d^2 = \frac{3\varphi_d^2 + \phi^2(1+\rho_x)^2\varphi_e^2 + \phi^2\varphi_e^2}{3+(1+\rho_x)^2\varphi_e^2+\varphi_e^2}$$

Sixth, we match the autocorrelation and the unconditional variance of economic uncertainty σ_t^2. Iterating forward, we obtain an expression that relates variance in month t to the one in month $t+3$:

$$\sigma_{t+3}^2 - \sigma^2 = \nu_1^3(\sigma_t^2-\sigma^2) + \sigma_w\nu_1^2 w_{t+1} + \sigma_w\nu_1 w_{t+2} + \sigma_w w_{t+3}$$

By setting $\tilde{\nu}_1 = \nu_1^3$ and $\tilde{\sigma}_w^2 = \sigma_w^2(1+\nu_1^2+\nu_1^4)$, we match the autocorrelation and variance of the quarterly equation

$$\tilde{\sigma}_{t+1}^2 - \tilde{\sigma}^2 = \tilde{\nu}_1(\tilde{\sigma}_t^2 - \tilde{\sigma}^2) + \tilde{\sigma}_w\tilde{w}_{t+1}$$

C Results

We now discuss the estimation results in detail. The main text of the paper summarizes the findings, focussing on the intuition.

C.1 Standard LRR Model

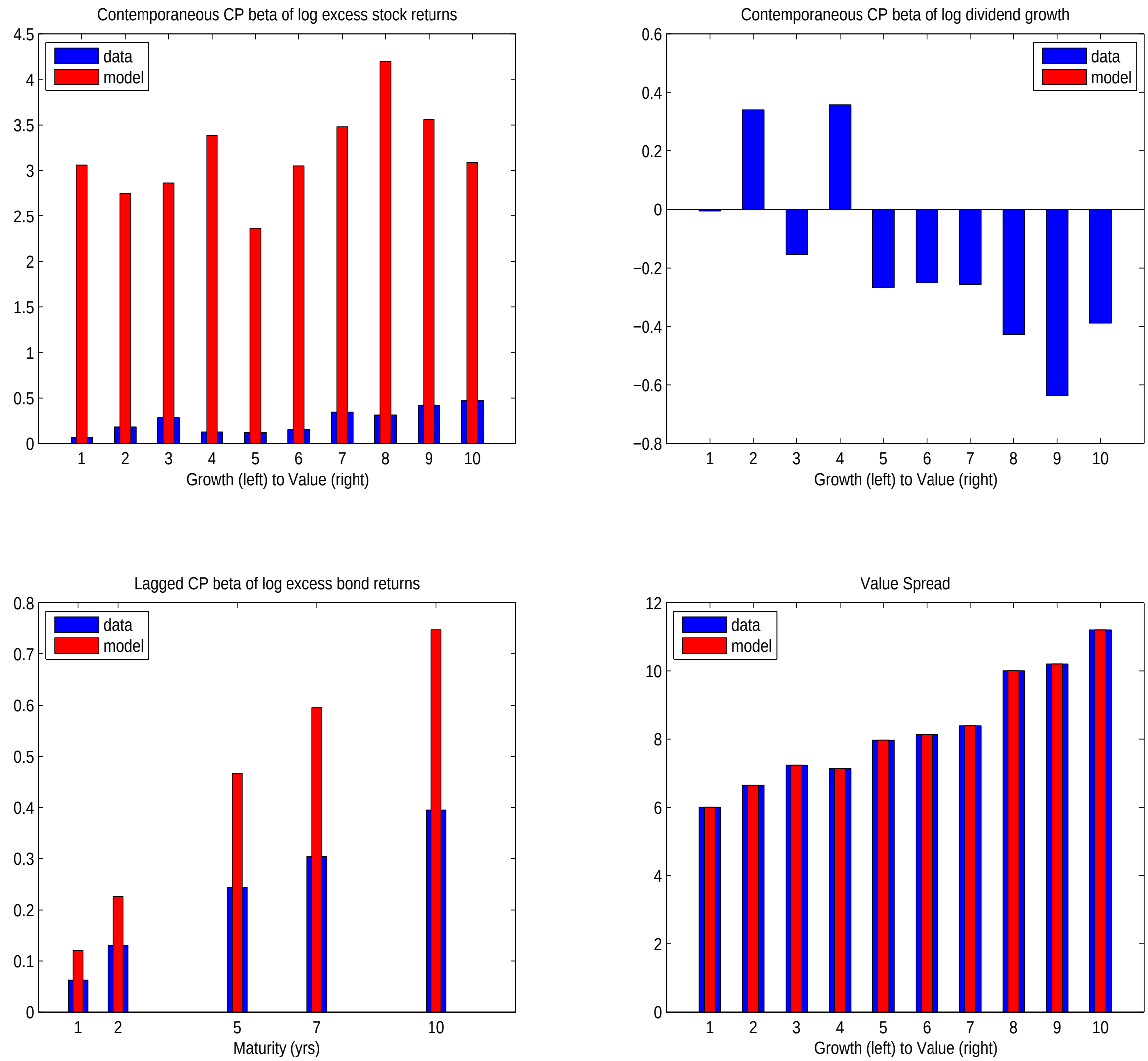
As a benchmark, Figure 1 shows these 38 moments for the Bansal and Shaliastovich (2007) calibration, henceforth B-S calibration. This calibration sets $\chi = 0$ and $\phi^i_{d,\sigma} = 0$. It also features a high value for ν_1, so that economic uncertainty is very persistent, in fact more persistent than long-run growth ($\nu_1 > \rho_x$). To give this model its best chance at matching the value premium, we estimate $\phi^i_{d,x}$ in order to match risk premia on each portfolio, following Bansal, Dittmar, and Lundblad (2005), Bansal, Kiku, and Yaron (2006), and Hansen, Heaton, and Li (2008). The bottom right panel shows that the model is flexible enough to exactly match not only the equity risk premium on the market portfolio but also the risk premia on each of the 10 book-to-market portfolios. The calibration accomplishes this by having value stocks carry more long-run risk: $\phi^i_{d,x}$ is higher for value than for growth. In other words, value stocks are risky because their dividend growth is high in states of the world where long-run consumption growth x_t is high. These are states of low marginal utility growth for the representative investor. Panel A of 2 shows that $\phi^i_{d,x}$ ranges from 3.3 for BM1 to 5.3 for BM10. It is 3.5 for the market portfolio. A higher $\phi^i_{d,x}$ translates in a higher A^i_1, which increases the equity risk premium through the second term in equation (**??**).

However, inspection of the two top panels of Figure 1 shows that this calibration fails to generate the observed relationship between the CP factor and excess stock returns and dividend growth rates. First, equation (43) shows that when both $\phi^i_{d,\sigma} = 0$ and $\chi = 0$, the contemporaneous dividend growth betas are zero for all portfolios. Because expected dividend growth does not depend on economic uncertainty σ^2_t, it is uncorrelated with CP (which perfectly correlates with economic uncertainty). Second, the excess stock return betas do not show the increasing pattern we found in the data, and they are at least a factor of five too big. Equation (37) helps us understand why. The innovation betas are all negative because $\chi = 0$ and $A^i_2 < 0$. They become increasingly negative from growth to value: from -8 to -16. The lagged excess return betas in (38) are the sum of $\phi^i_{d,\sigma}$, $-A^i_2(\kappa^i_1 - \nu_1)$, which increases from 360 to 630 from growth to value, and $\frac{\alpha-\rho}{1-\rho}A^c_2(\kappa^c_1 - \nu_1) + .5\Gamma(0) = -993 + 1,046 = 53$. Given $b_{CP} = 35$, when $\phi^i_{d,\sigma} = 0$, these lagged excess stock return betas range from 11 to 19. Combining the two results in contemporaneous betas that are too high without a pronounced pattern from growth to value. Finally, he first beta on the right hand side of equation (**??**) is zero when $\chi = 0$ and $\phi^i_{d,\sigma} = 0$. The second term is negative: high bond risk premia (or positive innovations in CP) coincide with negative innovations in expected future stock returns, which seems counter-intuitive. In addition, there is no clear pattern in this beta difference as opposed to the strongly increasing pattern in the data.

C.2 Results for Our Calibration

Our estimation sets $\chi = .2048 > 0$ and $\phi^i_{d,\sigma} < 0$ in order to match the 38 moments. It also substantially lowers the value for ν_1 from .9880 to .9296 ($\nu_1 < \rho_x$). The lower persistence we estimate in economic uncertainty facilitates the interpretation of σ^2_t (or CP_t) as a business cycle variable. Figure 2 shows that our calibration does a good job matching the patterns and magnitudes of the CP exposures of excess stock returns, dividend growth rates, while maintaining the good

Figure 1: Risk Premia and Betas for $\chi = 0$ and $\phi^i_{d,\sigma} = 0$



fit for the value premium. What drives these results? First, a sufficiently negative $\phi^i_{d,\sigma}$ directly generates a negative dividend growth beta. Panel B of Table 2 shows that the estimation chooses negative loadings $\phi^i_{d,\sigma}$ for all portfolios. They range from -537 for growth (BM1) and decrease monotonically to -883 for value (BM10). According to this channel, stocks, and in particular value stocks, have lower expected dividend growth when economic uncertainty, and therefore CP, is high. Second, this makes them risky because they have low cash flow growth exactly when marginal utility

Table 2: Cash-Flow Parameters and the Value Premium

In Panel A, we use the benchmark calibration of Bansal and Shaliastovich (2007). In panel B, we use our own calibration. The first column reports the unconditional mean dividend growth rate. The second column reports the loading of dividend growth on the long-run risk state variable x_t. The third column reports the loading of dividend growth on the economic uncertainty state variable σ_t^2. The fourth column reports the equity risk premium, the expected excess log stock return (including a Jensen adjustment). The last column reports the mean price-dividend ratio.

Panel A: B-S Calibration

	μ_d^i	$\phi_{d,x}^i$	$\phi_{d,\sigma}^i$	Risk premium	PD
market	1.79	3.52	0	6.82	16.56
BM1	0.60	3.30	0	6.01	15.13
BM2	1.65	3.45	0	6.65	16.53
BM3	2.08	3.70	0	7.24	16.54
BM4	1.41	3.82	0	7.14	15.07
BM5	3.16	3.85	0	7.97	18.21
BM6	2.69	4.09	0	8.14	16.48
BM7	2.49	4.31	0	8.39	15.57
BM8	3.17	5.17	0	10.01	14.83
BM9	3.86	5.05	0	10.20	16.16
BM10	4.89	5.33	0	11.21	17.39

Panel B: Own Calibration

	μ_d^i	$\phi_{d,x}^i$	$\phi_{d,\sigma}^i$	Risk premium	PD
market	1.79	4.42	-575.17	6.81	17.17
BM1	0.60	4.17	-536.69	6.09	15.30
BM2	1.65	4.35	-558.70	6.66	17.05
BM3	2.08	4.71	-615.94	7.32	17.11
BM4	1.41	4.74	-614.86	7.13	15.64
BM5	3.16	4.92	-655.90	8.02	19.31
BM6	2.69	5.16	-687.54	8.18	17.43
BM7	2.49	5.39	-716.69	8.43	16.49
BM8	3.17	6.28	-854.99	9.97	16.30
BM9	3.86	6.22	-851.68	10.22	17.96
BM10	4.89	6.39	-883.53	11.00	20.48

growth is high. Hence the value premium. This $\phi_{d,\sigma}^i$ effect reinforces the $\phi_{d,x}^i$ effect. Panel B shows that value stocks continue to have higher loadings on long run growth x_t than growth stocks, just as in Panel A. The dividend growth properties of the market portfolio resemble those of the third growth decile portfolio, consistent with the finding in Jurek and Viceira (2006) that growth stocks make up about 70% of the U.S. market capitalization.

Because χ is estimated to be .20, x_t and σ_t^2 have a conditional correlation of 20%. Positive correlation between the two state variables implies that times with higher growth are also times with more economic uncertainty. Such an economy is less risky relative to the one with $\chi = 0$ because the shocks to uncertainty (which causes prices to fall) are hedged by a higher long-term growth rate. The interest rate increases (slightly) because the demand for bonds is lower because of reduced precautionary savings motives. Because the economy is less risky, the price dividend ratios on all assets are higher and the risk premia lower. This reduction in risk is why all $\phi^i_{d,x}$ values are somewhat higher in Panel B ($\chi = 0.2$) than in Panel A ($\chi = 0$). Third, the positive χ helps to increase the excess stock return innovation betas, while the negative $\phi^i_{d,\sigma}$ helps to lower the lagged excess stock return betas. Both help to match the observed magnitude of the contemporaneous excess stock return betas. A positive χ suggests that economic uncertainty is highest at the end of a recession, when growth is about to resume. Such a positive correlation arises endogenously in models of information production (Van Nieuwerburgh and Veldkamp 2006). It also increases the correlation between expected returns and expected dividend growth. (Binsbergen and Koijen 2007) estimate this correlation for the aggregate stock market and conclude it is positive. In both our model and the B-S calibration, the lagged excess bond return betas are somewhat too high. This is not surprising, in the model bond returns and CP are both driven by σ^2_t, with no offsetting effect from cash-flow growth as in the case of stocks. Finally, our calibration predicts a strongly increasing pattern in the beta difference of equation (**??**), just as in the data. The first beta on the right hand side is positive and increasing from growth to value. A high bond risk premium (at the end of a recession or the beginning of a boom) implies good news about future dividend growth, and more so for value stocks than for growth stocks. The second term is also positive and increasing. A high bond risk premium (at the end of a recession) coincides with a high equity risk premium, and more so for value than for growth stocks. The cash-flow effect dominates the discount rate effect.

Matching these betas does not come at the expense of matching the moments of consumption, interest rates, or inflation, for the most part because we kept these parameters at their standard values. Table 3 plots unconditional means, standard deviations (both expressed in percent per year), and quarterly autocorrelation in the benchmark B-S calibration (first 3 columns) and in our calibration (last 3 columns). The moments in our model are both close to the ones in the B-S calibration and close to the data.

Figure 2: Risk Premia and Betas for $\chi > 0$ and $\phi^i_{d,\sigma} < 0$

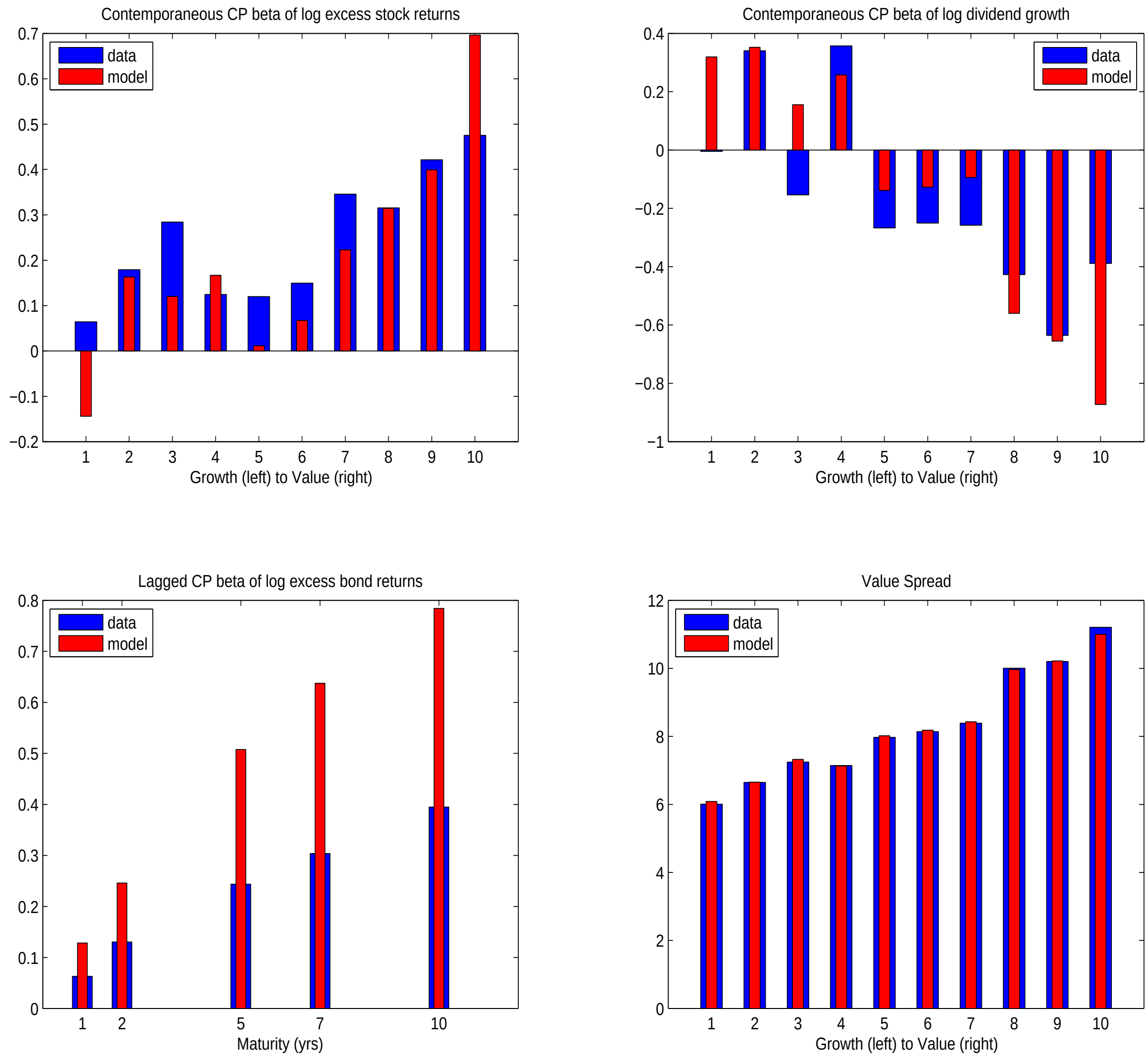


Table 3: Other Moments of Interest

This table reports unconditional means (expressed in percent per year), standard deviations (expressed in percent per year), and the quarterly first-order autocorrelation in the benchmark B-S calibration (first 3 columns) and in our calibration (last 3 columns). The moments reported for stocks are for the market portfolio, and are denoted with a ^{m} superscript. The notation follows the main text.

	Panel A: B-S Calibration			**Panel B: Our Calibration**		
	mean	stdev	ac(1)	mean	stdev	ac(1)
x	0.00	0.78	0.97	0.00	0.78	0.97
σ^2	0.0123	0.0026	0.99	0.0123	0.0011	0.93
$\bar{\pi} - \mu_\pi$	3.84	0.84	0.84	3.84	0.84	0.84
Δc	1.92	1.36	0.32	1.92	1.36	0.32
Δd^m	1.79	7.20	0.14	1.79	7.50	0.21
π	3.84	1.52	0.29	3.84	1.52	0.29
$y(1)$	2.19	0.54	0.97	2.34	0.52	0.97
$y^\$(1)$	6.18	0.52	0.63	6.33	0.51	0.62
$y^\$(20)$	6.30	0.15	0.92	6.48	0.14	0.91
$y^\$(20) - y^\$(1)$	0.12	0.73	0.11	0.15	0.73	0.11
$E_t\left[r_{t+1}^{b,e,\$}(20)\right]$	-0.94	0.20	0.99	-0.76	0.07	0.93
$E_t\left[r_{t+1}^{b,e,\$}(20)\right]$	0.21	0.04	0.99	0.26	0.02	0.93
CP	0.45	0.09	0.99	0.57	0.04	0.93
WC	67.75	5.66	0.98	74.83	4.29	0.97
PD^m	16.56	32.77	0.98	16.89	34.77	0.97
$r_{t+1}^m - y_t(1)$	5.68	15.15	0.01	5.42	17.23	0.00
$E_t[r_{t+1}^m - y_t(1)]$	5.68	1.17	0.99	5.42	0.73	0.95
$E_t[\Delta d_{t+1}^m]$	1.79	2.75	0.97	1.79	3.46	0.97
$E_t[r_{t+1}^{e,m}]$	6.82	1.42	0.99	6.90	0.60	0.93

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