

Financial Distress and the Cross Section of Equity Returns*

Lorenzo Garlappi[†]
University of Texas at Austin

Hong Yan[‡]
University of South Carolina

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[†]Department of Finance, McCombs School of Business, University of Texas at Austin, Austin, TX 78712. Email: lorenzo.garlappi@mcombs.utexas.edu

[‡]Department of Finance, Moore School of Business, University of South Carolina, Columbia, SC 29208. Email: yanh@moore.sc.edu.

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Abstract

In this paper, we provide a new perspective for understanding cross-sectional properties of equity returns. We explicitly introduce financial leverage in a simple equity valuation model and consider potential shareholder recovery upon the resolution of financial distress. Our model demonstrates the amplifying effect of leverage on the book-to-market effect and generates two novel predictions about the cross section of equity returns: (i) the value premium is hump-shaped with respect to default probability, and (ii) momentum profits are concentrated among high credit-risk firms with significant expected shareholder recovery upon financial distress. These results are robust in a more general model with endogenous dynamic investment and financing decisions and are supported by our empirical analysis. Using data simulated from a calibrated version of the general model in which a conditional CAPM holds, we illustrate how our framework can accommodate the appearance of significant alphas in the cross section of stock returns.

JEL Classification Codes: G12, G14, G33

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1 Introduction

A firm’s financial leverage and the ensuing possibility of financial distress crucially affect its equity returns, making their dynamics fundamentally different from that of asset returns. In the empirical asset pricing literature, the risk of financial distress has frequently been invoked to justify the presence of cross-sectional CAPM “anomalies” such as the size effect and the value premium (e.g., Fama and French (1996)). The existing empirical evidence, however, appears to elude a coherent and unifying explanation. On the one hand, Griffin and Lemmon (2002) show that the value premium is most significant among firms with high probability of financial distress, and Vassalou and Xing (2004) demonstrate that both the size and the book-to-market effects are concentrated in high default risk firms. On the other hand, Dichev (1998) and Campbell, Hilscher, and Szilagyi (2008) document that high-default-risk firms tend to have lower future stock returns, casting doubt on the notion of a market compensation for distress risk. Furthermore, Avramov, Chordia, Jostova, and Philipov (2007) provide evidence that momentum profits are concentrated among firms with low credit ratings, thus establishing a link between financial distress and momentum profitability in stock returns.

In this paper we show theoretically that the above empirical patterns can be accounted for in a simple equity valuation model in which a levered firm may face financial distress and, upon its resolution, shareholders may recover part of the firm’s assets. At the heart of this mechanism is the fact that the potential shareholder recovery upon financial distress fundamentally alters the risk structure of equity as default probability increases. For low levels of default probability, higher leverage increases equity beta. At high levels of default probability, however, the possibility of debt renegotiation and asset redistribution during financial distress gradually de-levers equity beta, and thus reduces equity risk. This mechanism causes the equity beta, and hence the expected return on equity, to be *hump-shaped* in default probability. We argue that this feature is capable of simultaneously explaining two documented empirical regularities: (i) the inverse relationship between expected return and default probability (see *e.g.*, Dichev (1998), Campbell, Hilscher, and Szilagyi (2008), and Garlappi, Shu, and Yan (2006)); (ii) the concentration of momentum profits in low-credit-quality firms documented in Avramov, Chordia, Jostova, and Philipov (2007). Moreover, we show that the same mechanism predicts a hump-shaped relation-

ship between the value premium and default probability, a novel prediction that we confirm in our empirical analysis.

Our work builds on the growing literature, stemming from Berk, Green, and Naik (1999), that provides rational explanations for value premia or momentum profits by showing that optimal firm-level investment decisions generate asset betas that are naturally dependent on firm characteristics, such as the book-to-market ratio.¹ By ignoring financial leverage, however, these asset-based models are not equipped to explain the recent empirical evidence on the relationships between cross-sectional stock return patterns and financial distress.

We explicitly introduce financial leverage in an equity valuation framework similar to Berk, Green, and Naik (1999) and show that, while the book-to-market effect is similarly embedded in the equity beta, the presence of financial leverage amplifies its magnitude as the likelihood of financial distress increases. Intuitively, this happens because equity is *de facto* a call option on the firm's assets, and hence its beta increases with leverage and, *ceteris paribus*, with default probability.² The presence of leverage therefore provides an explanation for the concentration of the book-to-market effect in low-credit-quality firms (Griffin and Lemmon (2002) and Vassalou and Xing (2004)) and justifies the absence of such an effect in the returns of unlevered assets (Hecht (2000)).

An important novel feature of our modeling framework is an explicit consideration of the potential shareholder recovery upon the resolution of financial distress. Most cases of financial distress do not end in bankruptcy and instead involve varying degrees of strategic interactions between debtholders and shareholders that lead to redistribution of assets among claim-holders (e.g., Fan and Sundaresan (2000)). Indeed, in a structural estimation of dynamic capital structure choices of U.S. firms, Morellec, Nikolov, and Schürhoff (2008) find a wide range of such redistributions with a mean estimate of shareholder recovery around 20% of firm value.³ This evidence highlights the pervasive and persistent presence of shareholder recovery upon financial

¹The related literature includes Gomes, Kogan, and Zhang (2003), Carlson, Fisher, and Giammarino (2004), Zhang (2005), Cooper (2006), Gala (2006), Sagi and Seasholes (2007) and Novy-Marx (2008).

²Default refers to financial distress, which includes instances of missed payments, modified terms and structure of debt in private workouts, and, ultimately, bankruptcy filings. In this paper, we use the terms "default" and "financial distress" interchangeably.

³Even in the case of bankruptcy filings, there have been deviations from the absolute priority rule as documented in Franks and Torous (1989), Eberhart, Moore, and Roenfeldt (1990), Weiss (1991), and Betker (1995). While Bharath, Panchapegesan, and Werner (2007) argue that such deviations have become less prevalent in recent years, Weiss and Capkun (2007) show that the length of the bankruptcy process with no violation of absolute priority tends to be long, potentially encouraging the avoidance of such costly procedures in resolving financial distress.

distress that has been largely ignored in the asset pricing literature thus far, except for Garlappi, Shu, and Yan (2006) who use this feature to explain the observed negative relationship between default probability and stock returns.

We show that the hump-shaped relationship between equity beta and default probability due to potential shareholder recovery can simultaneously affect the cross-sectional properties of both the value premium and momentum profits. Specifically, a hump shape in expected returns implies a similar pattern in the value premium as a function of default probability. The intuition is simple. Consider two firms with identical book value and default probability. One of them experiences a positive shock to its equity price and the other an equivalent negative shock. The former will see its book-to-market ratio decrease while the latter will see it increase. Suppose we form a portfolio long high book-to-market firms and short low book-to-market firms, all with a similar level of default probability. The expected return of this portfolio, which is the spread of the long and short positions, will depend on where these firms are located on the default probability spectrum. For firms with low levels of default probability, the relationship between expected return and default probability is upward sloping and hence the spread will be positive. Otherwise, for firms with relatively high level of default probability, the relationship is downward sloping and hence the spread will be negative. This suggests that, in general, the value premium will be conditional on the default probability of the firms in the spread portfolio, high and possibly increasing at low levels of default probability and low and possibly decreasing at high levels of default probability.

The hump-shaped relationship between default probability and expected return has also interesting implications for momentum profits in stock returns. All else being equal, as a firm's profitability and stock price decline, its probability of default increases. Because the hump shape implies that at high levels of default likelihood the equity expected return is decreasing in default probability, low (high) realized returns are followed by low (high) expected returns. In other words, our model predicts that return continuation is more likely to be concentrated within the group of firms with high default probabilities. This finding is consistent with the recent evidence in Avramov, Chordia, Jostova, and Philipov (2007). It is important to note that our model generates momentum in equity returns *without* assuming predictability (e.g., mean reversion) of the underlying fundamental process of revenues. Moreover, our mechanism

for generating momentum is different from that proposed in Sagi and Seasholes (2007), which relies on growth options.⁴

In order to check the robustness of the above results and make the implications more relevant for empirical work, we develop a general model in which a firm endogenously makes investment and financing decisions based on its existing capital level and debt load over the business cycle. Similar modeling frameworks have been used by Gomes and Schmid (2008), Lidvan, Sapriza, and Zhang (2008) and Obreja (2006) who examine the impact of financing frictions, leverage and investment on stock returns, without considering shareholder recovery. We confirm that the effects of shareholder recovery on the cross section of equity returns are robust and strong in this more general framework.

In our theoretical framework, cross-sectional stock returns are described by a conditional CAPM with a time-varying beta. However, the empirical literature usually refers to cross-sectional “anomalies” as the part of returns unexplained by beta and loadings on other presumed risk factors, i.e., anomalies are about patterns of alpha. Our general model offers a laboratory setting for reconciling these conflicting views and assessing the validity of the conventional risk adjustment when factor loadings are time-varying. Using a panel of simulated returns from our model to examine the relationship between stock returns and default probability, we find significant alphas with the traditional Fama and French (1993) risk-adjustment procedure. This means that an empiricist presented with our data would conclude in favor of the presence of an anomaly even though by construction the data come from a conditional CAPM model. Moreover, and quite strikingly, the risk-adjusted return from our simulated data demonstrates patterns of alpha and factor loadings similar to those documented in Campbell, Hilscher, and Szilagyi (2008). These findings imply that a proper characterization of the conditional equity beta is essential for understanding the equity return dynamics and for distinguishing true market mispricing from unwittingly born risk in equity investment.

The main contribution of our paper is to provide a unifying economic mechanism that can explain cross-sectional variations of the book-to-market effect and momentum profits simultaneously. A unique feature of our work is to highlight the important role of shareholder recovery upon financial distress in affecting equity risk. In light of the recent empirical evidence on the sig-

⁴We note that the mechanism proposed in Sagi and Seasholes (2007) may be more applicable to the momentum in growth stocks, such as high-tech firms, which do not usually have significant financial leverage. Our mechanism applies more suitably to firms with substantial leverage, similar to the subset of stocks studied in Avramov, Chordia, Jostova, and Philipov (2007).

nificance of shareholder recovery among U.S. firms (see Morellec, Nikolov, and Schürhoff (2008)), our paper offers a new perspective for analyzing cross-sectional features of equity returns.

The rest of the paper proceeds as follows. In the next section, we present a simple equity valuation model with financial leverage in order to develop an intuition for the unique role of financial distress. We generalize the modeling framework to a setting with endogenous investment and financing decisions in Section 3, and perform numerical analysis of the model in Section 4. We conclude in Section 5. All proofs are collected in Appendix A. The numerical procedure and parameter choice for the simple model are described in Appendix B and empirical evidence is presented in Appendix C.

2 A simple model of levered equity returns

In order to understand the risk structure that contributes to the cross-sectional properties of equity returns, we begin with a simple valuation model of levered equity. The model of this section is intentionally stylized in order to develop the main intuition in the cleanest possible way. We take a firm’s capital structure as given and ignore financing frictions. In Section 3 we generalize this setup by allowing for endogenous investment and financing decisions at the firm level over the business cycle.

We consider two types of firms: a “growth” firm, which has the option to make an investment to expand its scale, and a “mature” firm, which cannot change its operating scale and will produce at capacity for as long as the firm is alive. We assume that the price P of the output produced by each firm follows a geometric Brownian motion

$$dP = \mu P dt + \sigma P dW, \tag{1}$$

where μ is the growth rate of the product price, σ its volatility, and dW denotes the increment of a standard Brownian motion.⁵ Both μ and σ are firm-specific constants. This price process may be thought of as a revenue stream for a standard productive unit and is the sole source of uncertainty. We further assume that the risk premium associated with the price P is a positive constant λ , which is also firm-specific. Hence, under the risk-neutral measure, the risk-adjusted

⁵The process for the product price is the same as in the monopolistic setting studied in Carlson, Fisher, and Giammarino (2004). While the presence of competition in the product market may lead the product price to follow a mean-reverting process, our simplifying assumption allows for analytical tractability for the case of mature firms.

revenue stream from production obeys the following process:

$$dP = (\mu - \lambda)Pdt + \sigma PdW^*, \quad (2)$$

where dW^* is a standard Brownian motion under the risk-neutral measure. For ease of notation we will denote by δ the difference between the quantity $r + \lambda$ and the growth rate μ , i.e.,

$$\delta \equiv r + \lambda - \mu, \quad (3)$$

where r is the constant risk-free rate.

In the cross section, firms differ in their cost structure, financial leverage, operating scale, and growth opportunity. Our main focus is to examine how, in general, firms' characteristics affect equity expected returns and momentum, and, in particular, the effects of leverage and default probability on these relations. The product price P represents the state variable in our model, and we denote by $V(P)$ the market value of equity. The systematic risk of equity with respect to P is measured as

$$\beta = \frac{\partial \ln V(P)}{\partial \ln P}. \quad (4)$$

Hence, the expected return on equity is given by:⁶

$$\text{Expected Return} = r + \beta\lambda. \quad (6)$$

Equity exhibits positive *return autocorrelation* if its expected return increases with realized returns. The instantaneous return autocorrelation can thus be expressed as:⁷

$$\text{AutoCorr} = \lambda \frac{\partial \beta / \beta}{\partial P / P}. \quad (7)$$

Because return autocorrelation is a sufficient condition for the profitability of momentum strategies, we follow Johnson (2002) and Sagi and Seasholes (2007) and use the autocorrelation defined

⁶Note that β in expression (6) is *not* the CAPM beta, and our model is silent about the systematic risk structure of the product price process. The corresponding CAPM beta would be $\beta^{\text{CAPM}} = \beta \cdot \lambda / \lambda_m$ where λ_m is the risk premium of the market portfolio. Following Duffie and Zame (1989), we can obtain express the risk premium λ as

$$\lambda = SR \cdot \rho \cdot \sigma. \quad (5)$$

where SR is the maximal Sharpe ratio attainable in the economy, and $-\rho$ is the correlation of the price process P with the price kernel in the economy.

⁷See Proposition 1 in Sagi and Seasholes (2007).

in (7) as an indicator of momentum. In the simple model, momentum profits are associated with positive autocorrelation and reversals occur when autocorrelation is negative.

In the rest of this section we derive analytical expressions for the equity value of mature and growth firms and study cross-sectional properties of equity expected returns.

2.1 Mature firms

Mature firms have no access to growth options. Their value derives from the revenue stream generated by production for as long as the firm is alive. Producing one unit of goods requires an operating cost of c per unit of time. We assume that the firm operates at a fixed scale ξ . Hence the net profit from operation per each unit of time is equal to $\xi(P_t - c)$. The capital structure of the firm is characterized by a single issue of perpetual debt with a continuous and constant coupon payment of l . The profit after interest service is thus $\xi(P_t - c) - l$. We ignore tax considerations. In our analysis, we take the cross-sectional distribution of leverage levels l as given and do not consider the optimal capital structure decision of the firm.

Equity holders receive the stream of profits $\xi(P_t - c) - l$ as long as the firm is operating. When the firm encounters financial distress and defaults on its debt, we assume that equity holders can recover a fraction η of the residual firm's value $X^m(P)$, a generic non-negative quantity that can potentially depend on the underlying price process P . This assumption is a reduced-form representation of the potential asset redistribution as outcome of strategic renegotiations between creditors and shareholders upon financial distress (e.g., Fan and Sundaresan (2000)).⁸ Most cases of financial distress are resolved through debt reorganization in private workouts, with only a small portion of them going through bankruptcy filings. In their structural estimation of a dynamic capital structure model that incorporates such debt renegotiation, Morellec, Nikolov, and Schürhoff (2008) find that the parameter η has a wide cross-sectional variation among U.S. firms with a mean around 20% of firm value.

We model the residual value as a linear function of the underlying product price, $X^m(P) = a + bP$, $a, b > 0$. This choice includes situations in which, upon the resolution of financial distress, equity holders receive either a fixed payout (as we will argue later) or a stake in the restructured

⁸In the structural model of Fan and Sundaresan (2000), η is the product of shareholder bargaining power and liquidation costs, both taken to be deterministic quantities. While it is possible to consider the case of a stochastic η , such a choice does not seem to have a solid economic or empirical foundation. Moreover, adding this layer of complexity does not alter the basic intuition.

firm (as in Fan and Sundaresan (2000) and Garlappi, Shu, and Yan (2006)). The choice, however, does not affect the underlying intuition, but only the complexity of the formulation.

The equity value of a mature firm can therefore be expressed as follows:

$$V^m(P_t) = \mathbb{E}^* \left[\int_0^{\tau_L} (\xi(P_{t+s} - c) - l)e^{-rs} ds + \eta X^m(\underline{P}^m) e^{-r\tau_L} \right] \quad (8)$$

where $\tau_L = \inf \{t : P_t = \underline{P}^m\}$ denotes the first time price P hits the threshold \underline{P}^m , at which point the firm defaults. The threshold \underline{P}^m is chosen optimally by shareholders.⁹ The expectation \mathbb{E}^* is taken under the risk-neutral probability measure. The integrand in equation (8) represents the stream of profits received by equity holders until default. The last term represents the salvage value of equity upon default, which is a fraction η of the residual value $X^m(P)$. The following proposition characterizes the equity value of a mature firm and its endogenous default boundary.

Proposition 1 *Assume the residual firm value upon default is $X^m(P) = a + bP$, where $a > 0$, $0 \leq b < \xi/(\eta\delta)$. The equity value of a mature firm is given by*

$$V^m(P) = \begin{cases} \xi \left(\frac{P}{\delta} - \frac{c}{r} \right) - \frac{l}{r} + A_1 P^{\phi_1}, & \text{if } P > \underline{P}^m \\ \eta(a + bP), & \text{if } P = \underline{P}^m \end{cases}, \quad (9)$$

where $\delta = r + \lambda - \mu$, $\phi_1 < 0$ is the negative root of the characteristic equation

$$\frac{1}{2}\sigma^2\phi(\phi - 1) + (r - \delta)\phi - r = 0, \quad (10)$$

and

$$A_1 = \frac{1}{\phi_1} \left(\eta b - \frac{\xi}{\delta} \right) (\underline{P}^m)^{1-\phi_1} > 0, \quad \underline{P}^m = \frac{\eta a + \frac{\xi c + l}{r}}{\left(\frac{\xi}{\delta} - \eta b \right) \left(1 - \frac{1}{\phi_1} \right)} > 0. \quad (11)$$

The condition $0 \leq b < \xi/(\eta\delta)$ in the above proposition is imposed to guarantee that the limited liability condition is satisfied ($A_1 \geq 0$) and that the price at which the firm endogenously defaults is strictly positive, $\underline{P}^m > 0$.¹⁰ Substituting the expression of A_1 in (9) we obtain

$$V^m(P) = \xi \left(\frac{P}{\delta} - \frac{c}{r} \right) - \frac{l}{r} + \frac{\pi}{\phi_1} \left(\eta b - \frac{\xi}{\delta} \right) \underline{P}^m > 0, \quad (12)$$

⁹The endogenous choice of default boundary by shareholders is a common feature in theoretical models (see, e.g., Black and Cox (1976) and Leland (1994)). Empirically, Brown, Ciochetti, and Riddiough (2006) show that default decisions are endogenous responses to anticipated restructuring outcomes.

¹⁰We can think of A_1 as the position in put options representing the downside insurance to shareholders provided by the limited liability option.

where

$$\pi \equiv \pi(P, l, c, \sigma, \eta) = \mathbb{E}^* [e^{-r\tau_L}] = \left(\frac{P}{\underline{P}^m} \right)^{\phi_1} \quad (13)$$

is the risk-neutral probability of default. Expression (12) explicitly links equity value to financial leverage l and to a measure of default probability π . While for our theoretical derivations we will refer to π as the “probability of default”, in our numerical analysis we adhere to the industry practice and adopt a definition derived under the real probability measure, provided in Lemma 1 of Appendix A (equation (A8)). Notice, however, that the use of the risk-neutral probability of default π does not alter any of the properties we derive in this section since the two quantities are monotonically related.

From the expression of the equity value (12) it appears that financial leverage l does not have a substantially distinct role from operating leverage ξc . This observational equivalence between the two forms of leverage stems from the exogenous nature of both c and l in this simple model and will be resolved in the general version we develop in Section 3. However, it is important to point out that, even with exogenous operating and financial leverages, financial leverage serves an entirely different contractual role than that of operating leverage. The contractual obligation of shareholders to bondholders is binding and the outcome of the strategic interaction between them crucially determines the potential payoff to shareholders upon financial distress. In the absence of financial leverage, this interaction is absent and with it all the implications we derive in the rest of the paper.

Equity beta and the book to market effect

From the equity value derived in Proposition 1 and definition (4), we obtain the beta of a mature firm, as described in the following corollary.

Corollary 1 *The beta of a mature firm is given by*

$$\beta = 1 + \underbrace{\left(\frac{(\xi c - l)/r}{V^m} \right)}_{\text{BE/ME}} \underbrace{\left(\frac{(\xi c + l)}{(\xi c - l)} \left(1 - \pi \frac{\eta ar + \xi c + l}{\xi c + l} \right) \right)}_{\text{Leverage effect}}. \quad (14)$$

The firm’s revenue beta is normalized to 1. The term labeled “BE/ME” represents the equity book to market ratio. Because of the lack of an explicit account for capital in this simple model, as in Carlson, Fisher, and Giammarino (2004), we take the capitalized operating cost, $\xi c/r$,

as a proxy for the book value of assets. We also use the capitalized value of coupons, l/r , as a proxy for the book value of debt. The quantity $(\xi c - l)/rV^m$ can hence be thought of as a proxy for the equity book-to-market ratio. In the general model of Section 3, we explicitly account for installed capital and obtain a measure of book-to-market closer to the quantity used in empirical work. The part in (14) labeled “Leverage effect” captures the impact of financial leverage and distress on equity beta. As the expression in (14) indicates, financial leverage impacts the “book-to-market” effect—i.e., the dependence of equity beta on the book-to-market ratio—both directly, through the multiplicative factor “Leverage effect”, and indirectly, through its effect on the equity value V^m and hence on the book-to-market ratio itself.

Financial leverage directly affects equity beta through the implicit put option that the limited liability provision provides. This is reflected in the negative sign appearing in front of the default probability π in (14). At first sight, this negative sign might suggest that equity risk is *always* declining with default probability. This impression, however, is not accurate because it neglects the indirect impact of financial leverage on the default probability π itself and on equity value V^m . In fact, it can be proven that when $\eta = 0$, as the firm approaches default, i.e., $\pi \rightarrow 1$, the equity value V^m approaches zero at a faster rate than the quantity $1 - \pi$, causing β to be *increasing* with default probability and to explode to infinity as π tends to one. On the contrary, if $\eta > 0$, the equity value V^m is bounded away from zero as $\pi \rightarrow 1$. If the beta of the assets is finite, and equityholders receive a fraction of these assets upon the resolution of distress, equity will progressively become *less risky* as the default boundary is approached. This implies that, for sufficiently high levels of default probability, the equity risk *has* to decline with π . Therefore, as stated in the following corollary, both direct and indirect effects of financial leverage impact equity risk.

Corollary 2 *In the cross section, possible shareholder recovery in distress, as represented by η , affects the relationship between a firm’s equity beta and default probability. Specifically,*

1. *If $\eta = 0$, equity beta and expected return are monotonically increasing in default probability π , with $\lim_{\pi \rightarrow 1} \beta = \infty$.*
2. *If $\eta > 0$, equity beta and expected return are hump-shaped, i.e., increasing in default probability π when π is low, and decreasing in default probability when π is high.*

The corollary above provides a *local* characterization of equity beta valid for either very low or very high levels of default probability. Because the default probability π is a function of several variables, it is not immediate to infer the global dependence of beta on default probability from expression (14) and a numerical analysis becomes necessary. For this purpose, we generate a cross section of firms differing by operating costs c , leverage l , scale of operation ξ , volatility of profits σ , degree of correlation ρ between the price process and the pricing kernel in the economy (see footnote 6), and expected recovery for equity in financial distress η . We choose the salvage value $X^m(P)$ to be the book value of asset $\xi c/r$, as defined above. This choice allows us to interpret the value of the parameter η as a fraction of the book value of assets, as it is frequently reported in empirical studies (see, e.g. Eberhart, Moore, and Roenfeldt (1990)). This assumption amounts to choosing $a = \xi c/r$ and $b = 0$ in the definition of the salvage value $X^m(P)$. As we will argue later, imposing a constant salvage value does not affect the qualitative nature of the results. Appendix B contains details of the numerical analysis and Table 1 summarizes the parameters used to generate cross-sectional data from our model.

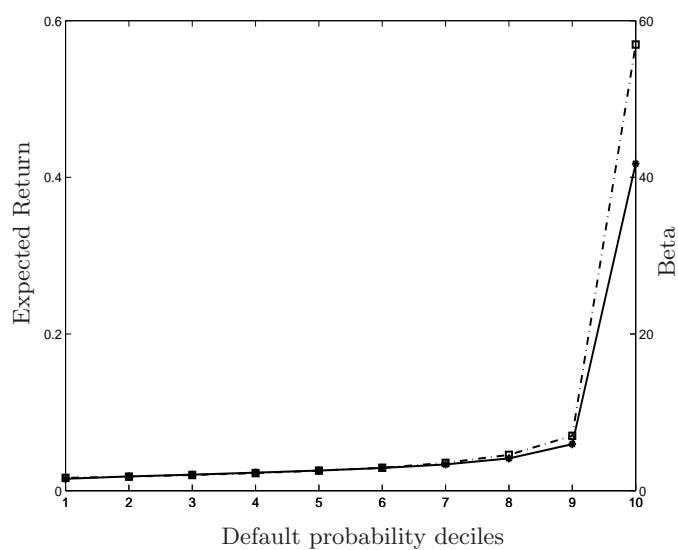
In Figure 1 we report expected returns (solid line, left axis) and equity beta (dash-dotted line, right axis) as a function of default probability, based on the simulated cross-sectional data. Beta is given in equation (14) and the expected return is computed according to (6). Firms are ranked in deciles based on their default probability computed according to equation (A8) in Appendix A, which refers to the likelihood of a firm defaulting within a year. Within each default probability decile, we obtain the average expected return and average β by equally weighting each firm in the decile. Panel A of Figure 1 shows that, when $\eta = 0$, i.e., when there is no equity recovery upon financial distress, the expected return is monotonically increasing in default probability, so is the risk of equity measured by β .

Panel B of Figure 1 shows that when $\eta = 10\%$, i.e., the expected recovery by equity holders upon financial distress is set at a modest level of 10% of the asset value, both the expected return and β exhibit a hump shape with respect to default probability. Empirically, there is a discernible hump in the stock return pattern documented by Dichev (1998). The intuition for this result in our model is as follows. Both financial and operating leverages increase the risk of equity until the default probability reaches a relatively high level. At this point, the magnitude of the book-to-market effect is several times stronger than that at the lower end of the default probability spectrum. Beyond this point, however, the prospect of recovering a fraction of the

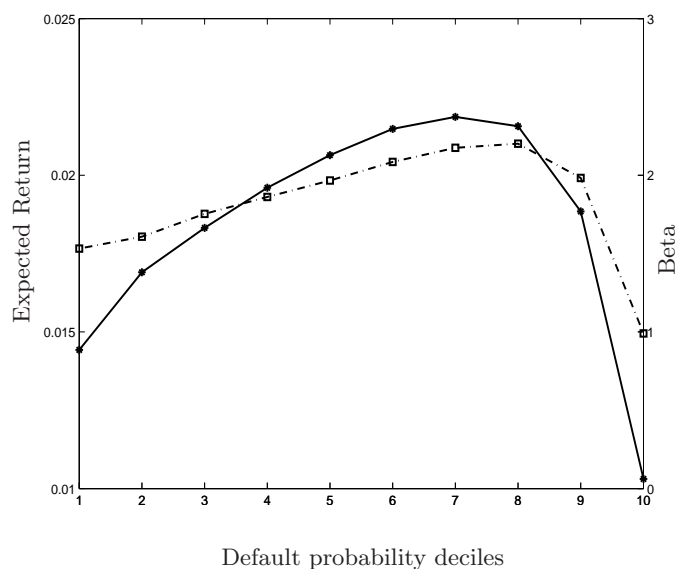
Figure 1: Mature firms' equity return and β versus default probability

The figure reports the monthly expected return (solid line, left axis) and equity β (dash-dotted line, right axis) of mature firms as a function of default probability, with β described in equation (14) and the expected return defined in (6). The graphs are obtained from a cross section of firms by varying firm-level characteristics as described in Appendix B. Firms are ranked in deciles based on their default probability computed according to equation (A8) in Lemma 1 of Appendix A and refers to the likelihood of the firm defaulting within a year. Panel A refers to the case of no expected shareholder recovery upon financial distress, $\eta = 0$, while Panel B refers to the case in which $\eta = 10\%$.

Panel A: No shareholder recovery ($\eta = 0$)



Panel B: With shareholder recovery ($\eta = 10\%$)



assets, which have lower risk than the equity, outweighs the leverage effect in determining the risk of equity. This prospect grows stronger and the equity risk is further reduced as the firm inches closer to the point of default.

It is important to note that the hump shape in Panel B is not an artifact of our assumption that shareholders recover a fraction of the book value of assets upon default. This is because when $\eta = 0$ the equity beta explodes for high levels of default probability as the equity value goes to zero. In contrast, when $\eta > 0$ the equity beta is finite as long as the risk of the assets recovered by shareholders in distress is finite. At lower levels of default probability, the amplifying effect of leverage will still drive up the equity beta for all η cases. Therefore, when $\eta > 0$, the relations between expected return and beta and default probability is bound to be hump-shaped, regardless the form of the shareholder recovery.

The value premium

The analysis above indicates that the cross-sectional variation in the beta of mature firms is attributed to the book-to-market effect. Holding the book-to-market ratio (BE/ME) constant, the sensitivity of β to BE/ME , i.e., the part labeled “Leverage Effect” in (14), is positive except for very high π and η . Hence the patterns of equity beta in Figure 1 are also indicative of the variation in the book-to-market effect across different levels of default probability. Interestingly, the way in which beta depends on default probability has implications for the relationship between default probability and the value premium, i.e., the return spread between firms with high book-to-market ratios and those with low book-to-market ratios. If beta is monotonically increasing in default probability, as in Panel A of figure 1, then the value premium is also positive and increasing. Alternatively, if beta is hump-shaped in default probability, as in Panel B, then the value premium is positive for low levels of default probability and negative otherwise. The intuition for this pattern of value premium is as follows: Suppose we have two firms with identical book value and default probability. One of them experiences a negative shock to its stock price; the other a positive shock. The first stock will then have a higher book-to-market ratio than the second stock. If both stocks have $\eta = 0$, then from Panel A of Figure 1, the difference in expected returns of these two stocks, i.e., the value premium, will be positive and, especially for high levels of default probability, upward sloping. On the other side, if both firms have $\eta > 0$, then Panel B of Figure 1 indicates that the return spread between high book-to-market

stocks and low book-to-market stocks should be positive for low levels of default probability and negative for high levels. Empirically this can be consistent with a value spread that is either declining or hump-shaped in default probability. We verify this intuition after solving the general model of Section 3 and in our empirical analysis in Appendix C.

The momentum effect

The relation between equity beta and default probability also plays an important role in understanding momentum in equity returns. From the equity value in equation (9) and the definition of autocorrelation in (7), the equity return autocorrelation for a mature firm can be expressed as follows:

$$\text{AutoCorr} = \lambda \left[1 - \beta - \left(\frac{\phi_1 \pi}{\beta V^m} \right) \left(\frac{l + \xi c(1 + \eta)}{r} \right) \right], \quad (15)$$

where λ is the risk premium associated with the output price P . The next proposition provides a formal link between default probability and autocorrelation.

Proposition 2 *In the absence of shareholder recovery in distress, $\eta = 0$, the equity return of a mature firm always exhibits a negative autocorrelation. In the presence of shareholder recovery, $\eta > 0$, the return on equity of a mature firm exhibits a positive autocorrelation only for high levels of default probability.*

This proposition highlights the crucial role of financial distress for levered equity—and the ensuing potential recovery for equity-holders—in the determination of momentum in equity returns. The intuition behind this result stems from the humped relationship between expected return and default probability, as shown in Panel B of Figure 1. Because of the potential equity recovery, as the firm edges toward default with a declining stock price, the ex-ante risk level of equity *decreases* too, as does the expected return for the future period. Similarly, as the firm moves away from the brink of bankruptcy, its stock price rises, but the risk of its equity *increases* because of the debt burden, as does the expected return in the future period. Both scenarios depict a return pattern that exhibits positive autocorrelation. Notice that this mechanism applies *only* to firms with high default probability and $\eta > 0$. For this reason, the risk dynamic we highlighted is consistent with the recent empirical finding of Avramov, Chordia, Jostova, and

Philipov (2007), who document that the momentum effect in stock returns is driven primarily by firms with low credit ratings.¹¹

There are two more points worth noting. First, in our model, momentum in equity returns endogenously arises among firms with low credit quality and high expected shareholder recovery upon default even though the fundamental revenue processes of these firms are not predictable, since P follows a geometric Brownian motion. Second, our model is able to generate momentum for mature firms which have no growth options. In Sagi and Seasholes (2007), growth options are instrumental for inducing return momentum.

To examine how growth options may affect our results on the book-to-market and the momentum effects, we turn to growth firms in the next subsection.

2.2 Growth firms

We define a growth firm as a firm which currently produces one unit of product but has a perpetual option to expand its operating scale to ξ (> 1) units of product upon making a one-time investment of I . In other words, a growth firm is the predecessor, in the life-cycle of firms, to the mature firm discussed in the previous subsection. In the current framework, we abstract away from the endogenous financing decision and assume for simplicity that the investment is financed by new equity. Consistent with the case of mature firms, a growth firm has an existing level of leverage that is represented by a consol bond paying a continuous coupon of l . The general model in Section 3 relaxes these assumptions by allowing the firm to optimally choose its investment and financing levels.

A growth firm maintains its status until it either defaults or exercises its growth option and becomes a mature firm. As for the mature firm case, we allow for possible equity recovery upon financial distress that enables equity holders to receive a fraction η of the residual value, $X^g(P)$. The equity value of a growth firm is given by

$$V^g(P_t) = \mathbb{E}^* \left[\int_0^{\tau_L \wedge \tau_G} (P_{t+s} - c - l) e^{-rs} ds \right] + \eta X^g(\underline{P}^g) \mathbb{E}^* [e^{-r\tau_L} \mathbf{I}_{\{\tau_L < \tau_G\}}] + (V^m(\bar{P}) - I) \mathbb{E}^* [e^{-r\tau_G} \mathbf{I}_{\{\tau_G < \tau_L\}}], \quad (16)$$

¹¹This mechanism is also consistent with the reversal in the momentum in stock returns. As the fortune of a low-credit-quality firm improves, its default probability is reduced and its expected return may shift over the hump in Panel B of Figure 1. When this happens, its autocorrelation turns negative and the momentum in stock returns is reversed.

where \underline{P}^g and \bar{P} are the prices at which the growth firm defaults or expands, respectively; τ_L and τ_G the times at which these two events take place; $X^g(\underline{P}^g)$ is the residual value of the growth firm upon default; and $V^m(\bar{P})$ is the equity value of the corresponding mature firm, derived in (12).¹²

Equation (16) states that the equity value of a growth firm is equal to the present value of its stream of profits, net of operating and interest costs, until the firm is no longer operative as a growth firm, i.e., until the arrival of the smaller of the two stopping times τ_L and τ_G . If the firm defaults before it expands ($\tau_L < \tau_G$), equity holders receive a fraction η of the residual asset value, $X^g(\underline{P}^g)$. If, on the other hand, the firm expands before it defaults ($\tau_L > \tau_G$), equity holders pay I and receive the equity value of the mature firm it transforms into. The boundaries \underline{P}^g and \bar{P} are chosen optimally by shareholders. The following proposition characterizes the equity value of such a growth firm.

Proposition 3 *The equity value of a growth firms is given by*

$$V^g(P) = \frac{P}{\delta} - \frac{c+l}{r} + AP^{\phi_1} + BP^{\phi_2} + \eta X^g(\underline{P}^g)f(P) + (V^m(\bar{P}) - I)g(P), \quad (17)$$

where $\phi_2 > 1$ is the positive root of the characteristic equation (10), $f(P) = \mathbb{E}^* \left[e^{-r\tau_L} \mathbf{I}_{\{\tau_L < \tau_G\}} \right]$ is the price of a perpetual barrier option that pays off one dollar if the price P reaches the default boundary before the growth option is exercised, and $g(P) = \mathbb{E}^* \left[e^{-r\tau_G} \mathbf{I}_{\{\tau_G < \tau_L\}} \right]$ is the price of a perpetual barrier option that pays off one dollar if the price P reaches the expansion boundary before the firm defaults. Their expressions are given in equation (A28) of Appendix A. The four unknowns A , B , \underline{P}^g , and \bar{P} are obtained from the value-matching and smooth-pasting conditions (A30) and (A31) in Appendix A.

The first three terms in (17) are similar to those for the value of a mature firm in (9). Growth options appear *directly* through the increasing and convex term BP^{ϕ_2} and through the intrinsic value of the investment option $(V^m(\bar{P}) - I)g(P)$, and *indirectly* through the limited liability term $\eta X^g(\underline{P}^g)f(P)$. Using the expression of equity value in Proposition 3, the following corollary characterizes the equity beta of a growth firm.

¹²If the new investment is financed through debt or via a mix of debt and equity, the expression for the equity value (16) remains unaltered except at the growth boundary, but the default and growth thresholds will be affected by the corresponding capital structure.

Corollary 3 *The beta of a growth firm is given by*

$$\begin{aligned} \beta &= 1 + \frac{(l+c)/r}{Vg(P)} \\ &+ \frac{1}{Vg(P)} \left((\phi_1 - 1)(\underline{P}^g)^{\phi_1} A' + (\phi_2 - 1)\overline{P}^{\phi_1} B' \right) f(P) \\ &+ \frac{1}{Vg(P)} \left((\phi_1 - 1)(\underline{P}^g)^{\phi_2} A' + (\phi_2 - 1)\overline{P}^{\phi_2} B' \right) g(P), \end{aligned} \quad (18)$$

where A' and B' are constants defined in equations (A33) and (A34) of Appendix A.

The corollary shows that a growth firm's beta is composed of three parts: The first part, $1 + \frac{(l+c)/r}{Vg(P)}$, represents the revenue beta and includes the effect of both operating and financial leverage. The second part, i.e., the second line of expression (18), contains the "limited liability option" captured by the term $f(P)$, while the third part relates to the "growth option" captured by the term $g(P)$. Note, however, that this decomposition is only a convenient way of interpreting the expression for beta, given that the prices $f(P)$ and $g(P)$ of the double barrier options described in Proposition 3 are simultaneously affected by the growth and limited-liability options. With this caveat, we can think of the first two parts as conceptually similar to the mature firm beta derived in Corollary 1. The illustration of the relationship between these components of β and default probability is very similar to that in Figure 1. All else being equal, the effect of η on the beta of growth firm is smaller than that of mature firms due to the difference in operating scale, ξ .

The last term in (18) represents a unique component of the beta of a growth firm that is directly related to the likelihood of exercising the growth option. As this likelihood, $g(P)$, increases, the weight of the growth option in the equity value gets larger, and hence the equity risk increases. This component is ascribed to capturing the size effect by Berk, Green, and Naik (1999) and others, and it is only present for growth firms.

As for the case of mature firms, we can obtain the following measure of autocorrelation for equity returns of a growth firm:

$$\text{AutoCorr} = \lambda \left[1 - \beta - \left(\frac{1}{\beta V^m} \right) \left(\phi_1(\phi_1 - 1)A'P^{\phi_1} + \phi_2(\phi_2 - 1)B'P^{\phi_2} \right) \right], \quad (19)$$

where λ is the risk-premium associated with the output revenues P , and A' and B' are constants defined in equations (A33) and (A34) of Appendix A.

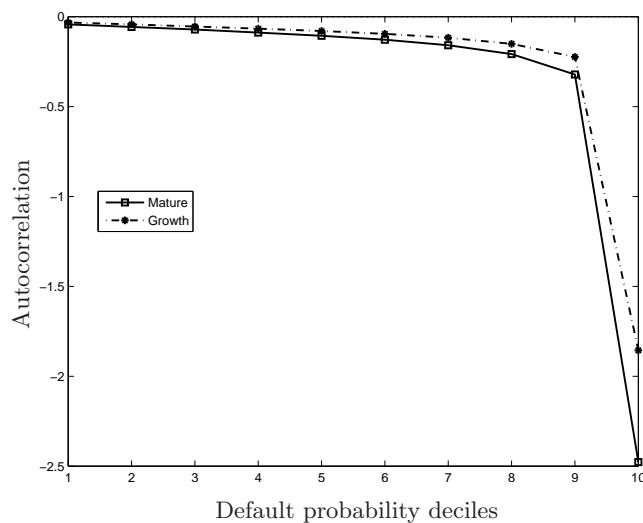
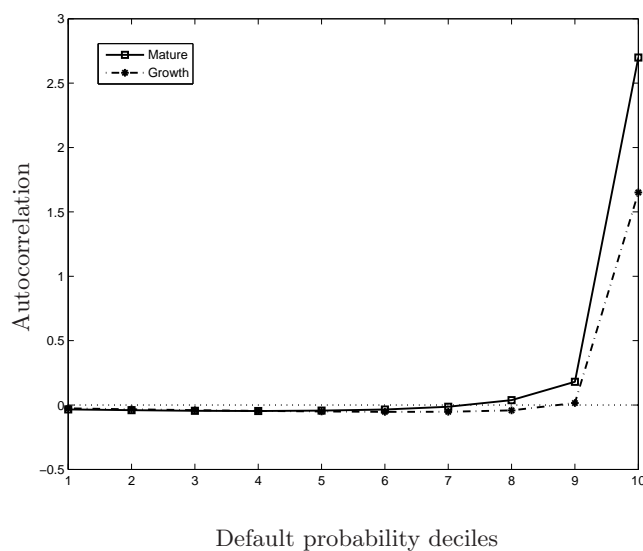
Because the expression for the equity value of growth firms is not available in closed-form, we resort to a numerical analysis of the relationship between autocorrelation and default probability. We generate the cross section of both mature and growth firms using the procedure described earlier in Subsection 2.1 and compute the average return autocorrelation in each default probability decile. The results are reported in Figure 2. As shown in Panel A, in the absence of shareholder recovery upon financial distress, i.e., $\eta = 0$, the autocorrelation in equity returns for both mature and growth firms is negative across the entire spectrum of default probability. This result is interesting when compared to the findings of Sagi and Seasholes (2007). In the absence of shareholder recovery our setup is similar to theirs with two notable exceptions: (i) the inclusion of financial leverage, and (ii) a non-mean-reverting price process P . As pointed out by Sagi and Seasholes (2007), while the presence of growth options leads to a positive autocorrelation in equity returns, when these growth options are valuable financial leverage diminishes the impact of growth options. This is what happens in Panel A. However, in the presence of shareholder recovery in distress, i.e., $\eta > 0$, and for sufficiently high default probability, we find significantly positive autocorrelations in stock returns, as illustrated in Panel B, for both mature and growth firms. This persistence in returns will enhance momentum profits. Notice that these patterns are driven by financial leverage and the expected shareholder recovery upon financial distress and not by the presence of growth options. This is consistent with the intuition we developed earlier in the context of mature firms and with the empirical evidence in Avramov, Chordia, Jostova, and Philipov (2007).

2.3 Discussion

The simple model discussed in this section yields insights into several puzzling pieces of empirical evidence on the cross section of equity returns. First, because of the interaction between leverage and book-to market in the determination of beta, for most firms—with the exception of low credit quality firms for which shareholders expect a non-zero recovery value in distress renegotiations—the risk of assets-in-place to equity holders is amplified by high levels of financial leverage, implying that the magnitude of the book-to-market effect is stronger for more heavily levered firms. This is consistent with the evidence that the value premium is most significant for firms

Figure 2: Return autocorrelation and default probability

The figure reports the momentum measure for mature and growth firms (equations (15) and (19)) as a function of default probability. The graphs are obtained from a cross section of firms by varying firm-level characteristics as described in Appendix B. Firms are ranked based on their default probability computed according to equation (A8) in Lemma 1 of Appendix A and refers to the likelihood of the firm defaulting within a year. Panel A refers to the case of no expected shareholder recovery upon financial distress, $\eta = 0$, while Panel B refers to the case in which $\eta = 10\%$.

Panel A: No shareholder recovery ($\eta = 0$)Panel B: With shareholder recovery ($\eta = 10\%$)

with high default probability (see, e.g., Griffin and Lemmon (2002), Vassalou and Xing (2004), and Chen (2006)).

Second, our model provides a perspective for understanding the results of Hecht (2000), that firm-level asset returns do not exhibit strong cross-sectional patterns, such as the book-to-market and momentum effects. As our model shows, these patterns are generally enhanced by the presence of leverage, and their magnitude may be too small in asset returns to be statistically detectable. The nonlinearity in the equity payoff introduced by the limited liability and growth options in our settings helps provide a plausible justification for the relationship between conditional skewness and stock returns documented in Harvey and Siddique (2000). This relationship is also hinted at in the results of Ferguson and Shockley (2003), who argue that the SMB and HML factors in the Fama-French three-factor model are instruments for measurement errors in equity beta that surface when debt is ignored in the construction of the market portfolio. While compelling, the argument of Ferguson and Shockley (2003) ignores the time-varying nature of beta as well as its dependence on firms' characteristics, as highlighted in our framework.

More importantly, our simple model shows that accounting for potential shareholder recovery upon financial distress produces a rich set of implications for the cross-sectional properties of stock returns. The resulting hump-shaped relationship between expected return and default probability leads to a testable new prediction of a humped value premium with respect to default probability. It also provides an explanation for the recently discovered evidence on the concentration of momentum profits in low credit quality firms and further predicts that the momentum profits will be stronger for nearly distressed firms with higher expectation of shareholder recovery. Together with the work of Garlappi, Shu, and Yan (2006) regarding the inverse relationship between stock returns and default probability, the notion of shareholder recovery provides a promising perspective for understanding several features of stock returns simultaneously.

While the simple model of this section is useful for developing a basic intuition for a number of cross-sectional patterns in stock returns, its streamlined nature may cast doubt on the robustness of this intuition. To address this concern, we present in the next section a general model with endogenous investment and financing decisions that is more suitable for calibration to real data. Besides providing a way of determining the robustness of our claims, the realistic features of the

model will also allow us to assess the effectiveness of traditional risk-adjustment procedures for identifying “anomalous” returns.

3 A general model of levered equity returns

We build a general model of a firm’s investment and financing decisions in the presence of systematic and idiosyncratic shocks. The model is in the spirit of the neoclassical Lucas-Prescott framework and shares its basic structure with several recent papers.¹³ Our goal is to construct a *stationary* economic environment that can be calibrated to real data and use such an environment as a laboratory for analyzing the effect of shareholder recovery upon financial distress on the cross section of equity returns. Because it endogenizes financing and investment decisions, the model in this section nests the simple model of Section 2 as a special case.

3.1 The firm’s problem

The model is set up in discrete time. The production technology generates cash flows whose level depends on the capital input and its productivity which is subject to both systematic, economy-wide and firm-specific shocks. The firm’s manager acts in the interests of shareholders and chooses investment and financing decisions by maximizing equity value.

Production technology

The production output $Y_{i,t}$ of firm i at time t is

$$Y_{i,t} = K_{i,t}^\alpha e^{X_t + Z_{i,t}}, \quad (20)$$

where $K_{i,t}$ is the firm’s capital level at the beginning of period t , α is the capital share in total output, chosen to be between 0 and 1 in order to obtain decreasing return to scale. The variables X_t and $Z_{i,t}$ in (20) represent, respectively, the aggregate and firm-specific shocks to the output. These shocks are modeled as stationary Markov processes evolving according to the following auto-regressive processes

$$X_{t+1} = X_t + (1 - \rho_x)(\bar{X} - X_t) + \sigma_x \varepsilon_{t+1}^x, \quad (21)$$

¹³See, for example, Gomes and Schmid (2008), Li (2008), Lidvan, Saprizza, and Zhang (2008), and Obreja (2006).

$$Z_{i,t+1} = Z_{i,t} + (1 - \rho_z)(\bar{Z} - Z_{i,t}) + \sigma_z \varepsilon_{i,t+1}^z, \quad (22)$$

where \bar{X} and \bar{Z} are the long-run averages, ρ_x and ρ_z the autocorrelation coefficient and σ_x and σ_z the volatility coefficients. The innovations ε_{t+1}^x and $\varepsilon_{i,t+1}^z$ are normally distributed with mean zero and unit variance. Because $Z_{i,t}$ is a firm-specific shock, we require it to be uncorrelated both with the aggregate shock X_t and across firms, i.e., $E[\varepsilon_{i,t}^z \varepsilon_t^x] = 0$ for all i , and $E[\varepsilon_{i,t}^z \varepsilon_{j,t}^z] = 0$ for all $i \neq j$. In period t , the firm's after-tax profit is

$$\Pi_{i,t} = (1 - \tau)(Y_{i,t} - fK_{i,t} - F) \quad (23)$$

where τ is the corporate tax rate, and f and F the proportional and fixed costs, respectively.

Investment

Each period, the firm makes an investment decision that affects its capital stock next period according to the capital accumulation equation

$$K_{i,t+1} = I_{i,t} + (1 - \kappa)K_{i,t}, \quad (24)$$

where $I_{i,t}$ is the amount of new investment at time t , and κ is the depreciation rate of the installed capital. Following Lucas (1967), we assume a quadratic adjustment cost for new investment, i.e.,

$$h(I_{i,t}) = \frac{\theta}{2} \left(\frac{I_{i,t}}{K_{i,t}} \right)^2 K_{i,t}, \quad (25)$$

where $\theta > 0$ is the adjustment cost coefficient.

Financing

In order to finance new investment and distribution to shareholders, the firm also makes a choice of issuing either new equity (i.e., negative dividends) or new debt, or a combination of both. As in Li (2008) and Gomes and Schmid (2008), we assume that the only debt instrument available to the firm is a one-period bond. At each date t the firm decides to issue a bond with promised principal $B_{i,t+1}$ and coupon $b_{i,t+1}$, to be repaid at time $t + 1$. The debt is assumed to be issued at par, so at time t firm i is raising an amount equal to $B_{i,t+1}$. The firm is implicitly allowed to refinance all its liability in each period. Accounting for tax deductibility of the coupon payment,

we define firm i 's *total debt commitment* at time t as

$$D_{i,t} = B_{i,t} + (1 - \tau)b_{i,t}. \quad (26)$$

This quantity represents the amount needed to service the debt issued by the firm in the *previous* period and coming due at time t . If we assume that the net cash flow to equity is paid out as dividend to equity-holders, the dividend at time t is then

$$c_{i,t} = \Pi_{i,t} + \tau\kappa K_{i,t} - I_{i,t} - h(I_{i,t}) - D_{i,t} + B_{i,t+1}. \quad (27)$$

If $c_{i,t} < 0$, the firm can raise external financing through a seasoned equity offering. Following Gomes (2001) and Hennessy and Whited (2007), we assume that it is costless to increase debt, but costly to raise new equity. The cost of raising new financing through seasoned equity offering is assumed to be

$$\Lambda(c_{i,t}) = (\lambda_0 + \lambda_1(-c_{i,t}))\mathbf{1}_{c_{i,t} < 0}, \quad (28)$$

where λ_0 is the fixed cost and λ_1 represents the proportional cost. Therefore, the net dividend is

$$d_{i,t} = c_{i,t} - \Lambda(c_{i,t}). \quad (29)$$

3.2 Equity valuation

A firm's equity value is the maximum present value of the discounted stream of dividends that the firm can achieve by altering its investment and financing policy. To evaluate cash flow, we assume a process for the pricing kernel $\mathbb{M}_{t,t+1}$ similar to that in Berk, Green, and Naik (1999) and Zhang (2005),

$$\mathbb{M}_{t,t+1} = \beta \exp \{ \Gamma_t (X_t - X_{t+1}) \}, \quad \Gamma_t = \gamma_0 + \gamma_1 (X_t - \bar{X}), \quad (30)$$

where $0 < \beta < 1$ is the time discount factor, and γ_0 and γ_1 constants.¹⁴

At any point in time a firm is entirely described by four state variables: the aggregate shock X_t , the firm-specific shock $Z_{i,t}$, the capital level K_t and the total debt commitment $D_{i,t}$ defined in (26). We denote by $S_{i,t} = \{X_t, Z_{i,t}, K_{i,t}, D_{i,t}\}$ the vector of state variables. Equity value,

¹⁴The market price of risk is equal to $\lambda_{m,t} = \text{Var}_t[\mathbb{M}_{t,t+1}]/E_t[\mathbb{M}_{t,t+1}]$. Given the assumption of normality in the innovations of X_t , $\lambda_{m,t} = \beta(e^{\sigma_m^2} - 1)$, where $\sigma_m = \sigma_x[\gamma_0 + \gamma_1(X_t - \bar{X})]$.

$V(S_t)$, is the solution of a dynamic programming problem with optimal investment, financing and default choices. Unless the company optimally defaults at time t , these choices will result in a new level of capital $K_{i,t+1}$ and a new level of total debt commitment $D_{i,t+1}$. Hence the future levels of capital $K_{i,t+1}$ and total debt commitment $D_{i,t+1}$ are *control* variables at time t , and become *state* variables at time $t + 1$.

In the absence of shareholder recovery upon financial distress, the firm is financially viable as long as equity has a positive value, i.e., $V(S_{i,t}) > 0$ and default occurs when $V(S_{i,t}) = 0$. Here we relax this assumption by allowing for possible equity recovery upon financial distress. Specifically, we assume that the recovery is a fraction, η , of the residual asset value upon financial distress, $R(S_t)$. Following Hennessy and Whited (2007), we model the residual asset value as

$$R(S_{i,t}) = \max \{ \Pi_{i,t} + \tau \kappa K_{i,t} + \xi_1 (1 - \delta) K_{i,t} - \xi_0, 0 \}, \quad (31)$$

where $\Pi_{i,t}$ is the after-tax profit defined in (23), and $1 - \xi_1$ and ξ_0 are the proportional and fixed distress costs, respectively. Firm i 's equity value is therefore determined by the solution of the following Bellman equation:

$$V(S_{i,t}) = \max \left\{ \eta R(S_{i,t}), \max_{\{K_{i,t+1}, D_{i,t+1}\}} \{ d(S_{i,t}) + \mathbb{E}_t [\mathbb{M}_{t,t+1} V(S_{i,t+1})] \} \right\}. \quad (32)$$

One potential difficulty in computing the net cash flow— $c_{i,t}$ in (27), and, in turn, $d_{i,t}$ in (29)—needed for the solution of the dynamic programming problem (32), resides in the determination of the face value $B_{i,t+1}$ (and the coupon $b_{i,t+1}$) of the newly issued debt. As argued in Li (2008), the use of the total debt commitment $D_{i,t}$ as a state variable simplifies the problem considerably because we can avoid keeping track of the coupon. Denoting by

$$\chi_{i,t+1} = \mathbf{1}_{\{V(S_{i,t+1}) > \eta R(S_{i,t+1})\}} \quad (33)$$

the indicator function for the firm's solvency, we can in fact evaluate the market value of the bond as follows,

$$\begin{aligned} B_{i,t+1} &= \mathbb{E}_t [\mathbb{M}_{t,t+1} (\chi_{i,t+1} (b_{i,t+1} + B_{i,t+1}) + (1 - \chi_{i,t+1}) (1 - \eta) R(S_{i,t+1}))] \\ &= \frac{\mathbb{E}_t \left[\mathbb{M}_{t,t+1} \left\{ \chi_{i,t+1} \frac{D_{i,t+1}}{1 - \tau} + (1 - \chi_{i,t+1}) (1 - \eta) R(S_{i,t+1}) \right\} \right]}{1 + \frac{\tau}{1 - \tau} \mathbb{E}_t [\mathbb{M}_{t,t+1} \chi_{i,t+1}]}, \end{aligned} \quad (34)$$

where the first equality considers the debt value in cases of solvency and default, respectively, and the last equation uses the definition of total debt commitment (26) to express the coupon $b_{i,t+1}$ as a function of $D_{i,t+1}$ and $B_{i,t+1}$. The bond pricing equation (34) involves only knowledge of the evolution of the state variables $S_{i,t}$ and will be used to determine the cash flows net of investment and financing defined in (27).

4 Numerical analysis of the general model

The model is solved numerically via value function iteration after discretization of the state space. We consider 100 grid points each for the capital level and the debt commitment level, both of which are assumed to take values between 0 and 20. We follow Tauchen (1986)'s quadrature approximation for the systematic and firm-specific shocks which are discretized with 17 and 39 grid points, respectively. Given the computationally intensive nature of the problem, it is not feasible to use the simulated method of moments (SMM) to calibrate the parameter set. Instead, we follow the calibration strategy of Lidvan, Sapriza, and Zhang (2008) and Gomes and Schmid (2008) who base their parameter choice on the values used in the existing macro and finance literature (e.g., Gomes (2001), Cooley and Quadrini (2001), Cooper and Ejarque (2003), Zhang (2005) and Hennessy and Whited (2005, 2007)). Our choice of parameters is summarized in Table 2. The model is calibrated on a monthly basis.

4.1 Equity expected return

Based on the stationary solution of the model we can analyze the dynamics of expected returns and the impact of the expected shareholder recovery, η . To simplify notation, in the following, we will drop the subscript i from firm-specific quantities. The equity expected return is defined as

$$E_t[R_{t+1}] = \frac{\mathbb{E}_t[V(S_{t+1})]}{V(S_t) - d(S_t)} \quad (35)$$

where the expectation \mathbb{E}_t is taken with respect to the probability measure induced by the Markov processes (21) and (22) for the systematic and idiosyncratic shocks and by the optimal investment and financing policies. We subtract the dividend $d(S_t)$ from the equity value in the denominator of (35) because the equity value in (32) is *cum dividend*. From the stationary

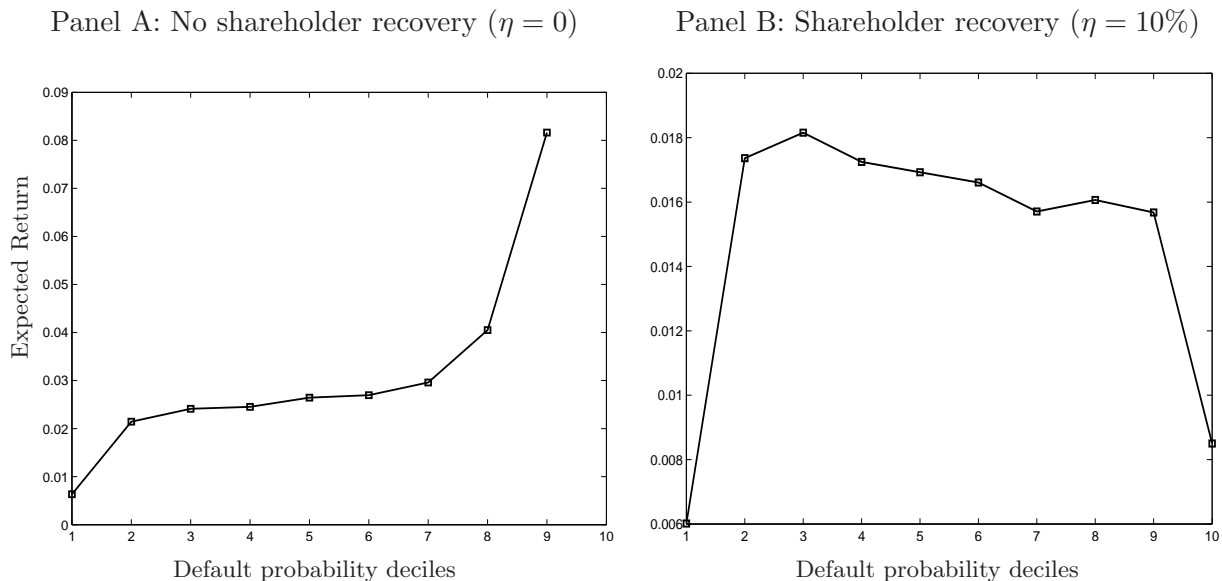
solution we compute recursively the τ -month ahead *probability of default* as follows

$$p_\tau(S_t) = (1 - \chi(S_t))\mathbb{E}_t[p_{\tau-1}(S_{t+1})], \quad p_0(S_{t+1}) = 1 - \chi(S_{t+1}), \quad (36)$$

where χ is the stationary default boundary (33) obtained from the solution of (32).

Figure 3: Expected return and default probability

The figure reports the monthly expected return as a function of default probability obtained from the general model of Section 3. Panel A refers to the case of no shareholder recovery, $\eta = 0$, while Panel B refers to the case in which shareholder recovery is set to $\eta = 10\%$ of the recovery value in (31).



According to the intuition developed in Section 2, the absence of shareholder recovery leads to expected returns that are increasing in default probability while the presence of recovery leads to expected returns that are hump-shaped in default probability. Figure 3 confirms these findings in the general model of Section 3.

The figure is obtained by bootstrapping expected returns from the stationary distribution. Specifically, according to the processes (21), the unconditional distribution for X_t is normal with mean \bar{X} and variance $\sigma_x^2/(1 - \rho_x^2)$. Similarly, the unconditional distribution for Z_t is normal with mean \bar{Z} and variance $\sigma_z^2/(1 - \rho_z^2)$. A firm is characterized by a point $S_t = \{X_t, Z_t, K_t, D_t\}$ in the state space. We construct a representative panel of 30,000 firms by randomly drawing: 3 points from the unconditional distribution of X_t , representing three phases of the business cycle; 100 points from the unconditional distribution of Z_t ; 100 points each for K_t and D_t chosen uniformly

from their support. After forming this panel, we sort firms into 10 portfolios according to their default probability (36) and compute the equally-weighted *expected* returns of the portfolios thus obtained. We repeat this procedure 500 times. Figure 3 report the average expected return across the 500 repetitions. Panel A presents the case of no shareholder recovery ($\eta = 0$), while Panel B reports the result for the case with expected shareholder recovery equal to 10% of the residual value $R(S_t)$ defined in (31).

As it emerges clearly from the picture, the case of no recovery leads to a monotonically increasing relation between expected return and default probability that “explodes” when default becomes almost certain, in the highest decile. On the other hand, in the presence of expected shareholder recovery upon distress, the relation between expected return and default probability is humped, increasing at low levels of default probability and decreasing at high levels of default probability. These patterns are consistent with those obtained in Figure 1 from the analysis of the simple equity valuation model.

4.2 The value premium

The simple model of Section 2 predicts that the value premium should be increasing in default probability in the absence of shareholder recovery and declining or hump-shaped in default probability in the presence of shareholder recovery. We now assess the robustness of this prediction in the context of the general model of Section 3.

From the stationary solution of the general model, we can construct the book-to-market ratio $BM(S_t)$ at each point S_t of the state space as

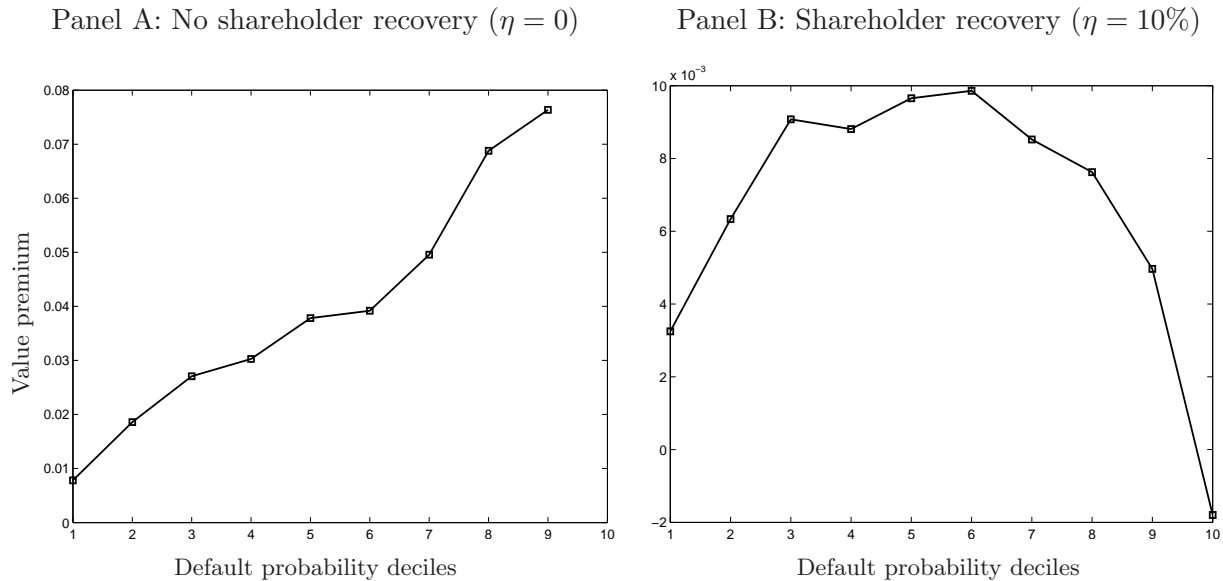
$$BM(S_t) = \frac{K_t - D_t}{V(S_t)}. \quad (37)$$

To study the structure of the value premium in the cross section, we follow the bootstrap methodology in the previous subsection. Specifically, we start by randomly drawing a panel of 30,000 firms from the state space. We then sort firms into ten portfolios based on their default probability (36). We finally sort each of these portfolios into five sub-portfolios according to the firms’ book-to-market ratio. Within each default probability decile, we compute the value premium as the spread between the expected returns of the highest and lowest book-to-market

quintile. We repeat this procedure 500 times and report in Figure 4 the average value premium in each default probability decile across the 500 repetitions.

Figure 4: Value premium and default probability

The figure reports the monthly spread between the average expected returns of high book-to-market firms and that of low book-to-market firms within each decile of default probability in the cross section of firms generated from the stationary solution of the general model of Section 3. Panel A refers to the case of no shareholder recovery, $\eta = 0$, while Panel B refers to the case in which shareholder recovery is set to $\eta = 10\%$ of the recovery value in (31).



Panel A reports the value premium on a monthly basis for the case of no shareholder recovery while Panel B considers the case of expected shareholder recovery equal to 10% of the residual value $R(S_t)$ defined in (31). The presence of shareholder recovery substantially affects the pattern of the value premium conditional on default probability. The value premium is positive and increasing in the absence of shareholder recovery while it is hump-shaped when shareholder recovery is present. The figure confirms that the intuition developed in the simple model of Section 2 is preserved in a more general model with endogenous investment and financing. The behavior of the value premium conditional on default probability is a novel prediction from our model for which we find empirical support in the data, as discussed below in Subsection 4.4.

4.3 Momentum profits

According to the simple model of Section 2, in the presence of possible shareholder recovery, the humped relationship between expected return and default probability implies the concentration of momentum profits among low credit-quality firms, and no momentum in the absence of shareholder recovery. The pattern of expected returns in Figure 3 suggests that this intuition holds also in the general model. To verify this conjecture, we examine the momentum profits, defined as the spread between the expected returns of winner and loser portfolios, constructed from the stationary solution of the general model.

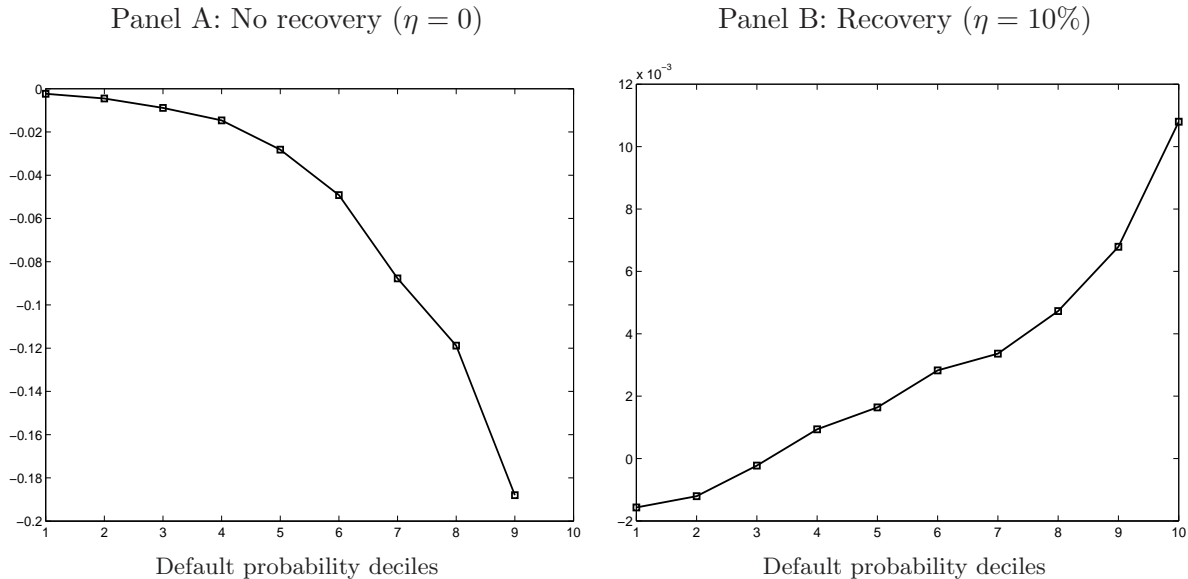
To construct momentum portfolios we generate a time series of realized returns that will determine winners and losers in each period. For this purpose, we follow the bootstrapping procedure of the previous two subsections. Specifically, we draw a panel of 30,000 firms by choosing points, $S_t = (X_t, Z_t, K_t, D_t)$, in the state space. Based on the dynamics of the state variables X_t and Z_t , and the optimal investment and financing strategies derived from the solution of the model, each state S_t will evolve to a *future* state S_{t+1} . The realized return is hence $\frac{V(S_{t+1})}{V(S_t) - d(S_t)}$, which will be used to separate winners from losers. The expected return in the state S_{t+1} can be subsequently deduced directly from the stationary solution.

We construct momentum profits by sorting the panel of firms in state S_{t+1} into ten portfolios based on their default probability and, independently, into five portfolios based on the realized return from state S_t to state S_{t+1} . The bottom quintile represents the portfolio of *losers* and the top quintile is the portfolio of *winners*. The expected momentum profits are calculated as the difference in the equally weighted expected returns of winners and losers in each default probability decile. We repeat this procedure 500 times and report in Figure 5 the average monthly momentum profits in each default probability decile across the 500 repetitions.

As before, Panel A considers the case of no shareholder recovery while Panel B refers to the case of shareholder recovery equal to 10% of the residual value (31). The figure illustrates that momentum profits are positive and significant only for firms with shareholder recovery and with high default probability. The range of momentum profits runs from 2% to 12% annually depending on the level of default probability, comparable to empirical estimates. For firms without shareholder recovery, the momentum strategy does not work, as it would only generate

Figure 5: Momentum profits and default probability

The figure reports the monthly momentum profits as a function of default probability generated from the general model of Section 3. Panel A refers to the case of no shareholder recovery, $\eta = 0$, while Panel B refers to the case in which shareholder recovery is set to $\eta = 10\%$ of the recovery value in (31).



losses, as indicated in Panel A.¹⁵ This therefore confirms that the intuition about momentum profits derived from the simple model of Section 2 is robust in a more general setting with endogenous investment and capital structure choices.

4.4 Empirical evidence

In our empirical investigation, we test the predictions regarding value premium and momentum profits by using a market-based measure of default probability, the *Expected Default Frequency* (EDF), obtained directly from *Moody's KMV*. We match the EDF database with the CRSP and COMPUSTAT databases. Details of our data and empirical design are contained in Appendix C.1.

¹⁵The large magnitude of losses is attributable to the explosive nature of expected returns as default probability approaches 1.

Value premium

The existing empirical evidence (see, e.g., Griffin and Lemmon (2002) and Vassalou and Xing (2004)) seems to suggest that the value premium is monotonically increasing in default probability. While this finding is consistent with our predictions in the absence of shareholder recovery, it seems at odds with the case when there is potential shareholder recovery, which is shown to be significant in the data by Morellec, Nikolov, and Schürhoff (2008). It is important to note, however, that the above mentioned empirical studies of the value premium usually exclude stocks with share price lower than \$5, which tend to be those with high default probabilities. Once these stocks are included in the sample and proper measures are taken to control for liquidity issues, we indeed find that, consistent with the prediction of our theory, the value premium is humped with respect to EDF, i.e., it increases initially with EDF, but decreases with EDF at high levels of EDF. Details of the empirical analysis are contained in Appendix C.2.

Momentum profits

Our model predicts that momentum in stock returns is strongest for firms with high levels of default probability. This is consistent with the empirical evidence documented in Avramov, Chordia, Jostova, and Philipov (2007). Moreover, our model provides a refinement to this prediction, based on the degree of the expected shareholder recovery. Adopting the proxies for shareholder recovery developed in Garlappi, Shu, and Yan (2006), we indeed verify that momentum profits are enhanced by the combination of high default probability *and* high level of expected shareholder recovery. Details of the empirical analysis supporting this conclusion are in Appendix C.3.

4.5 Risk adjustment and the presence of alpha

Because our model suggests that the cross section of equity returns is driven by the cross-sectional difference in the timing-varying β in a conditional CAPM setting, it is important to understand how the empirically documented cross-sectional “anomalies”—as expressed by risk-adjusted returns or alphas—may be accommodated in our model. We address this issue by simulating data from our model and use them as a laboratory to analyze the efficacy of the conventional risk-adjustment procedure used in the empirical asset pricing literature.

In our simulations, we generate a cross section of 2,000 firms with 40 years of monthly observations.¹⁶ As indicated in Morellec, Nikolov, and Schürhoff (2008), there is a wide range of expected shareholder recovery, represented by η , in the cross section of U.S. firms. Unfortunately, we do not have good knowledge of the distribution of the parameter η in the data. In our simulated sample we assume that there are three equally likely levels of shareholder recovery $\eta : 0, 0.1$ and 0.25 .¹⁷ The size of this panel roughly matches that of typical empirical studies conducted on the CRSP dataset. Table 3 reports the implied moments of some key statistics obtained from our simulations and compares them to those from the data. Overall, our sample provides a reasonable representation of the empirical data.

In the empirical literature, cross-sectional return anomalies are usually characterized by alphas after a risk-adjustment procedure following the Fama and French (1993) methodology of supplementing the market excess return (MKT) with the SML and HML factors to capture the size and book-to-market effects. We follow this risk-adjustment procedure and examine the relationship between “risk-adjusted” returns and default probability. We choose to examine this cross-sectional feature because, according to our theory, it underlies the results on the value premium and the momentum effect. It is also a puzzling anomaly investigated recently by Dichev (1998), Griffin and Lemmon (2002), and Campbell, Hilscher, and Szilagyi (2008). For our exercise, we construct the MKT, SML and HML factors from the simulated data following the Fama-French procedure.

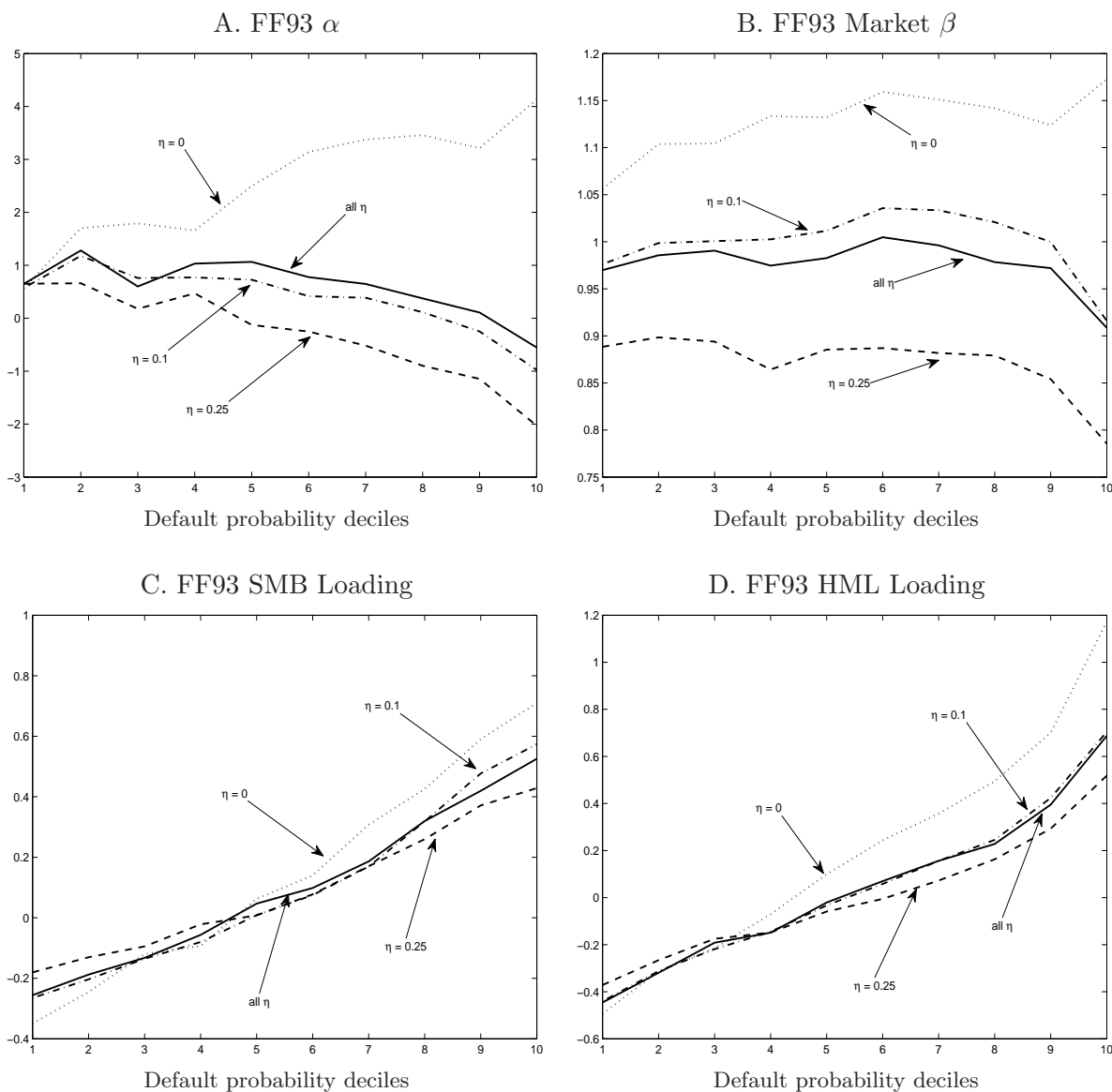
Figure 6 displays the Fama and French (1993) alphas and the corresponding factor loadings of the decile portfolios sorted by default probability. The results presented in Panel A include alphas, in annualized percentage terms, for the full simulated sample as well as for the three subsamples with $\eta = 0, 0.1$ and 0.25 , respectively. Despite the fact that our data, by construction, do not contain alpha, the figure exhibits significant alphas across deciles of default probability. Interestingly, the pattern of alpha seems to inherit the pattern of expected returns, i.e. positively sloped in default probability when there is no shareholder recovery and hump-shaped or downward sloped in the presence of shareholder recovery. The pattern of alphas for the full sample (dark solid line) resembles that documented in the empirical literature.

¹⁶In order to maintain the panel balanced, once a firm defaults, it is assumed that a new firm is born, whose characteristics are drawn at random from the distributions of state variables describing firm-specific profitability (Z_t), capital level (K_t) and debt commitment (D_t).

¹⁷The empirical estimation of Morellec, Nikolov, and Schürhoff (2008) finds that the average η among the firms they examine is about 0.20. Therefore, our choices of η may still be considered conservative.

Figure 6: Risk-adjusted return and factor loadings in the simulated data

The figure reports the annualized risk-adjusted return and factor loadings from our simulations. Risk adjustment is accomplished via a three-factor model as in Fama and French (1993). Panel A presents the factor-adjusted alphas, in annual percentage point, for the full sample and for the three η -subsamples. Panel B presents the corresponding market betas in the three-factor model, Panel C the loadings on the SMB factor, and Panel D the loadings on the HML factor.



Campbell, Hilscher, and Szilagyi (2008) point out that the puzzle of a downward relationship between stock returns and default probabilities is deepened by the fact that factor loadings used in the risk-adjustment procedure are usually *increasing* in default probability, except perhaps

for the market beta. To examine closely this point, we report in Figure 6 the Fama-French loadings on MKT, SMB and HML factors used to control for risk, in each default probability decile. Panel B shows the market beta for the full sample and the three η -subsamples, and Panels C and D report the loadings on the SMB and HML factors, respectively. As these graphs illustrate, the market beta is humped with respect to default probability, consistent with the findings of Campbell, Hilscher, and Szilagyi (2008). The SML and HML loadings are increasing in default probability, also consistent with the empirical evidence. In addition, in an unreported analysis, we find that firm-level equity volatilities are increasing with default probabilities in our simulations.¹⁸

As stated above, we know that the data used to obtain these results are generated from a model in which there is no mispricing and returns are driven by the risk exposure summarized in a time-varying and characteristics-dependent beta. The results in this section suggest therefore that (i) our conditional beta model can accommodate stock return “anomalies”, and (ii) conventional risk adjustment procedures, which rely on *static* factor loadings in a linear factor model, are too coarse to capture the complexity of beta in our model, and thus the appearance of alpha may be the consequence of mis-specification in the risk structure.

5 Conclusion

Recent empirical evidence strongly suggests that financial distress is instrumental in explaining the cross section of stock returns. While this seems to confirm the conjecture of Fama and French (1996) that the book-to-market effect is related to financial distress, efforts toward finding a distress risk factor have produced puzzling empirical patterns.

In this paper, we propose a new perspective for understanding the empirical regularities in the cross section of equity returns. We explicitly introduce financial leverage in a simple equity valuation model and investigate the impact of the likelihood of a firm defaulting on its debt obligations on stock returns in the presence of possible shareholder recovery upon the resolution of financial distress. In this simple framework, we derive two important insights. First, since financial leverage distinguishes equity from firm assets, the option feature of equity amplifies the cross-sectional patterns in stock returns. Therefore, introducing financial leverage validates

¹⁸Results available upon request.

the intuition of investment-based models for explaining cross-sectional returns by enhancing the magnitude of these effects, especially for firms with high default probabilities.

Second, we show that the presence of potential shareholder recovery upon financial distress alters the risk structure of equity and causes the expected return to be hump-shaped in default probability. This non-monotonic relationship between risk and default probability in turn leads to hump-shaped value premia with respect to default probability and predicts a concentration of momentum profits among firms with both poor credit quality and high expected shareholder recovery upon financial distress.

These insights are shown to be robust in a general model with endogenous investment and financing choices, and are supported by new empirical evidence. The analysis of data simulated from our structural model suggests that our theoretical framework is capable of simultaneously accommodating a number of empirical anomalies in the cross section of stock returns, and it also reveals the inadequacy of traditional risk-adjustment measures when the true equity risk is represented by a time-varying beta that depends on firm characteristics.

Our work underscores the importance of accounting for the time-varying nature of risk and enhances our understanding of conditional CAPM models championed by Ferson and Harvey (1991), Jagannathan and Wang (1996), Gomes, Kogan, and Zhang (2003), and many others. For both empirical tests and practical applications, the ability of measuring conditional beta becomes crucial. While Ferson and Harvey (1997, 1999) advocate modeling a firm's beta as a linear function of macro-economic and firm-specific variables, the findings in this paper indicate that the non-linear dependence of equity beta on firm-specific characteristics and the inherent forward-looking nature of a conditional beta present new econometric challenges for future work.

A Appendix: Proofs

Proof of Proposition 1

A standard argument allows to rewrite the dynamic programming problem (8) as $V^m(P_t) = e^{-rdt} \mathbb{E}^* [V^m(P_t + dP)]$. Using Ito's lemma we obtain the following ODE

$$\frac{1}{2} \sigma^2 P^2 \frac{\partial^2 V^m}{\partial P^2} + (r - \delta) P \frac{\partial V^m}{\partial P} - r V^m + \xi(P - c) - l = 0. \quad (\text{A1})$$

The solution of (A1) is

$$V^m(P) = \xi \left(\frac{P}{\delta} - \frac{c}{r} \right) - \frac{l}{r} + A_1 P^{\phi_1} + A_2 P^{\phi_2}, \quad (\text{A2})$$

where $\phi_1 < 0$ and $\phi_2 > 1$ are the two roots of the characteristic equation

$$\frac{1}{2} \sigma^2 \phi(\phi - 1) + (r - \delta)\phi - r = 0. \quad (\text{A3})$$

As $P \rightarrow \infty$, the probability of the firm not meeting the cost/coupon requirement is zero. This imposes the following no bubble condition on the solution (A2):

$$\lim_{P \rightarrow \infty} V^m(P) = \xi \left(\frac{P}{\delta} - \frac{c}{r} \right) - \frac{l}{r}, \quad (\text{A4})$$

which implies $A_2 = 0$. The final value of the firm with operating and financial leverage is therefore

$$V^m(P) = \begin{cases} \xi \left(\frac{P}{\delta} - \frac{c}{r} \right) - \frac{l}{r} + A_1 P^{\phi_1}, & \text{if } P \geq \underline{P}^m \\ \eta \frac{\xi c}{r}, & \text{if } P < \underline{P}^m. \end{cases} \quad (\text{A5})$$

The numbers of perpetual limited liability (put) options A_1 , and the default threshold \underline{P} , are determined by the following value-matching and smooth-pasting conditions:

$$V^m(\underline{P}^m) = \eta(a + b\underline{P}^m) \quad (\text{A6})$$

$$\left. \frac{\partial V^m}{\partial P} \right|_{P=\underline{P}^m} = \eta b. \quad (\text{A7})$$

Solving these two conditions yield the expressions for A_1 and \underline{P}^m in (11). ■

Definition of default probability

Lemma 1 *Let P_0 be the current product price, evolving according to the process described in (1), and \underline{P} the endogenous determined default trigger. The time 0 cumulative real default probability $\Pr_{(0,T]}$ over the time period $(0, T]$ is given by*

$$\Pr_{(0,T]}(P_0) = \mathcal{N}(h(T)) + \left(\frac{P_0}{\underline{P}}\right)^{-\frac{2\omega}{\sigma^2}} \mathcal{N}\left(h(T) + \frac{2\omega T}{\sigma\sqrt{T}}\right), \quad (\text{A8})$$

with $\omega = \mu - \frac{1}{2}\sigma^2 > 0$, $h(T) = \frac{\log(P/P_0) - \omega T}{\sigma\sqrt{T}}$, and $\mathcal{N}(\cdot)$ the cumulative standard normal function.

Proof: Direct application of the property of hitting time distribution of a geometric Brownian motion, e.g., Harrison (1985), equation (11), p. 14. ■

Proof of Corollary 1

The β of a mature firm that is still operating, i.e., for $P > \underline{P}^m$, is

$$\beta = \frac{\partial \ln V^m(P)}{\partial \ln P} = \frac{1}{V^m} \left(\frac{\xi}{\delta} P + \pi \left(\eta b - \frac{\xi}{\delta} \right) \underline{P}^m \right) \quad (\text{A9})$$

$$= \frac{1}{V^m} \left[\xi \left(\frac{P}{\delta} - \frac{c}{r} \right) - \frac{l}{r} + A_1 P^{\phi_1} + \frac{\xi c + l}{r} - A_1 P^{\phi_1} + \pi \left(\eta b - \frac{\xi}{\delta} \right) \underline{P}^m \right], \quad (\text{A10})$$

$$= 1 + \frac{1}{V^m} \left[\frac{\xi c + l}{r} + \pi \left(1 - \frac{1}{\phi_1} \right) \left(\eta b - \frac{\xi}{\delta} \right) \underline{P}^m \right], \quad (\text{A11})$$

where the second equality follows by using the definition of risk-neutral probability of default (13), the third equality simply re-writes (A9) by isolating the expression of V^m in (9) and the last equality follows from using the expression of A_1 in (11). The corollary follows after substituting the expression of \underline{P}^m in (11) and rearranging. ■

Proof of Corollary 2

We consider separately the cases of small and large default probabilities π .

- i. For low levels of default probability, $\pi \approx 0$, or alternatively $P \gg \underline{P}^m$. In this case the equity value $V^m \approx \xi(P/\delta) - (\xi c + l)/r > 0$ and the equity beta is approximated by

$$\beta|_{\pi \approx 0} \approx 1 + \frac{1}{V^m} \left(\frac{\xi c + l}{r} \right) = 1 + \frac{1}{\frac{\xi P/\delta}{(\xi c + l)/r} - 1}. \quad (\text{A12})$$

It is immediate to see that, at a very low level of default probability π , equity beta is increasing in the leverage l and cost c , and decreasing in P . By (11) the default boundary \underline{P}^m is increasing in c and l , and, by (13), the probability of default π is increasing in the default boundary \underline{P}^m and decreasing in P . Hence, for low levels of π equity betas are increasing in the default probability, independently of the value of η .

- ii. For very high levels of default probability, $\pi \approx 1$, or alternatively $P \approx \underline{P}^m$. We consider the effect of a change in the default threshold \underline{P}^m induced by a change in either c or l on the default probability π and on equity value V^m . We use a Taylor approximation around $\pi = 1$ and study the effect of an infinitesimal decrease $x > 0$ in leverage l on the level of default probability π and equity value V^m . From (13) we can locally express the probability of default π as

$$\pi|_{\pi \approx 1^-} \approx 1 + \left. \frac{\partial \pi}{\partial l} \right|_{\pi=1} (-x) + \frac{1}{2} \left. \frac{\partial^2 \pi}{\partial l^2} \right|_{\pi=1} x^2 + o(x^3) \quad (\text{A13})$$

$$= 1 + \frac{\phi_1}{\eta ar + \xi c + l} x + \frac{\phi_1(\phi_1 + 1)}{2(\eta ar + \xi c + l)^2} x^2 + o(x^3), \quad (\text{A14})$$

where we used the expression of \underline{P}^m in (11). The equity value for $\pi \approx 1$ can be approximated via a Taylor expansion of (12) for small changes x in leverage:

$$V^m|_{\pi \approx 1^-} \approx V^m|_{\pi=1} + \left. \frac{\partial V^m}{\partial l} \right|_{\pi=1} (-x) + \frac{1}{2} \left. \frac{\partial^2 V^m}{\partial l^2} \right|_{\pi=1} x^2 + o(x^3) \quad (\text{A15})$$

$$= \eta(a + b\underline{P}^m) - \frac{1}{2} r \frac{\phi_1}{(\eta ar + \xi c + l)} x^2 + o(x^3). \quad (\text{A16})$$

To approximate the equity beta, we use the expansion of (A16) for V^m and the expansion of (A14) for π in (14). Hence,

$$\beta|_{\pi \approx 1^-} \approx 1 + \frac{-\eta ar - \phi_1 x + o(x^2)}{\eta r (a + b\underline{P}^m) - \frac{1}{2} \frac{\phi_1}{(\eta ar + \xi c + l)} x^2 + o(x^3)}. \quad (\text{A17})$$

When $\eta = 0$,

$$\beta|_{\pi \approx 1^-} \approx 1 + \frac{2(\xi c + l)}{x}. \quad (\text{A18})$$

Hence, $\beta \rightarrow \infty$ as $\pi \rightarrow 1$, i.e., as $x \rightarrow 0^+$. Moreover, locally, β is decreasing in x , or, equivalently, increasing in leverage l .

When $\eta > 0$, from (A17), as $\pi \rightarrow 1$, i.e., as $x \rightarrow 0^+$, $\beta \rightarrow b\underline{P}^m/(a + b\underline{P}^m) \geq 0$, since $b \geq 0$. Moreover, it can be shown that

$$\left. \frac{\partial \beta}{\partial x} \right|_{\pi \approx 1^-} = -\frac{\phi_1}{\eta r(a + b\underline{P}^m)} > 0, \quad (\text{A19})$$

because $\phi_1 < 0$. Hence, for high levels of default probability, if $\eta > 0$, β is increasing in x , or equivalently, *decreasing* in leverage l . ■

Proof of Proposition 2

Because the risk premium λ associated with the price process is positive and constant, according to (7), positive autocorrelation in returns exists if the quantity $\theta \equiv \frac{P}{\beta} \frac{\partial \beta}{\partial P} > 0$. Using the fact that $\beta = \frac{P}{V^m} \frac{\partial V^m}{\partial P}$, together with expression (9) of V^m , we obtain:

$$\begin{aligned} \theta &= \frac{P}{\beta} \frac{\partial}{\partial P} \left(\frac{P}{V^m} \frac{\partial V^m}{\partial P} \right) \\ &= \frac{1}{\beta} \left(\frac{P}{V^m} \frac{\partial V^m}{\partial P} - \left(\frac{P}{V^m} \frac{\partial V^m}{\partial P} \right)^2 + \frac{P^2}{V^m} \frac{\partial^2 V^m}{\partial P^2} \right) \\ &= \frac{1}{\beta} \left(\beta - \beta^2 + \frac{A_1 \phi_1 (\phi_1 - 1) P^{\phi_1}}{V^m} \right) \\ &= 1 - \beta - \frac{1}{\beta V^m} \frac{\phi_1 (\eta a r + \xi c + l)}{r} \pi, \end{aligned} \quad (\text{A20})$$

where the second equality follows from the definition of β in (4), the third equality relies on the definition of V^m in (9), and the last equality uses the definitions of A_1 , \underline{P}^m in (11) and π in (13). When $\pi \rightarrow 0^+$, $\theta < 0$ because $\beta > 1$, independently of η . When $\pi \rightarrow 1^-$, as in the proof of Corollary 2, we use a Taylor approximation around $\pi = 1$ and consider an infinitesimal reduction x in the level of leverage. This allows us to locally approximate the values of V^m and β as in (A16) and (A17) and obtain, as $x \rightarrow 0^+$, i.e., as $\pi \rightarrow 1^-$,

$$\beta V^m|_{\pi \approx 1} = \eta b \underline{P}^m - \frac{\phi_1}{r} x + o(x^2). \quad (\text{A21})$$

When $\eta = 0$, using (A18) and (A21) we can rewrite (A20) as

$$\theta|_{\pi \approx 1^-} \approx -\frac{\xi c + l}{x}. \quad (\text{A22})$$

Hence, as $\pi \rightarrow 1^-$, i.e., as $x \rightarrow 0^+$, $\theta \rightarrow -\infty$. When $\eta > 0$, from Corollary 2, $\beta \rightarrow b\underline{P}^m / (a + b\underline{P}^m) \geq 0$ as $\pi \rightarrow 1^-$. Substituting (A21) into (A20), and setting $\beta = b\underline{P}^m / (a + b\underline{P}^m)$, yields

$$\theta|_{\pi \approx 1^-} \approx \frac{a}{a + b\underline{P}^m} - \frac{\phi_1(\eta ar + \xi x + l)}{\eta r b \underline{P}^m - \phi_1 x}. \quad (\text{A23})$$

Hence, as $\pi \rightarrow 1^-$, i.e., $x \rightarrow 0^+$, $\theta \rightarrow \frac{a}{a + b\underline{P}^m} - \frac{\phi_1(\eta ar + \xi x + l)}{\eta r b \underline{P}^m} > 0$. ■

Proof of Proposition 3

To solve (16), let us define the following expectations:

$$F(P) = \mathbb{E}^* \left[\int_0^{\tau_L \wedge \tau_G} [P_{t+s} - c - l] e^{-rs} ds \right] \quad (\text{A24})$$

$$f(P) = \mathbb{E}^* \left[e^{-r\tau_L} \mathbf{I}_{\{\tau_L < \tau_G\}} \right] \quad (\text{A25})$$

$$g(P) = \mathbb{E}^* \left[e^{-r\tau_G} \mathbf{I}_{\{\tau_G < \tau_L\}} \right]. \quad (\text{A26})$$

By the analysis above, $F(P)$ can be solved via dynamic programming and yields

$$F(P) = \frac{P}{\delta} - \frac{c + l}{r} + AP^{\phi_1} + BP^{\phi_2}, \quad (\text{A27})$$

with $\phi_1 < 0$ and $\phi_2 > 1$ solutions of (A3), and A and B arbitrary constants. For *given* \underline{P}^g and \overline{P} , the solutions of (A25) and (A26) are:¹⁹

$$f(P) = \frac{P^{\phi_1} (\overline{P})^{\phi_2} - P^{\phi_2} (\overline{P})^{\phi_1}}{(\overline{P})^{\phi_2} (\underline{P}^g)^{\phi_1} - (\overline{P})^{\phi_1} (\underline{P}^g)^{\phi_2}}, \quad g(P) = \frac{P^{\phi_2} (\underline{P}^g)^{\phi_1} - P^{\phi_1} (\underline{P}^g)^{\phi_2}}{(\overline{P})^{\phi_2} (\underline{P}^g)^{\phi_1} - (\overline{P})^{\phi_1} (\underline{P}^g)^{\phi_2}}. \quad (\text{A28})$$

Notice that, as it should be, $f(\underline{P}^g) = 1$ and $g(\underline{P}^g) = 0$. Similarly, $f(\overline{P}) = 0$ and $g(\overline{P}) = 1$. Using the fact that the salvage value $V_A(\underline{P}^g) = \eta X^g(\underline{P}^g)$, the value of a growing firm can be

¹⁹See Geman and Yor (1996).

expressed as

$$V^g(P) = \frac{P}{\delta} - \frac{c+l}{r} + AP^{\phi_1} + BP^{\phi_2} + \eta X^g(\underline{P}^g)f(P) + (V^m(\bar{P}) - I)g(P) \quad (\text{A29})$$

where $f(P)$ and $g(P)$ are given in (A28). The above expression contains four unknowns, A , B , \underline{P}^g , and \bar{P} . These can be obtained by imposing the following two pairs of value-matching and smooth-pasting conditions:

$$V^g(\underline{P}^g) = \eta X^g(\underline{P}^g), \quad \left. \frac{\partial V^g(P)}{\partial P} \right|_{P=\underline{P}^g} = 0, \quad (\text{A30})$$

$$V^g(\bar{P}) = V^m(\bar{P}) - I, \quad \left. \frac{\partial V^g(P)}{\partial P} \right|_{P=\bar{P}} = \left. \frac{\partial V^m(P)}{\partial P} \right|_{P=\bar{P}}, \quad (\text{A31})$$

where $\left. \frac{\partial V^m(P)}{\partial P} \right|_{P=\bar{P}} = \frac{\xi}{\delta} + A_1 \phi_1 \bar{P}^{\phi_1 - 1}$, by (9) and (11). ■

Proof of Corollary 3

Let us write the equity value in the following form

$$V^g(P) = \frac{P}{\delta} - \frac{c+l}{r} + A'P^{\phi_1} + B'P^{\phi_2}, \quad (\text{A32})$$

with

$$A' = A + \eta X^g(\underline{P}^g) \frac{\bar{P}^{\phi_2}}{\Gamma} - (V^m(\bar{P}) - I) \frac{(\underline{P}^g)^{\phi_2}}{\Gamma}, \quad (\text{A33})$$

$$B' = B - \eta X^g(\underline{P}^g) \frac{\bar{P}^{\phi_1}}{\Gamma} + (V^m(\bar{P}) - I) \frac{(\underline{P}^g)^{\phi_1}}{\Gamma}, \quad (\text{A34})$$

where A and B are obtained from the solution of the value-matching and smooth-pasting conditions (A30)–(A31) in Proposition 3, and $\Gamma = (\bar{P})^{\phi_2} (\underline{P}^g)^{\phi_1} - (\bar{P})^{\phi_1} (\underline{P}^g)^{\phi_2}$. The β of a growth firm is

$$\begin{aligned} \beta &= \frac{\partial \ln V^g(P)}{\partial \ln P} \\ &= \frac{1}{V^g(P)} \left(\frac{P}{\delta} + \phi_1 A' P^{\phi_1} + \phi_2 B' P^{\phi_2} \right) \\ &= 1 + \frac{1}{V^g(P)} \left[\frac{l+c}{r} + (\phi_1 - 1) A' P^{\phi_1} + (\phi_2 - 1) B' P^{\phi_2} \right]. \end{aligned} \quad (\text{A35})$$

From equations (A28), we have

$$P^{\phi_1} = (\underline{P}^g)^{\phi_1} f(P) + (\bar{P})^{\phi_1} g(P), \quad P^{\phi_2} = (\underline{P}^g)^{\phi_2} g(P) + (\bar{P})^{\phi_2} f(P). \quad (\text{A36})$$

Substitution in the expression for β yields

$$\begin{aligned} \beta = & 1 + \frac{1}{Vg(P)} \left[\frac{l+c}{r} + ((\phi_1 - 1)A'(\underline{P}^g)^{\phi_1} + (\phi_2 - 1)B'(\bar{P})^{\phi_2}) f(P) \right] \\ & + \frac{1}{Vg(P)} \left[((\phi_1 - 1)A'(\bar{P})^{\phi_1} + (\phi_2 - 1)B'(\bar{P})^{\phi_2}) g(P) \right]. \end{aligned} \quad (\text{A37})$$

■

B Appendix: Numerical analysis of the simple model of Section 2

There are a total of twelve parameters, out of which two are common economy-wide (i.e., the risk-free rate r and the maximal Sharpe ratio SR); five refer to the firm's output price process and can be thought of as industry-specific (i.e., the growth rate in the price process μ , the parameter δ , the volatility σ , the correlation with the pricing kernel in the economy ρ , and the initial output price P_0); and five are firm specific (i.e., the operating cost c , the financial leverage l , the scale of operation of mature firms ξ , the investment cost I , and the level of shareholder recovery η). To construct the cross section we need to select a set of parameter combinations characterizing each firm and a set of initial values for the price of the firm's output. Below we provide a description of our choice of parameters. A summary of our parameter choices is reported in Table 1.

We select the risk-free rate r to be 3% per annum to roughly match empirical estimates of the short rate and choose the maximal Sharpe ratio SR to be 0.5, in line with other studies (e.g., Campbell (2003)). Given the non-stationarity of the price process (1), we cannot rely on long-run properties to determine the growth rate $\mu = r - \delta + \lambda$. From equation (5) $\lambda = \rho SR \sigma$. We choose a benchmark value ρ of 0.7, and a benchmark value of σ of 0.3, both consistent with Sagi and Seasholes (2007). In constructing the cross section, we allow for two additional value for ρ , 0.5 and 0.9, and two additional value for σ , 0.2 and 0.4. This yields 9 different values for λ . δ is a "free" parameter which has to be less than the risk-free rate in order to insure that the growth option is ever exercised. The role of this parameter is to act as a scaling factor and does not qualitatively affect the results. We set it to be 1%. In addition, we choose 21 different levels of the initial output price P_0 , ranging from 0.1 to 0.3. The different initial values of P_0 represent differences across industries due to idiosyncratic shocks.

To choose the level of operating expenses c , we rely on the functional form of equity value in Propositions 1 and 3. Absent leverage and the limited liability/growth options, the net value of equity would be 0 if $P/\delta = c/r$. We use this as a reference point for the range of values of c to consider. Given the range of initial prices and the selected values of r and δ , the implied range of c is $r/\delta \times [0.1, 0.3] = [0.3, 0.9]$. We choose four different values of c in this range. This choice guarantees that at least for some firms in the cross section, the limited liability option is

valuable. The financial leverage is chosen as a fraction of the operating cost c so that $(c - l)/r$ is not negative. We choose six different levels of financial leverage, ranging from 40% to 90% of operating costs. We select three values for the scale of operation for mature firms: $\xi = 1.5, 2,$ and 2.5 . The investment cost I is linked to the size of growth and is chosen to be equal to the increase in the scale of operation $(\xi - 1)$ times the capitalized value of operating costs c/r , a proxy for the book value of assets. Moreover, we select two different values for the expected shareholder recovery in financial distress: $\eta = 0$, and 10% of asset value. Finally, we choose a horizon of one year to compute default probability according to equation (A8).

In total, for any given value of η , our cross section of firms at time 0 consists of 27,216 firms equally split between growth and mature firms: $(2 \text{ types of firms}) \times (21 \text{ initial prices}) \times (4 \text{ levels of } c) \times (6 \text{ levels of } l) \times (3 \text{ levels of } \xi) \times (3 \text{ levels of } \sigma) \times (3 \text{ levels of } \rho)$.

C Appendix: Empirical analysis

C.1 Data and summary statistics

In our empirical investigation, we use a market-based measure of default probability, the *Expected Default Frequency* (EDF), obtained directly from *Moody's KMV* (MKMV hereafter). This measure is constructed from the Vasicek-Kealhofer model (Kealhofer (2003a,b)) which adapts the Black and Scholes (1973) and Merton (1974) framework and is mapped with a comprehensive database of historical default experiences.²⁰

We match the EDF database with the CRSP and COMPUSTAT databases, i.e., a stock needs to have data in all three databases to be included in our analysis. Specifically, for a given month, we require a firm to have an EDF measure and an implied asset value in the MKMV dataset; stock price, shares outstanding, and returns data from CRSP; and accounting numbers from COMPUSTAT for firm-level characteristics. We limit our sample to non-financial US firms.²¹ We also drop from our sample stocks with a negative book-to-market ratio. Our baseline sample contains 1,430,713 firm-month observations and spans from January 1969 to December 2003.²² This is the same data sample used in Garlappi, Shu, and Yan (2006).

Summary statistics for the EDF measure are reported in Table 4. The average EDF measure in our sample is 3.44% with a median of 1.19%. The table shows that there are time-series variations in the average as well as in the distribution of the EDF measure, and that the majority of the firms in our dataset have an EDF score below 4%. One caveat is that MKMV assigns an EDF score of 20% to all firms with an EDF measure larger than 20%. Around 5% of the firms are in this group at any given time.

Since the EDF measure is based on market prices, in order to mitigate the effect of noisy stock prices on the default score, we use an exponentially smoothed version of the EDF measure based on a time-weighted average (see, e.g., RiskMetric Group (2006)). Specifically, for default

²⁰See Crosbie and Bohn (2003) for details on how MKMV implements the Vasicek-Kealhofer model to construct the EDF measure.

²¹Financial firms are identified as firms whose industrial code (SIC) are between 6000 and 6999.

²²We follow Shumway (1997) to deal with the problem of delisted firms. Specifically, whenever available, we use the delisted return reported in the CRSP datafile for stocks that are delisted in a particular month. If the delisting return is missing but the CRSP datafile reports a performance-related delisting code (500, 520-584), then we impute a delisted return of -30% in the delisting month.

probability in month t , we use

$$\overline{EDF}_t = \frac{\sum_{s=0}^5 e^{-s\nu} EDF_{t-s}}{\sum_{s=0}^5 e^{-s\nu}}, \quad (\text{C38})$$

where ν is chosen to satisfy $e^{-5\nu} = 1/2$, such that the five-month lagged EDF measure receives half the weight of the current EDF measure. The empirical results are reported based on \overline{EDF}_t , which we will still refer to as EDF for notational convenience. Our results are not sensitive to the smoothing scheme for EDF.

C.2 Value premium and default probability

We first examine how value premium changes with default probability. We sort *all* stocks in our sample each month into ten deciles according to their EDF scores and, independently, into three terciles according to their book-to-market ratios. We then record both value-weighted and equal-weighted returns of each portfolio in the second month after portfolio formation to avoid possible market microstructure effects (Da and Gao (2008)). Panel A of Table 5 presents the time-series average value premium in each EDF decile.

The results show that in the full sample value premium initially rises with default probability and then starts to decline at high levels of default probability. For value-weighted returns, the value premium rises with EDF scores until the seventh EDF decile and then turns and drops to a lower level in the last decile. This hump-shaped pattern is particularly pronounced with equal-weighted returns, with a clear decline starting from the seventh decile all the way to the highest level of EDF scores. The more pronounced pattern is due to the fact that stocks with higher EDF scores, which usually have lower market capitalizations, within each EDF decile take more weight in equal-weighted portfolio returns. This hump shape in the relationship between value premium and default probability is precisely the prediction of our model as a consequence of potential shareholder recovery upon default. The patterns in raw returns persists after we control for risk according to the Fama and French (1993) three-factor model (see the rows labeled “FF- α ”).

The results presented here seem to contradict the existing evidence in the literature that the value premium is larger for firms with higher default probability (e.g., Griffin and Lemmon (2002)). Note that in these empirical studies, a customary sample filtering rule is to exclude

stocks with a price per share less than \$5 to avoid market microstructure issues. As we discussed earlier, filtering out these stocks excludes the stocks with very high levels of default probability. Therefore, it is likely that the extant empirical evidence reflects the variation of the value premium over a limited range of default likelihood where the value premium increases in default probability, as indicated by our theory.

To test this notion, we restrict our sample of stocks to those with stock prices larger than \$5 per share, or with market capitalization larger than the breakpoint of the lowest size decile of NYSE stocks, and redo the same portfolio formation and return recording as for the full sample. We again report only the second-month portfolio returns to mitigate liquidity and market microstructure concerns. The results, presented in Panel B of Table 5, confirm that for this subset of stocks, the value premium is indeed monotonically increasing in EDF scores, consistent with the existing evidence in the literature. Taken together, results in Table 5 provide a solid confirmation of the prediction of our model for the relationship between value premium and default probability, and hence validate the importance of potential shareholder recovery for the cross section of stock returns.

C.3 Momentum profits, shareholder recovery and default probability

Our model also predicts that momentum in stock returns is strongest for firms with high levels of default probability. This is consistent with the evidence in Avramov, Chordia, Jostova, and Philipov (2007). Furthermore, our model yields an additional unique prediction regarding how the expected shareholder recovery, as represented by η in our model, can affect the cross-sectional pattern of momentum. A verification of this prediction will be a strong piece of supporting evidence that our model provides a valid mechanism for understanding the momentum phenomenon.

In order to test the model prediction, we need to have proxies for the role of η . We use three proxies: asset size, R&D expenditure, and industry concentration measured by the Herfindahl index of sales. The justification of these proxies is discussed in detail in Garlappi, Shu, and Yan (2006). Basically, as documented by Franks and Torous (1994) and Betker (1995), firm size is a persistent determinant of the deviation from the absolute priority rule. Opler and Titman (1994) show that firms with high costs of R&D suffer the most in financial distress and may be subject to liquidity shortage that diminishes the bargaining power of shareholders in financial

distress (e.g., Fan and Sundaresan (2000)). Therefore, firms with smaller asset bases or higher R&D expenditures are likely to have a smaller η . Moreover, Shleifer and Vishny (1992) argue that specificity of a firm’s assets increases liquidation costs when the firm is in financial distress. High liquidation costs can motivate creditors to negotiate with shareholders in the resolution of financial distress and therefore increase the chance of shareholder recovery. Firms in a more concentrated industry are likely to have more specific assets, and hence a larger η .

To test the prediction that stocks with high η and high default probability have stronger momentum, we sort all stocks in each month independently into terciles of a proxy for η , terciles of EDF scores, and quintiles of losers and winners based on the past six-month returns. We then record the equal-weighted portfolio returns over the six-month period, starting from the second month after portfolio formation. We report in Table 6 the results of monthly momentum returns, averaged over the sample period, for portfolios in the top tercile of EDF scores.

Panel A of Table 6 shows that for firms with high EDF scores and large asset bases, winners outperform losers by 1.20% per month over the next six-month period. In contrast, among those high EDF firms with small asset bases, past winners do not outperform past losers, i.e., the momentum profits are not statistically different from zero for this group of firms. This difference is consistent with our prediction and is significant with a t -statistic of 3.96. Panel B demonstrates that firms with high EDF scores and low R&D expenditures experience strong momentum in stock returns, but firms with high R&D expenditures with similar credit profiles do not. Moreover, high EDF firms in a more concentrated industry are more likely to have momentum in stock returns than similar firms in a more competitive industry, albeit the statistical significance may be attenuated because of the coarse nature of the Herfindahl measure for our purpose. These conclusions are generally unaffected, and in some cases stronger, after we control for risk according to the Fama and French (1993) three-factor model (see the rows labeled “FF- α ”). All these results directly confirm the prediction of our model regarding the role of expected shareholder recovery for financially distressed firms in enhancing momentum in equity returns.

Table 1: Parameters for the simple model of Section 2

Parameter	Values
r	3%
SR	0.5
δ	1%
σ	20%, 30%, 40%
ρ	0.5, 0.7, 0.9
μ	$r - \delta + \rho SR \sigma$
P_0	[0.1, 0.3], 21 values equally spaced
c	0.3, 0.5, 0.7, 0.9
l	[0.4 c to 0.9 c], 6 values equally spaced (0.1 c interval)
ξ	1.5, 2, 2.5
η	0, 2.5%, 5%
I	$\frac{(\xi-1)c}{r}$

Table 2: Parameter for the general model of Section 3

Parameter	Description	Value
\bar{X}	Long-run average of aggregate productivity	-3.100
σ_x	Conditional volatility of aggregate productivity	0.002
ρ_x	Persistence of aggregate productivity	0.983
\bar{Z}	Long-run average of firm-specific productivity	0.000
σ_z	Conditional volatility of firm-specific productivity	0.100
ρ_z	Persistence of firm-specific productivity	0.900
α	Capital share	0.650
κ	Capital depreciation	0.010
f	Variable cost of production	0.000
F	Fixed cost of production	0.034
α	Capital share	0.650
θ	Adjustment cost	15.000
γ_0	Constant price of risk	50
γ_1	Time-varying price of risk	-1000
β	Time-preference coefficient	0.995
τ	Tax rate	0.350
ξ_0	Fixed bankruptcy cost	0.120
ξ_1	Liquidation value per unit of capital	0.900
λ_0	Fixed equity issuance cost	0.080
λ_1	Variable equity issuance cost	0.025

Table 3: Unconditional sample moments

The table reports the average moments across 200 simulations of the stationary policy obtained by solving the general model in Section 3. Each simulation consists of a cross section of 2,000 firms for which we construct a time series of 500 monthly observations.

Moments	Model	Data
Annual risk-free rate	0.021	0.018
Annual Sharpe ratio	0.404	0.430
Annual volatility of real interest rate	0.028	0.030
Annual equity premium	0.092	0.060
Annual investment rate	0.108	0.150
Market-to-book ratio	1.224	1.492
Market leverage	0.285	0.290
1-year default probability	0.049	0.040

Table 4: Summary statistics of the EDF measure

At the beginning of every three-year interval (starting from January 1970), the table reports the number of firms in our sample, the mean, standard deviation, median, and first and third quartiles of the EDF distribution. Sample period: January 1969–December 2003. EDF quantities are expressed in percent units.

Month	# Firm	Mean	Std.	Median	Quart 1	Quart 3
Jan-70	1,455	1.19	1.76	0.56	0.17	1.50
Jan-73	1,894	2.00	3.20	0.83	0.23	2.25
Jan-76	2,945	3.87	4.77	2.06	0.88	4.58
Jan-79	3,149	2.57	4.21	0.97	0.31	2.56
Jan-82	3,116	3.19	4.60	1.42	0.59	3.40
Jan-85	3,566	3.21	5.17	0.98	0.34	3.18
Jan-88	3,745	4.25	5.83	1.68	0.48	5.02
Jan-91	3,627	5.48	7.11	1.80	0.37	8.08
Jan-94	3,916	2.73	4.56	0.85	0.22	2.82
Jan-97	4,541	2.72	4.61	0.78	0.18	2.82
Jan-00	4,246	3.68	5.11	1.53	0.52	4.26
Jan-03	3,572	5.23	6.52	2.03	0.59	7.39
Full Sample	1,430,713	3.44	5.22	1.19	0.35	3.75

Table 5: Relationship between value premium and default probability

Each month, stocks are sorted independently into terciles of book-to-market ratios (BM) and deciles of MKMV's EDF scores (EDF). The value-weighted (VW) and equal-weighted (EW) returns of each portfolio in the second month after portfolio formation are recorded and averaged over time. Panel A includes all stocks in our sample, while Panel B excludes those with stock prices less than \$5 per share or with a market capitalization smaller than the NYSE bottom size decile breakpoint. FF- α refers to the value premium after controlling for risk according to the Fama-French three-factor model. *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively.

	Low EDF	1	2	3	4	5	6	7	8	9	High EDF	High-Low	t-value
Panel A: Full sample													
BM													
<i>Low</i>	0.96	1.03	1.03	0.71	0.73	0.63	0.63	0.05	-0.02	-0.19	0.19	-0.78	-1.51
<i>Medium</i>	1.05	1.11	1.20	1.20	1.15	1.10	1.10	1.11	1.32	0.81	0.47	-0.58	-1.11
<i>High</i>	0.96	1.14	1.29	1.29	1.41	1.33	1.27	1.74	1.36	1.54	1.42	0.45	1.03
High - Low	-0.00	0.11	0.59**	0.69**	0.70**	0.64**	0.64**	1.69***	1.38***	1.72***	1.23***		
t-value	-0.01	0.48	2.26	2.49	2.49	2.57	2.37	5.67	3.98	4.50	3.17		
FF- α	-0.42**	-0.28*	0.24	0.38*	0.38*	0.36*	0.34	1.38***	0.96***	1.35***	0.87**		
t-value	-2.57	-1.92	1.42	1.92	1.83	1.83	1.59	5.35	3.25	3.72	2.29		
EW Returns													
<i>Low</i>	1.13	1.01	0.86	0.86	0.79	0.62	0.72	0.33	0.55	0.66	1.75	0.63	1.21
<i>Medium</i>	1.15	1.27	1.33	1.33	1.33	1.46	1.35	1.38	1.34	1.34	1.79	0.63	1.43
<i>High</i>	1.32	1.34	1.42	1.42	1.66	1.58	1.65	1.78	1.79	1.87	2.13	0.80**	2.10
High - Low	0.20	0.33	0.56**	0.87***	0.96***	0.96***	0.93***	1.45***	1.24***	1.21***	0.37		
t-value	1.04	1.56	2.55	3.72	3.72	4.14	4.13	6.12	4.44	4.27	1.22		
FF- α	-0.07	0.10	0.38***	0.67***	0.67***	0.72***	0.70***	1.29***	1.06***	1.05***	0.31		
t-value	-0.58	1.00	3.28	5.17	5.05	5.05	4.67	6.83	4.47	4.03	1.05		
EW Returns													
<i>Low</i>	0.96	1.04	1.04	1.11	0.48	0.81	0.65	0.61	0.68	0.35	-0.47	-1.43***	-3.72
<i>Medium</i>	0.97	1.05	1.07	1.07	1.12	1.15	1.07	1.27	1.21	0.96	0.96	-0.01	-0.04
<i>High</i>	1.04	1.00	1.23	1.23	1.18	1.28	1.37	1.26	1.34	1.22	1.51	0.47	1.41
High - Low	0.08	-0.03	0.12	0.12	0.70**	0.47	0.72**	0.65**	0.66**	0.87***	1.98***		
t-value	0.32	-0.14	0.49	0.49	2.79	1.59	2.44	2.20	2.24	3.12	6.27		
FF- α	-0.38**	-0.34**	-0.25	0.36**	0.36**	0.07	0.32	0.37	0.34	0.57**	1.54***		
t-value	-2.23	-2.07	-1.48	2.08	2.08	0.96	1.53	1.54	1.45	2.46	5.66		
EW Returns													
<i>Low</i>	1.16	1.12	0.99	0.99	0.85	0.82	0.72	0.67	0.67	0.20	0.00	-1.16***	-3.68
<i>Medium</i>	1.08	1.13	1.30	1.30	1.27	1.32	1.29	1.26	1.41	1.20	1.08	-0.00	-0.00
<i>High</i>	1.19	1.24	1.33	1.33	1.41	1.44	1.54	1.45	1.58	1.44	1.40	0.20	0.75
High - Low	0.03	0.12	0.34	0.34	0.55**	0.62**	0.82***	0.77***	0.91***	1.23***	1.40***		
t-value	0.17	0.53	1.47	1.47	2.32	2.57	3.33	3.25	3.77	5.40	5.81		
FF- α	-0.30**	-0.12	0.11	0.32**	0.32**	0.35**	0.56***	0.55***	0.59***	0.98***	1.08***		
t-value	-2.39	-1.03	0.98	2.41	2.41	2.72	3.80	3.60	3.90	6.01	5.50		

Panel B: Subsample of stocks with per-share price greater than \$5

Table 6: Effect of shareholder recovery on momentum profits

Each month, all stocks are sorted independently into terciles of EDF scores, terciles of a proxy for expected shareholder recovery and quintiles of winners/losers according to past six-month returns. The returns of each portfolio for the next six-month period are recorded and averaged through time. Only portfolios in the top terciles are reported in the table. The proxies for the expected shareholder recovery are: asset size (AVL), R&D expenditure-asset ratio (R&D), and Herfindahl index of sales (SalesHfdl). FF- α refers to momentum profits after controlling for risk according to the Fama-French three-factor model. *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively.

Panel A: Momentum profits across AVL groups								
AVL	Loser	2	3	4	Winner	W-L	<i>t</i> -stat	
Low	2.11	1.90	1.94	2.01	2.06	-0.05	-0.24	
Med	0.63	1.15	1.35	1.46	1.70	1.06***	3.80	
High	0.34	1.01	1.27	1.29	1.62	1.20***	3.14	
High-Low						1.21***	3.96	
FF- α						1.14***	3.64	

Panel B: Momentum profits across R&D groups								
R&D	Loser	2	3	4	Winner	W-L	<i>t</i> -stat	
Low	0.99	1.34	1.72	1.67	1.82	0.76***	3.07	
Med	2.08	1.82	2.02	2.09	2.03	-0.04	-0.13	
High	2.76	2.55	2.40	2.61	2.47	-0.32	-1.16	
High-Low						-1.07***	-3.74	
FF- α						-1.14***	-3.89	

Panel C: Momentum profits across SalesHfdl groups								
SalesHfdl	Loser	2	3	4	Winner	W-L	<i>t</i> -stat	
Low	1.75	1.63	1.75	1.71	1.87	0.12	0.51	
Med	1.67	1.68	1.78	2.00	2.02	0.35	1.40	
High	1.37	1.51	1.63	1.59	1.84	0.47*	1.72	
High-Low						0.35	1.53	
FF- α						0.44*	1.89	

References

- Avramov, Doron, Tarun Chordia, Gergana Jostova, and Alexander Philipov, 2007, Momentum and credit rating, *Journal of Finance* 62, 2503–2520.
- Berk, Jonathan B., Richard C. Green, and Vasant Naik, 1999, Optimal investment, growth options, and security returns, *Journal of Finance* 54, 1553–1607.
- Betker, Brian, 1995, Management’s incentive, equity bargaining power, and deviation from absolute priority, *Journal of Business* 68, 161–184.
- Bharath, Sreedhar, Venky Panchapegesan, and Ingrid Werner, 2007, The changing nature of chapter 11, Working Paper, University of Michigan, GSAM and Ohio State University.
- Black, Fischer, and John C. Cox, 1976, Valuing corporate securities: Some effects of bond indenture provisions, *Journal of Finance* 31, 351–367.
- Black, Fischer, and Myron Scholes, 1973, The pricing of options and corporate liabilities, *Journal of Political Economy* 81, 637–659.
- Brown, David, Brian Ciochetti, and Timothy J. Riddiough, 2006, Theory and evidence on the resolution of financial distress, *Review of Financial Studies* 19, 1357–1397.
- Campbell, John Y., 2003, Consumption based asset pricing, in G. Constantinides, M. Harris, and R. Stultz, ed.: *Handbook of the Economics of Finance* . pp. 803–887 (North-Holland, Amsterdam).
- , Jens Hilscher, and Jan Szilagyi, 2008, In search of distress risk, *Journal of Finance* Forthcoming.
- Carlson, Murray, Adlai Fisher, and Ron Giammarino, 2004, Corporate investment and asset price dynamics: Implications for the cross-section of returns, *Journal of Finance* 59, 2577–2603.
- Chen, Jason, 2006, Firm life expectancy and the heterogeneity of the book-to-market effect, Working Paper, University of British Columbia.
- Cooley, Thomas F., and Vincenzo Quadrini, 2001, Financial markets and firm dynamics, *American Economic Review* 91, 1286–1310.

- Cooper, Ilan, 2006, Asset pricing implications of nonconvex adjustment costs and irreversibility of investment, *Journal of Finance* 61, 139–170.
- Cooper, Russel W., and João Ejarque, 2003, Financial frictions and investment: A requiem in q , *Review of Economic Dynamics* 6, 710–728.
- Crosbie, Peter J., and Jeffrey R. Bohn, 2003, Modeling default risk, *Moody's KMV* Available at http://www.defaultrisk.com/pp_model_35.htm.
- Da, Zhi, and Pengjie Gao, 2008, Clientile change, liquidity shock, and the return on financially distressed stocks, *Journal of Financial and Quantitative Analysis* forthcoming.
- Dichev, Iliia D., 1998, Is the risk of bankruptcy a systematic risk?, *Journal of Finance* 53, 1131–1147.
- Duffie, Darrell, and William Zame, 1989, The consumption based capital asset pricing model, *Econometrica* 57, 1279–1297.
- Eberhart, A. W., W. Moore, and R. Roenfeldt, 1990, Security pricing and deviation from absolute priority rule in bankruptcy proceedings, *Journal of Finance* 45, 1457–1469.
- Fama, Eugene F., and Kenneth R. French, 1992, The cross-section of expected stock returns, *Journal of Finance* 47, 427–465.
- , 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3–56.
- , 1996, Multifactor explanations of asset pricing anomalies, *Journal of Finance* 51, 55–84.
- Fan, Hua, and Suresh M. Sundaresan, 2000, Debt valuation, renegotiation, and optimal dividend policy, *Review of Financial Studies* 13, 1057–1099.
- Ferguson, Michael F., and Richard L. Shockley, 2003, Equilibrium ‘anomalies’, *Journal of Finance* 58, 2549–2580.
- Ferson, Wayne E., and Campbell R. Harvey, 1991, The variation of economic risk premiums, *Journal of Political Economy* 99, 385–415.
- , 1997, Fundamental determinants of international equity returns: A perspective on conditional asset pricing, *Journal of Banking and Finance* 21, 1625–1665.

- , 1999, Conditioning variables and the cross section of stock returns, *Journal of Finance* 54, 1325–1360.
- Franks, Julian R., and Walter N. Torous, 1989, An empirical investigation of u.s. firms in renegotiation, *Journal of Finance* 44, 747–779.
- , 1994, A comparison of financial restructuring in distressed exchanges and chapter 11 reorganizations, *Journal of Financial Economics* 35, 349–370.
- Gala, Vito D., 2006, Investment and returns, Working Paper, London Business School.
- Garlappi, Lorenzo, Tao Shu, and Hong Yan, 2006, Default risk, shareholder advantage, and stock returns, *Review of Financial Studies* forthcoming.
- Geman, H., and M. Yor, 1996, Pricing and hedging double-barrier options: A probabilistic approach, *Mathematical Finance* 4, 365–378.
- Gomes, João F., 2001, Financing investment, *American Economic Review* 90, 1263–1285.
- , Leonid Kogan, and Lu Zhang, 2003, Equilibrium cross-section of returns, *Journal of Political Economy* 111, 693–732.
- Gomes, João F., and Lukas Schmid, 2008, Levered returns, Working Paper, The Wharton School, University of Pennsylvania.
- Gomes, João F., Amir Yaron, and Lu Zhang, 2006, Asset pricing implications of firms financing constraints, *Review of Financial Studies* 19, 1321–1356.
- Griffin, John M., and Michael L. Lemmon, 2002, Book-to-market equity, distress risk and stock returns, *Journal of Finance* 57, 2317–2336.
- Harrison, Michael J., 1985, *Brownian Motion and Stochastic Flow Systems* (John Wiley and Sons: New York, N.Y.).
- Harvey, Campbell R., and Akhtar Siddique, 2000, Conditional skewness in asset pricing tests, *Journal of Finance* 55, 1263–1295.
- Hecht, Peter, 2000, The cross section of expected firm (not equity) returns, Working Paper, University of Chicago.

- Hennessy, Christopher, and Toni Whited, 2005, Debt dynamics, *Journal of Finance* 60, 1129–1165.
- , 2007, How costly is external financing? evidence from a structural estimation, *Journal of Finance* 62, 1705–1745.
- Jagannathan, Ravi, and Zhenyu Wang, 1996, The conditional capm and the cross-section of expected returns, *Journal of Finance* 51, 3–53.
- Johnson, Timothy C., 2002, Rational momentum effects, *Journal of Finance* 57, 585–608.
- Kealhofer, Stephen, 2003a, Quantifying credit risk I: Default prediction, *Financial Analysts Journal* January/February, 30–44.
- , 2003b, Quantifying credit risk II: Debt valuation, *Financial Analysts Journal* May/June, 78–92.
- Leland, Hayne E., 1994, Corporate debt value, bond covenants, and optimal capital structure, *Journal of Finance* 49, 1213–1252.
- Li, Erica X., 2008, Corporate governance, the cross section of returns, and financing choices, Working Paper, University of Michigan.
- Lidvan, Dimitry, Horacio Sapriza, and Lu Zhang, 2008, Financially constrained stock returns, *Journal of Finance* forthcoming.
- Lucas, Robert E. Jr., 1967, Adjustment costs and the theory of supply, *Journal of Political Economy* 75, 321–334.
- Merton, Robert C., 1974, On the pricing of corporate debt: The risk structure of interest rates, *Journal of Finance* 29, 449–470.
- Morellec, Erwan, Brosi Nikolov, and Norman Schürhoff, 2008, Dynamic capital structure under managerial entrenchment: Evidence from a structural estimation, Working Paper, University of Lausanne.
- Novy-Marx, Robert, 2008, Competition, productivity and the cross section of expected returns, Working Paper, University of Chicago.
- Obreja, Iulian, 2006, Financial leverage and the cross-section of stock returns, Working Paper, Carnegie Mellon.

- Opler, Tim, and Sheridan Titman, 1994, Financial distress and corporate performance, *Journal of Finance* 49, 1015–1040.
- RiskMetric Group, 2006, The riskmetric 2006 methodology, RiskMetric Technical Document.
- Sagi, Jacob S., and Mark S. Seasholes, 2007, Firm-specific attributes and the cross-section of momentum, *Journal of Financial Economics* 84, 389–434.
- Shleifer, Andrei, and Robert W. Vishny, 1992, Liquidation values and debt capacity: A market equilibrium approach, *Journal of Finance* 47, 1343–1366.
- Shumway, Tyler, 1997, The delisting bias in crsp data, *Journal of Finance* 52, 327–340.
- Tauchen, George, 1986, Finite state markov-chain approximations to univariate and vector autoregressions, *Economics Letters* 20, 177–181.
- Vassalou, Maria, and Yuhang Xing, 2004, Default risk in equity returns, *Journal of Finance* 59, 831–868.
- Weiss, Lawrence A., 1991, The bankruptcy code and violations of absolute priority, *Journal of Applied Corporate Finance* 4, 71–78.
- , and Vedran Capkun, 2007, Bankruptcy resolution: Priority claims with the secured creditor in control, Working paper, Georgetown and HEC Paris.
- Zhang, Lu, 2005, The value premium, *Journal of Finance* 60, 67–103.