

# Explaining the Returns of Chilean Equities: Are All Markets Created Equal?

Gonzalo Maturana\*

Department of Industrial Engineering, University of Chile

June, 2009

## Abstract

This paper studies the relevance in the Chilean market of factors such as the ones proposed in the Fama and French three factor model: SMB, the difference between the returns of diversified portfolios of small and big firms; HML, the difference between the returns of diversified portfolios of high and low book-to-market equity ratios; and the market's risk premium; plus a momentum factor (MOM). The results show that in the period between January 1992 and June 2006, the size (measured as the stock-exchange capitalization of the shares) and the book-to-market equity ratio of the companies have a significant incidence in explaining the differences between the returns of shares in the Chilean stock-market. On the other hand, the market factor does not explain the differences observed in the returns in the cross-section. Consistent with the international evidence, the CAPM Model is incomplete. The results also reveal that the momentum effect is not significant. Furthermore, there is an important proportion of the variance of the returns of the Chilean shares that is not explained by the studied factors. The coefficients of determination of the time-series regressions are quite lower than in the United States, suggesting that the Chilean stock-market would respond less to fundamental variables.

**Keywords:** Asset Pricing, Cross-section, Factor Models, Stock Returns.

**JEL Classification Numbers:** C31, C32, G12.

---

\*e-mail: gmaturan@dii.uchile.cl. I would like to thank José Miguel Cruz and Matías Braun for helpful discussions and comments. The comments and suggestions of Viviana Fernández, Andrés Liberman and the audience at the II Workshop of Stochastic Modeling in Finance and Risk Management (University of Chile) are also appreciated. All remaining errors are my own.

# 1 Introduction

In this paper we will apply the classical and standard empirical tests used in Asset Pricing to test factor models (Fama and Macbeth (1973) and Black, Jensen and Scholes (1972)). In particular, in the time-series approach of Black, Jensen and Scholes we use the Seemingly Unrelated Regressions (SUR) to account for the problem of correlated errors across portfolios<sup>1</sup>. We will study the relevance in the Chilean market of factors such as the ones proposed in the Fama and French three factor model (SMB, the difference between the returns of diversified portfolios of small and big firms; HML, the difference between the returns on diversified portfolios of high and low book-to-market equity ratios; and the market's risk premium) plus a momentum factor.

Since the beginning of the 70s, the asset pricing model of Sharpe (1964), Litner (1965) and Black (1972) - typically known as CAPM - has strongly influenced the way in which both academic and financial professionals think in terms of average returns and asset risk. Even today this model is commonly taught in MBA courses and widely used for various applications, such as determining the cost of capital of companies or evaluating the performance of managed portfolios. The main prediction of the CAPM is that the market portfolio of invested wealth is efficient in terms of mean-variance which implies that the slope (mostly known as market  $\beta$ ) in a regression of a security's return on the market's return would be positive; and that the  $\beta$ 's suffice to explain the cross-section of expected returns. The cross-section regression tests of Fama and MacBeth (1973), and the time-series regression tests of Gibbons (1982) and Stambaugh (1982) validated the model by finding that market  $\beta$ 's suffice to explain the expected returns and that the risk premium for  $\beta$  would be positive<sup>2</sup>.

Later on, scholars found the first empirical contradictions regarding the findings mentioned above, which weakened the CAPM. They found that, variables which until then had not had any relevance in the Theory of Asset Pricing, showed a tremendous ability to explain differences in cross-section of average stock returns. Basu (1977) presented the first of these variables, the earnings-price ratio (E/P). Banz (1981) recorded a size effect, being the proxy for size the market capitalization (ME) - price times shares outstanding - of the stocks. Statman (1980) and Rosenberg, Reid and Lanstein (1985) documented the value effect, associated to the book-to-market equity ratio (BE/ME). Finally, Bhandari (1988) found that the debt-equity ratio (D/ME) was also important in explaining stock returns in the cross-section. Since the price was implicit in all these variables, one could have expected at least one of them to be redundant. Fama and French (1993) concluded that the two variables, book-to-market equity ratio and market capitalization, explained the cross-sectional variation of average stock returns associated to the four variables mentioned above and introduced their famous three-factor model. In the same line, Fama and French (1998) found that the variables that invalidated the CAPM were the same for twelve non-U.S. mayor markets, suggesting that the failure of the model related to price ratios did not depend on the sample.

In relation to the empirical failures of the CAPM, one explanation would be the need for a more complex asset pricing model: The extra variables would capture a dimension of risk that would be missed by the market portfolio return (Fama and French (1993, 1996), Zhang (2005)). On the other hand, the behavioralists argue that the value strategies produce higher average returns because they exploit the suboptimal behavior of the standard investor. Investors overreact to good and bad times because they over-extrapolate past performances (DeBondt and Thaler (1987), Lakonishok, Shleifer and Vishny (1994)). Regardless of

---

<sup>1</sup>For more details on both methods, see the Appendix.

<sup>2</sup>In particular, they accepted the Black version of the CAPM and rejected the Sharpe-Litner version (which is more specific and requires that the premium for each unit of  $\beta$  is the expected market return minus the risk-free interest rate).

the explanation behind the empirical findings, Fama and French's three factor model is actually used in applications such as estimating the cost of equity capital.

Jegadeesh and Titman (1993) showed that following a strategy in which one buys stocks that have had a good performance and sells stocks that have done poorly, has a positive and significant return if the position is held between three to twelve months. The three factor model has failed in explaining this so-called momentum effect. One solution proposed by Carhart (1997), and the one adopted in part of this paper, is to simply add a momentum factor to the three factor model. There is still discussion about whether this effect exists or if it is exploitable after transaction costs (Carhart (1997)).

In Chile, in contrast to the developed world and especially the United States, there is not much literature on what factors would be decisive in explaining stock returns. Marshall and Walker (2000), using aggregated indexes and quintiles by size, concluded that the size effect would not be entirely clear. Subsequently, the same authors in 2002, working with weekly returns (from 1990 to 2000) and analyzing portfolios of stocks constructed by size and traded volumes, documented the existence of a liquidity effect and to a lesser extent, a size effect. On the other hand, authors such as Fuentes, Gregoire and Zurita (2005) have tried other macroeconomic factors such as GDP growth, copper price, oil prices and inflation in the context of Ross's (1976) arbitrage pricing theory. In summary, there are still no conclusive evidences regarding which factors explain the cross-sectional differences of Chilean stock returns. There are scarce publications and few methodologies have been explored. However the CAPM model is widely used in Chile in rate setting processes in sectors such as utilities and telecommunications.

This paper is organized as follows. Section 2 presents the data and specific considerations accounted to develop the portfolios used as input for the regressions. Section 3 presents the empirical results and Section 4 concludes.

## 2 Data, considerations and input for the regressions

For effects of this study we consider all the companies that have traded in the Santiago Stock-Exchange (SSE) from January of 1992 to June of 2006, whose stock-exchange and accounting information is registered in the data bases of Economática or the SSE<sup>3</sup>. We work with series of total returns for a sample of 261 shares. On the other hand, 33 of the shares that existed in 1992 are not present in the data base in 2006.

### 2.1 Liquidity in the Chilean market

Due to the shallow depth of the Chilean market, it was necessary to select a sub-sample of shares to analyze. If a share is not traded for a long period of time, its price might not reflect the true value of the company, then when traded, their returns might alter the results.

Table 1 illustrates the liquidity problem in the Chilean market. It may occur that some shares are not traded for more than a year. To avoid this problem, the sub-sample contains the 80 shares with highest presence during each period<sup>4</sup>. In this way we ensure that the less traded share in each period was at least exchanged once per period, allowing its price to be corrected by the market taking it to a value closer to the fundamental. The shares of the sub-sample represent in average an 84% of the total market capitalization of the stock-market.

---

<sup>3</sup>To control for survivor bias.

<sup>4</sup>We consider a stock as "present" one day if it was traded during the day.

## 2.2 Periods

The definition of the periods is important because both, the portfolios and the sub-sample of 80 shares, must be rebalanced at the beginning of these. We work with 3 month periods, given that the model we wish to test considers the momentum factor, which in Chile, eventually should occur in short periods of time (as shown in Table 2).

Finally, the sample shares considered in each period  $t$  were the 80 ones with highest presence in period  $t-1$ .

## 2.3 BE/ME, ME and Momentum

In order to assure that the data of the financial statements was well-known by the market before the returns that supposedly it would explain, we linked quarter  $t$  with the daily averages of the BE/ME ratios during the last month of quarter  $t-2$ . This is equivalent to a 3 month window between the time the information is known and it is having any effects.

On the other hand, the average stock-market capitalization (ME) for the last month of quarter  $t-1$  was associated to quarter  $t$ .

The allocation of the momentum was done by classifying in each quarter the shares in 2 groups according to its profitability: The most profitable and the less profitable. We considered that a share belongs to the group with momentum if it is one of the most profitable in quarter  $t-1$ .

## 2.4 Construction of the Portfolios

This paper is based on a portfolio approach to construct series of monthly returns to be explained and series of explanatory returns. The explanatory returns include the returns of the Chilean market, as well as the returns of mimicking portfolios of the size, BE/ME ratio and momentum factors. The returns to explain are those of 8 portfolios constructed on the basis of the same three characteristics just mentioned.

## 2.5 Portfolios to Explain (dependent)

To build the series of monthly returns of these portfolios we used the allocation indicated in the previous paragraphs. Each period: First, the shares were divided in 2 groups according to their ME size, the small ones ("S") and big ones ("B"). Then, the 2 groups were again separated in 2 categories according to their BE/ME: Those of low ratio and those of high ratio. Finally, each one of these 4 portfolios was divided into 2 groups according to momentum (without momentum and with momentum). Thus, 8 portfolios were generated. The returns of each series were weighted according to the capitalization of the shares in each portfolio. It is important to notice that the shares might change of group and/or classification in each period.

Fama and French (1993) worked with 25 dependent portfolios. This was possible because the NYSE, AMEX and NASDAQ have a larger number of stocks than the SSE and do not have the liquidity problem. Working with 80 shares does not allow the creation of many portfolios. The ideal is that each portfolio should have at least 8 shares and in this way be able to diversify the non-systematic risk. This is obtained with the distribution explained in the previous paragraph. In order to form more dependent portfolios it would be necessary to cut & slice the grouping factors (size, BE/ME or momentum) in 3 groups, generating 12 portfolios with around 6 shares each ( $80/12 = 6,7$ ), which would be insufficient for the purpose of diversification.

In Table 3, descriptive information of the 8 portfolios is presented. The relevance of the quarterly rebalance can be appreciated. In average, less than 40% of the shares in a portfolio are in the same portfolio in the following period.

## 2.6 Explanatory Portfolios (independent)

### 2.6.1 SMB and HML

These factors were constructed in the same way Fama and French (1993) did. Like in the case of the dependent portfolios, in each quarter the shares we divided in 2 groups according to its size, small (“S”) and big (“B”). At the same time, the companies were separated in 3 groups according to its BE/ME ratio, low, medium and high (“L”, “M” and “H”). With this classification it was possible to construct series of value weighted returns for 6 portfolios (from the intersections: S/L, S/M, S/H, B/L, B/M, and B/H). The SMB factor was fixed considering the simple average return from the series of portfolios S/L, S/M and S/H minus the simple average of portfolios B/L, B/M and B/H. Also, HML factor was created from the simple average of the S/H and B/H returns minus the simple average of S/L and B/L.

### 2.6.2 MOM

After classifying the shares in 2 groups of momentum in each period, this factor was calculated simply by subtracting the weighted average return by capitalization of the shares with momentum (“WM”), and the weighted average of the shares without momentum (“NM”).

### 2.6.3 Chilean Market Factor (MKT)

The market return was calculated every month based on the weighed average of the market capitalization of all the shares in sub-ample to which the Chilean risk free rate was deducted. The Monetary Policy Rate of the Central Bank of Chile was used as proxy of the risk free rate<sup>5</sup>.

Table 6 presents descriptive statistics of the monthly series corresponding to the constructed factors. The correlation matrix of the factors is shown in Table 7. Ideally, to favor the tests and the model, it would be desirable that SMB, HML and MOM factors, be independent among each other. The correlation between SMB and HML is the highest and the only one significantly different from zero, but as it will be seen further on, this is not a problem, since both SMB and HML explain different components of the stock returns.

### 2.6.4 The Portfolio Approach and Diversification in the Chilean Market

When using the Fama-MacBeth methodology, the portfolio approach is relevant. Portfolios have a lower residual variance than individual stocks, so the loadings of the time-series regressions are best measured. The portfolio betas are more stable than the betas of individual stocks, so their estimates are more accurate.

---

<sup>5</sup>It is important to mention that this rate was just turned into nominal terms on August 7th, 2001. Before that a real rate was fixed (since May, 1995). For the previous months we considered de rates of the PRBC (a 90 days Resettable Note, also in real terms). We nominalized the real rates adding to them the ex-post inflation rate, which would be consistent under the rational expectations assumption. we also computed a series of what might have been the expectations for inflation in each month using a simple VAR model (where 6 lags of the monthly variation of the CPI, IMACEC -an activity indicator- and of the M1 were considered). The comparison between the estimated inflation rate and the realized inflation rate is presented in Figure 1. The result of this study did not vary significantly when using the different measurements (see Table 4), so we decided to use the ex-post inflation rate. On the other hand, the nominalization of the risk free rate does not affect the explaining power of the market factor neither changes the results. This can be seen in Table 5, where we compare the pooled regression of the dependant portfolios on the four factors against a regression where we control for the nominalization (The F statistic of the comparison test is 5.77).

Another argument for the use of portfolios is that returns of individual stocks are too volatile. One cannot reject the hypothesis that the average stock returns are equal. In fact, the average annual volatility of the shares in the subsample of the 80 most traded stocks per quarter is 37.9%. The volatility of the 40 most traded shares is 37.8% on average. By grouping stocks according to common characteristics, a reduction in volatility is accomplished, making possible to observe differences in returns. Table 8 presents the average volatility of portfolios of different number of shares grouped by size. Figure 2 shows the rapid diversification of non-systematic risk of the shares in the subsample (portfolios conformed by 10 stocks eliminate 81.3% of non-systematic risk on average). The market’s annual volatility is 20.5%. All of these implies that the portfolios diversify risk well enough as to be used in the regressions.

### 3 Empirical Results

#### 3.1 The 4 factors and the cross-section

Table 9 presents the average excess return and the volatility of the excess returns resultant from the 8 dependent portfolios constructed from characteristics ME, BE/ME and momentum. We see that some portfolios have rented more than others. The low capitalization or small portfolios display a higher monthly average excess returns than those of high capitalization. The same happens to the portfolios with high BE/ME in comparison to those with low BE/ME and the portfolios with momentum, contrary to those without momentum. Historically, the most profitable portfolio is portfolio S/H/WM (shares of low stock-exchange capitalization, high book to market value and that present momentum), with an average monthly excess return of 1.6%. There is no indication that this extra profitability is due to a greater volatility.

In relation to the diversification of risk, as shown in Table 3, all 8 dependent portfolios in average have more than 8.5 shares. These would diversify in average around 79.9% of the nonsystematic risk. On the other hand, the root of the average covariance of the portfolios is 20.2%, very similar to the volatility of 20.5% shown by the market. All this indicates that the portfolios diversify the risk quite well, so the FM regressions should not present significant bias.

Table 10 presents descriptive statistics for the resulting betas after running the first stage of FM regressions<sup>6</sup>. It shows how the small types of portfolios have larger average  $\hat{\beta}_{SMB}$  than the rest. Furthermore, the average  $\hat{\beta}_{HML}$  is larger in the high types of BE/ME portfolios, the average  $\hat{\beta}_{MOM}$  for the portfolios with momentum is positive and negative in those without momentum, and the average  $\hat{\beta}_{MKT}$  is around 1 in all the portfolios.

The FM regressions are a refinement from running a simple regression of the excess returns against betas or “loadings”, controlling for time. The results of this cross - sectional regression is presented in Table 11. The coefficient associated to  $\beta_{HML}$  is significant at a 5% level, whereas the coefficient associated to  $\beta_{SMB}$  is significant at a 10% level. The rest of the factors are not statistically significant.

After carrying out the second stage of FM regressions, it is possible to observe that both the size and book to market factors explain the cross-sectional differences between the returns of Chilean stocks. The coefficient associated to  $\beta_{HML}$  is significant at a 5% level, whereas the p - value corresponding to the coefficient associated to  $\beta_{SMB}$  is 0.0507 (almost significant at a 5%). In opposition to this, we could not reject the null hypothesis that the coefficients are equal to 0 for the other factors (results are presented in Table 4). Neither the Chilean market, nor the momentum factor, would be determining in explaining why

---

<sup>6</sup>In the calculation of these loadings, 60 months rolling time-series regressions were considered.

some shares rent more than others. The results of the simple regression presented in Table 11 are reaffirmed.

One can see that the average  $R^2$  of the regressions in cross section is quite high (0.69). This indicates that the betas explain a high percentage of the cross-sectional variance of the returns in the Chilean market.

Figure 3 shows average contribution of each factor to the differences in the excess returns between portfolios S/H/WM and B/L/NM estimated from the simple cross-sectional regression. We can appreciate that the market factor does not have any incidence in the cross-sectional differences. On the contrary, HML factor, in average, has contributed in explaining the differences in the returns of the stocks. This contribution has become less important in the last years, losing relevance compared to the size and momentum factors. The size factor, which was the most relevant a decade ago, has regained importance. If the loadings to each factor of the extreme portfolios are compared (see Figure 4), that is to say, portfolios B/L/NM and S/H/WM, Figure 3 can be explained. In the graphs, it is possible to appreciate that the recent loss of importance of the HML factor that explains the differences of returns in the cross-section is due mainly by a fall of the beta associated to that factor in portfolio S/H/WM. In the same way, the loading to SMB factor of portfolio S/H/WM is more volatile than the one corresponding to portfolio B/L/NM. The curve of the contribution of the momentum factor also is explained mainly by the sensitivity of portfolio S/H/WM to the momentum factor, although in a less categorical way than in the other two cases.

### 3.2 Time-Series Analysis

In order to understand the influence of the factors analyzed in this paper, this section includes a time-series analysis of the returns. The momentum factor will be excluded from this analysis because it is an effect that only has sense in the cross-section. Due to how momentum is defined in time-series, it could be interpreted as a measure of dispersion or variance of the returns through time, more than a prize to higher returns in the previous period.

Nine new dependent portfolios were calculated using the same methodology described in the previous section, but only considering the variables of size and book to market equity ratio<sup>7</sup>.

Table 13 shows the results of a regression of the excess returns of the dependent portfolios against the factors, controlling for portfolio classification. Both, the factor associated to the size and the market, are significant in explaining the average returns in time-series (p-value equal to 0). The  $R^2$  is 0.67.

Figure 5 shows the evolution of the estimated contribution of each factor to the differences in the excess returns of portfolios S/H/WM and B/L/NM (in time-series). We can appreciate that the most important one is the market factor. The SMB factor also fulfills an important role.

The average estimated contribution of the factors to the excess returns, both in the cross-section and in time series, is not constant. The factors could be substituting each other in their role in explaining the returns. In particular the contribution of the factor associated to size varies through time and could be affected by events such as recessions or market crashes, specifically by the denominated Asian Crisis and the Dotcom Crash. As sustained by Perez-Quirós and Timmermann (2000): small companies, that tend to have smaller collateral, are more affected in times of credit restrictions than large companies (with greater collateral). Since smaller companies do not obtain financing for many of their projects, they have worse results. In the same line, after controlling for book-to-market equity ratio, these companies tend to have a lower income. Small companies usually show less productivity. On the other hand, as demonstrated by Campbell, Hilscher and Szilagyi (2006), when the market behaves in a more volatile way, low capitalization companies have a larger probability to go into bankruptcy. As shown in Figure 6, within the period in

---

<sup>7</sup>Descriptive statistics for these new portfolios are presented in Table 12.

study, the Chilean market displayed its higher levels of volatility between years 1999 and 2003. On the contrary, during years 2005 and 2006, when factor SMB recovers importance, the volatility of the market was at an historical minimum level. Another explanation for the rise of the importance of the factor can be associated to the high liquidity of the agents of the market in the period, which was canalized slightly more in investments in companies of lower capitalization. In fact, the total amounts of the shares classified as Small in the sample which were traded from 2003 to 2006, increased more than 5 times, going from 20.100 million pesos to 106.000 million pesos. The traded amounts of the shares classified as High capitalization increased less in relative terms, from 173.000 million to 750.000 million pesos (a little more than 4 times). This might be related to the creation and consolidation of several “Small Caps” funds and the strong participation of the Retirement Funds.

As mentioned previously in this paper, Fama and French (1993) used the time-series approach of Black, Jensen and Scholes. This consists in carrying out simple regressions of the monthly returns of each separate portfolio against the return on the market and mimicking portfolios. In an intent to understand the influence of each factor in the stock returns, 3 regressions are needed:

1. Portfolios against the market factor.
2. Portfolios against factors SMB and HML.
3. Portfolios against the market factor, SMB and HML.

The results of the 3 regressions (using SUR) for the 9 portfolios are presented in Tables 14, 15 and 16.

The results of the first regression show that the market factor, although explains important part of the variance of the excess of returns of the dependent portfolios, leaves quite a lot unexplained. The values of  $R^2$  are rather high for the big size portfolios (high stock-exchange capitalization). Also, the  $R^2$  of the regressions tend to go down as dependant portfolio increases its book to market ratio. In fact the  $R^2$  associated to the regression that considers the B/L portfolio (high ME, low ratio BE/ME) is 0.82, whereas the  $R^2$  of the regression of portafolio S/H (low ME, high BE/ME) is 0.44, a much lower value. The  $R^2$  is lower for the regressions that consider the portfolios of smaller size and higher book to market ratio, in which the other two factors, SMB and HML, could have a good opportunity to explain the returns.

The statistical value of  $R^2$  in the second regression (which only considers factors SMB and HML as independent variables) are quite lower than those in the first regression. Only in 4 cases the  $R^2$  is higher than 0.1 (with a case above 0.3). The remarkable thing is that the higher coefficients of determination correspond to the regressions where the dependent portfolios are of smaller stock-exchange capitalization and greater BE/ME. The SMB and HML factors explain part of the variation in the returns of this type of portfolios.

On the other hand, the values of  $R^2$  in the third regression increase significantly. The average  $R^2$  observed is of 0.77, which implies a 15% improvement if it is confronted to the average  $R^2$  of 0.67 obtained in the regression that considers the market as the only explanatory variable. We note that the determination coefficient rises in the regressions that consider the portfolios of low capitalization and higher BE/ME ratio. Table 17 shows the result of the F test performed to compare the three-factor model (regression 3) with the first regression model, which considers only the market factor. The F test indicates that the factors SMB and HML complete model that considers only the market factor in 8 of the 9 cases.

Another aspect to mention is the fact that although an average  $R^2$  of 0.77 is quite high, it is lower than the one obtained by Fama and French (it exceeded 0.9). This may be due to the fact that in developing

countries the prices of the stocks, as sustained by Braun and Johnson (2005) tend to respond less to the fundamentals. There would be a greater “noise” in returns. On the other hand, it could simply be due to poorer quality data or because fundamentals would be less credible due to the possibility that firms have to make an “accounting adulteration” to their results product of the weaker legislation that tends to prevail in developing countries compared to developed countries. In fact, Chang, Khanna and Palepu (2000) show that the accuracy of analysts’ projections of the results of the firms they follow is positively correlated with the size of the stock market relative to GDP, the size of the firm and the quality of regulation. Another explanation could be the great importance in the Chilean market of institutional investors. These, who tend to copy each other, could distort prices, deviating them from their fundamental values.

Continuing with the analysis of the regressions, it is important to note that, in regression 1 and regression 3, the market factor is always significant. The coefficient associated with this factor tends to be closer to 1 in regression 3 (except in the case of portfolio S/L). This would be another sign that the SMB and HML factors are being mistakenly omitted in the first regression. On the other hand, when analyzing the intercepts, it can be seen that in the first regression, 2 of the 9 intercepts are statistically significantly different from zero, while in the third regression none of them are different from zero. As in Fama and French (1993), in the first regression the intercepts become greater as the regressions consider portfolios with smaller ME and higher BE/ME. In regression 2, the intercepts tend to be more similar to each other than in regression 1. Finally, in the third regression, all the intercepts become closer to zero. All of these supports the consideration of SMB and HML.

At last, regarding the significance of the coefficients associated with SMB and HML, it can be seen that in the third regression, both were significant in 7 of 9 cases. The coefficients associated with SMB are negative in the case of portfolios that include large (high ME) firms and positive in the case of portfolios which include small firms. In the same way, the coefficients associated with the HML factor are negative for low BE / ME ratio portfolios and positive otherwise.

After the analysis one can conclude that in Chile, as in the United States, the market factor explains the level of stock returns and why stocks have returned more than bonds (fixed income), while SMB and HML factors explain why certain shares have been more profitable than others. The CAPM is incomplete and fails in explaining the cross-section of average stock returns in the Chilean Market.

## 4 Conclusion

In this paper we have investigated if specific characteristics of the stock that trade in the SSE (such as size, book-to-market equity ratio and momentum) explain the cross-sectional differences in their average returns. The Fama-Macbeth regressions were complemented by a time-series analysis using SUR. A portfolio approach was considered, creating portfolios and factors according to the characteristics desired to test. The results show that in the period between January 1992 and June 2006, the size (measured as the stock-exchange capitalization of the shares) and the book-to-market equity ratio of the companies have a significant incidence in explaining the differences between the returns of shares in the Chilean market. The shares with low ME and high BE/ME ratio tend to return more than the rest. The market factor does not explain the differences observed in the returns in the cross-section. Consistent with the international evidence, in Chile the CAPM Model is incomplete and can be improved by adding factors that mimic for risk associated to a small share with a high BE/ME ratio. The results also reveal that the momentum effect is not significant. Furthermore, there is an important proportion of the variance of the returns of the Chilean shares that is not explained by

the studied factors. The coefficients of determination of the time series regressions are quite lower than in the United States, suggesting that the Chilean stock-market would respond less to fundamental variables. The findings of this study clearly help investors who operate in the Chilean market. The use of the traditional long term investment strategies based on the fundamentals of the companies (like BE/ME ratio) which have been “imported” from abroad would be valid in Chile. In the same way, there is an empirical base for the great number of investment funds that invest in Small Caps and offer average returns above the market for extensive periods of time. On the other hand, investing in shares whose prices have had a tendency to rise in a short term is not effective. Investors must be aware of the greater risks than entails the use of the Small - Value strategies in the Chilean market in comparison to the United States market. In Chile, the stock returns and the differences between returns respond less to the studied factors. This may be because of the existence of fewer regulations in less developed countries, the low depth of the Chilean stock-market and the existence of institutional entities which may cause distortions. Finally, in relation to the CAPM model which as mentioned is incomplete, its use in the Chilean market should be submitted to revision.

## Tables and Graphs

Table 1: Shares without stock-exchange presence per year

Year in the sample	Total shares	Non-traded Shares	Shares without presence (%)
1993	208	36	17.3
1994	213	31	14.6
1995	220	32	14.5
1996	232	31	13.4
1997	235	30	12.8
1998	236	24	10.2
1999	236	28	11.9
2000	234	22	9.4
2001	231	23	10
2002	224	23	10.3
2003	219	21	9.6
2004	212	13	6.1
2005	211	8	3.8
2006	210	1	0.5

Table 1 shows the total number of shares in the Chilean stock market and the number of shares per year that were not traded. One can see that a significant amount (11% on average) of Chilean shares is not even traded once per year.

Table 2: Momentum based portfolios rebalanced at different time windows

	Monthly Average Excess Returns (%)	
	Without Momentum	With Momentum
Annually rebalanced portfolios	0.5	0.4
Quarterly rebalanced portfolios	0.2	0.7
	Volatility of Monthly Excess Returns (%)	
	Without Momentum	With Momentum
Annually rebalanced portfolios	6.6	6.1
Quarterly rebalanced portfolios	6.3	5.9

Table 2 presents the monthly average excess return and the volatility of monthly excess returns of annually and quarterly rebalanced portfolios according to its momentum. The allocation of the momentum was done in the following way: in each period the shares were classified in 2 groups according to profitability. A share was considered as belonging to the group with momentum if it was one of the most profitable in period  $t-1$ .

Table 3: Descriptive information of the dependant portfolios

ME	<i>Without Momentum</i>		<i>With Momentum</i>	
	BE/ME		BE/ME	
	<i>Low</i>	<i>High</i>	<i>Low</i>	<i>High</i>
Average ME of each portfolio (in millions of CLP)				
<i>Small</i>	89,300	69,200	92,600	70,900
<i>Big</i>	658,000	670,000	713,000	694,000
BE/ME average of each portfolio				
<i>Small</i>	0.5	1.9	0.5	1.8
<i>Big</i>	0.4	0.9	0.4	1.0
Number of shares average in each portfolio per quarter				
<i>Small</i>	10.6	9.2	10.0	8.5
<i>Big</i>	10.1	9.0	9.5	8.5
Average % of shares of each portfolio that do not change between quarters				
<i>Small</i>	35.8	36.4	36.2	30.5
<i>Big</i>	38.4	41.6	43.2	34.3

Table 3 shows descriptive information of the dependant portfolios. For the construction of the series of capitalization weighted monthly returns of these portfolios, the following procedure was used for each quarter: First, the shares were divided in 2 groups according to their ME size, the small ones ("S") and big ones ("B"). Then, the 2 groups were again separated in 2 categories according to their ratio BE/ME, those of low ratio and those of high ratio. Finally, each one of these 4 portfolios was divided into 2 groups according to momentum (without momentum and with momentum). Thus, 8 portfolios were generated.

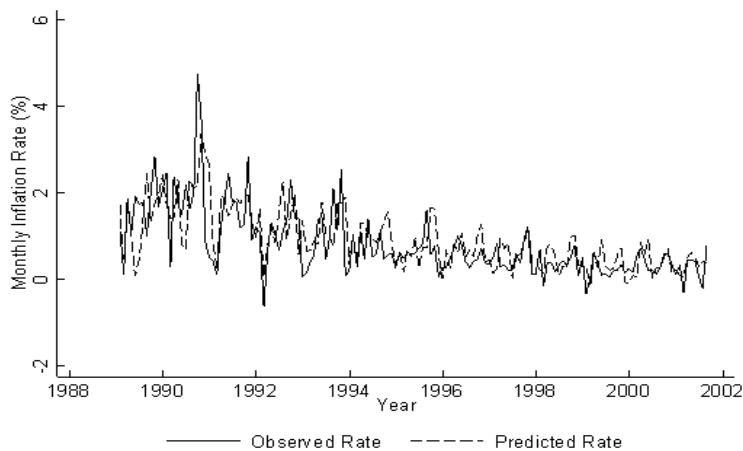


Figure 1: Predicted Inflation vs. Observed Inflation

Figure 1 compares the observed ex-post inflation rate with the estimated ex-ante inflation rate. A simple VAR model (where 6 lags of the monthly variation of the CPI, IMACEC -an activity indicator- and of the M1 were considered) for what might have been the expectations for inflation in each month was computed. One can see the similarity between both rates.

Table 4: t-tests to coefficients obtained from the second stage of FM regressions

Variable	(1)	(2)
$\gamma_{SMB}$	0.0071** (1.98)	0.0070** (1.98)
$\gamma_{HML}$	0.0096* (2.20)	0.0096* (2.21)
$\gamma_{MOM}$	0.0002 (0.06)	0.0002 (0.05)
$\gamma_{MKT}$	0.0018 (0.10)	0.0040 (0.21)
Intercept	0.0012 (0.06)	-0.0005 (-0.03)
Observations	108	108

\*significant at a 5% level

\*\*significant at a 10% level

Table 4 presents the results of the t-tests computed to the coefficients obtained after running the second stage of the Fama-Macbeth regressions. Column (1) considers the estimated inflation rate expectations when correcting the risk free interest rate. Column (2) considers the ex-post inflation rate. The results did not vary significantly.

Table 5: Pooled regression vs. Regression accounting for nominalization

Variable	(1)	(2)
SMB	0.40* (10.44)	0.42* (9.85)
HML	0.003 (0.12)	0.03 (0.80)
MOM	-0.01 (-0.36)	-0.01 (-0.44)
MKT	0.99* (53.87)	0.98* (49.43)
dINFL	-	0.0005 (0.20)
dINFLxSMB	-	-0.07 (-0.71)
dINFLxHML	-	-0.06 (-0.80)
dINFLxMOM	-	-0.02 (-0.16)
dINFLxMKT	-	0.07 (1.35)
intercept	-0.0005 (-0.44)	-0.0005 (-0.38)
$R^2$	0.7033	0.7046
observations	1344	1344

\*significant at a 5% level.

Table 5 compares the pooled regression of the dependant portfolios excess returns against SMB, HML, MOM and MKT with the same regression when controlling for nominalization (through dINFL). The F statistic when comparing both models is 5.77. Nominalization does not affect the results.

Table 6: Descriptive statistics of the factors excess returns

factor	average	standard deviation	minimum	maximum
SMB	0.52	3.56	-8.5	17.6
HML	0.91	4.60	-8.8	35.0
MOM	0.52	3.39	-8.3	11.1
MKT	0.53	5.96	-30.1	18.4

Table 6 shows descriptive statistics of the monthly series corresponding to the constructed factors. In each quarter the shares we divided in 2 groups according to size, small ("S") and big ("B"). Then, the companies were separated in 3 groups according to its BE/ME ratio, low, medium and high ("L", "M" and "H") constructing series of value weighted returns for 6 portfolios from the intersections S/L, S/M, S/H, B/L, B/M, and B/H. Factor SMB was fixed considering the simple average return from the series of portfolios S/L, S/M and S/H minus the simple average of portfolios B/L, B/M and B/H. Factor HML was created from the simple average of the S/H and B/H returns minus the simple average of S/L and B/L. MOM was applied simply by means of the subtraction of the weighted average return by capitalization of the shares with momentum ("WM") and the weighted average of the shares without momentum ("NM"). MKT was calculated based on the weighed average of the market capitalization of all the shares in sub-ample to which the Chilean risk free rate was deducted.

Table 7: Factor's Correlation Matrix

	SMB	HML	MOM	MKT
SMB	1			
HML	0.598*	1		
MOM	-0.011	-0.030	1	
MKT	-0.064	0.132	-0.106	1

\*significant at a 5% level.

Table 7 presents the different correlations within factors. In each quarter the shares we divided in 2 groups according to size, small ("S") and big ("B"). Then, the companies were separated in 3 groups according to its BE/ME ratio, low, medium and high ("L", "M" and "H") constructing series of value weighted returns for 6 portfolios from the intersections S/L, S/M, S/H, B/L, B/M, and B/H. Factor SMB was fixed considering the simple average return from the series of portfolios S/L, S/M and S/H minus the simple average of portfolios B/L, B/M and B/H. Factor HML was created from the simple average of the S/H and B/H returns minus the simple average of S/L and B/L. MOM was applied simply by means of the subtraction of the weighted average return by capitalization of the shares with momentum ("WM") and the weighted average of the shares without momentum ("NM"). MKT was calculated based on the weighed average of the market capitalization of all the shares in sub-ample to which the Chilean risk free rate was deducted.

Table 8: Average volatility of portfolios grouped by size

Number of stocks in the portfolio	Annual average volatility (%)
1	37.9
2	31.0
4	26.9
6	25.1
8	24.4
10	23.8
13	23.1

Table 8 shows the annual average volatility of portfolios sorted by size (and rebalanced on an annual basis).

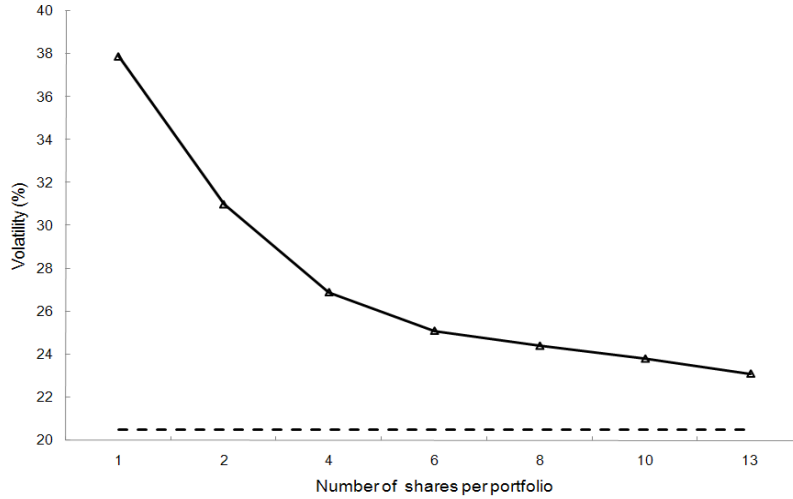


Figure 2: Diversification of risk in the Chilean Market

Figure 2 shows the rapid diversification of non-systematic risk of the shares in the subsample (portfolios conformed by 10 stocks eliminates 81.3% of non-systematic risk on average). The market's annual volatility is 20.5%.

Table 9: Monthly average excess return and volatility of the dependant portfolios

	<i>Without Momentum</i>		<i>With Momentum</i>	
	<b>BE/ME</b>		<b>BE/ME</b>	
	<i>Low</i>	<i>High</i>	<i>Low</i>	<i>High</i>
<b>ME</b>	Monthly Average Excess Returns(%)			
<i>Small</i>	0.4	0.8	1.1	1.6
<i>Big</i>	-0.1	0.3	0.3	0.9
	Volatility of Monthly Excess Returns (%)			
<i>Small</i>	7.2	9.0	6.2	7.9
<i>Big</i>	6.7	7.0	6.5	6.2

Table 9 presents the average excess return and the volatility of the excess returns resultant from the 8 dependent portfolios constructed from characteristics ME, BE/ME and momentum. For the construction of the series of monthly returns of these portfolios the allocation indicated in section 3 was used. The following procedure was used for each period: First, the shares were divided in 2 groups according to their ME size, the small ones ("S") and big ones ("B"). Then, the 2 groups were again separated in 2 categories according to their ratio BE/ME, those of low ratio and those of high ratio. Finally, each one of these 4 portfolios was divided into 2 groups according to momentum (without momentum - NM - and with momentum - WM -). Thus, 8 portfolios were generated. The returns of each series were weighed according to the capitalization of the shares in each portfolio.

Table 10: Descriptive statistics for the resulting betas after running the first stage of FM regressions

	Average				Standard Deviation			
	$\hat{\beta}_{SMB}$	$\hat{\beta}_{HML}$	$\hat{\beta}_{MOM}$	$\hat{\beta}_{MKT}$	$\hat{\beta}_{SMB}$	$\hat{\beta}_{HML}$	$\hat{\beta}_{MOM}$	$\hat{\beta}_{MKT}$
S/L/NM	1.24	-0.43	-0.05	1.08	0.17	0.14	0.06	0.04
S/H/NM	0.64	0.66	-0.31	1.02	0.09	0.22	0.16	0.08
B/L/NM	0.04	-0.33	-0.41	1.01	0.12	0.13	0.07	0.04
B/H/NM	-0.22	0.27	-0.42	1.01	0.13	0.06	0.09	0.07
S/L/WM	0.96	-0.38	0.07	0.85	0.21	0.14	0.10	0.10
S/H/WM	0.56	0.36	0.27	1.06	0.38	0.25	0.12	0.14
B/L/WM	-0.15	-0.12	0.39	1.04	0.09	0.13	0.08	0.03
B/H/WM	0.02	0.22	0.37	0.92	0.09	0.18	0.07	0.06

	Minimum				Maximum			
	$\hat{\beta}_{SMB}$	$\hat{\beta}_{HML}$	$\hat{\beta}_{MOM}$	$\hat{\beta}_{MKT}$	$\hat{\beta}_{SMB}$	$\hat{\beta}_{HML}$	$\hat{\beta}_{MOM}$	$\hat{\beta}_{MKT}$
S/L/NM	0.94	-0.71	-0.15	0.99	1.49	-0.25	0.06	1.14
S/H/NM	0.47	0.41	-0.64	0.83	0.84	1.09	-0.04	1.10
B/L/NM	-0.30	-0.47	-0.61	0.93	0.18	-0.01	-0.30	1.10
B/H/NM	-0.47	0.09	-0.63	0.88	0.05	0.45	-0.18	1.16
S/L/WM	0.62	-0.65	-0.15	0.69	1.34	-0.23	0.28	1.04
S/H/WM	0.06	-0.07	0.07	0.89	1.30	0.88	0.57	1.28
B/L/WM	-0.29	-0.31	0.20	0.98	0.05	0.11	0.51	1.09
B/H/WM	-0.10	-0.13	0.21	0.85	0.24	0.40	0.54	1.04

Table 10 shows descriptive statistics for the resulting betas after running 60 months rolling time-series regressions (the first stage of FM regressions).

Table 11: Regression of the excess returns against betas or "loadings", controlling for time

Variable	(1)
$\beta_{SMB}$	0.0049** (1.76)
$\beta_{HML}$	0.0083* (2.25)
$\beta_{MOM}$	0.002 (0.44)
$\beta_{MKT}$	-0.0060 (-0.41)
Intercept	0.011 (0.82)
$R^2$	0.69
Observations	864

\*significant at a 5% level.

\*\*significant at a 10% level.

Table 11 presents the results of a simple regression of the excess returns against betas or "loadings" obtained after running the 60 months rolling time-series regressions (the first stage of FM regressions), controlling for time.

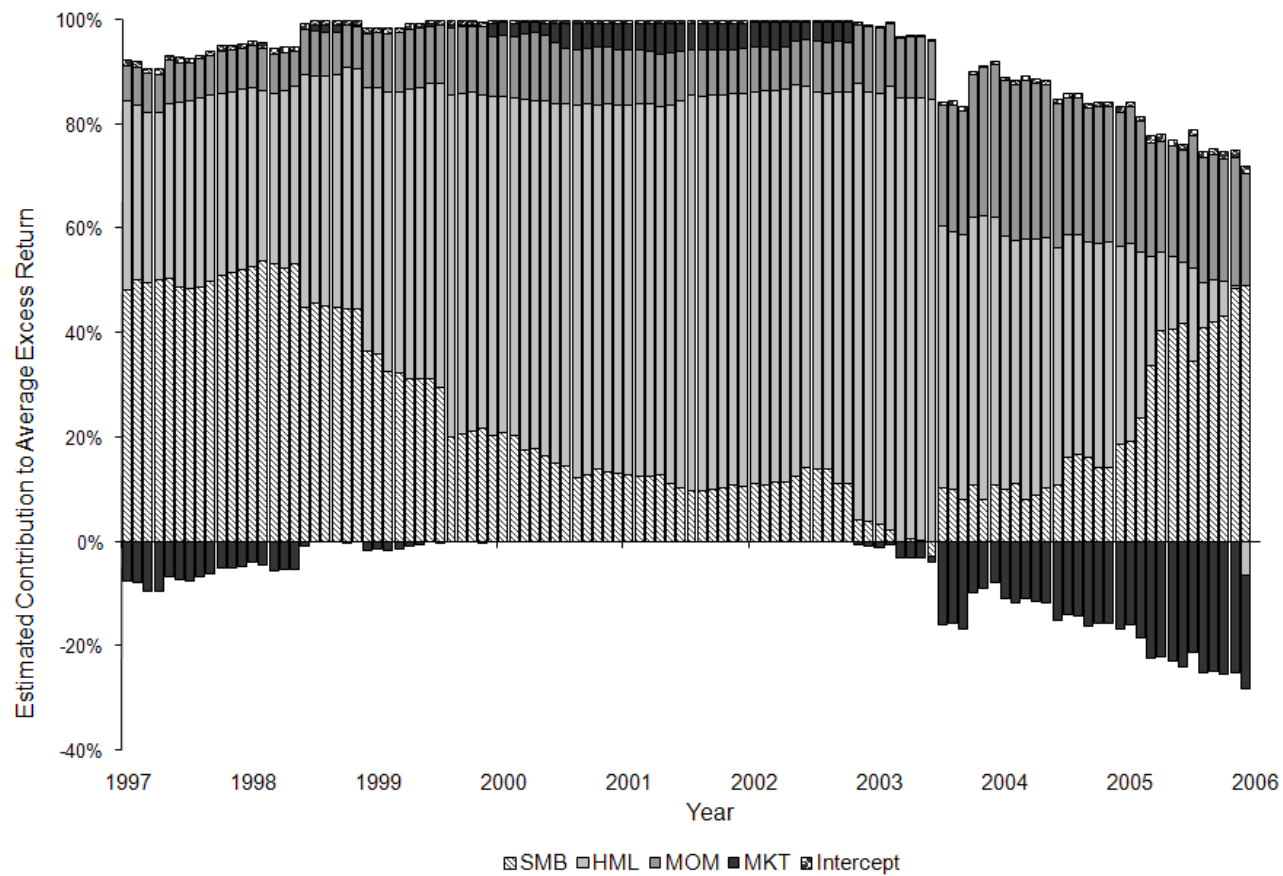


Figure 3: Average Contribution of Each Factor to the Differences in the Excess Returns Between Portfolios S/H/W/M and B/L/N/M

Figure 3 shows the average contribution of each factor to the differences in the excess returns between portfolios S/H/W/M and B/L/N/M estimated from the simple cross-sectional regression.

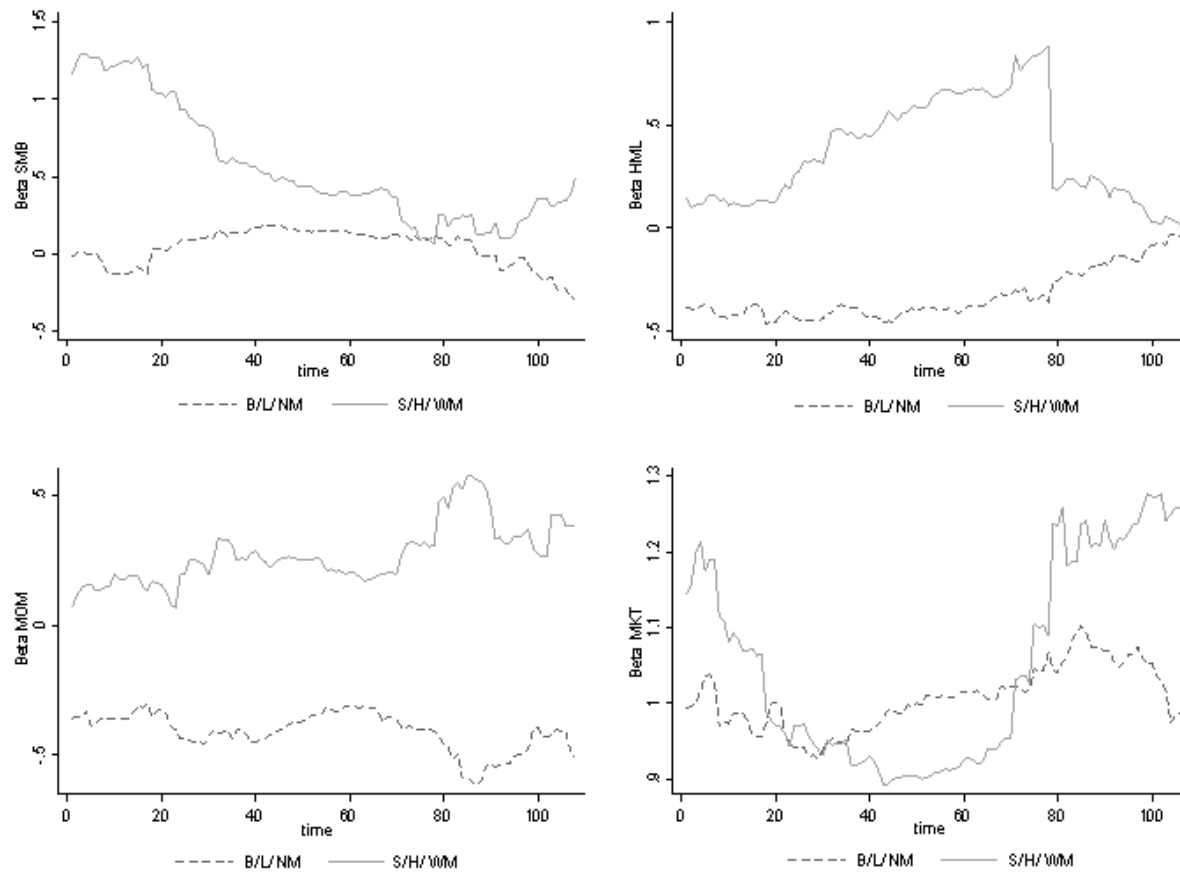


Figure 4: Average Contribution of Each Factor to the Differences in the Excess Returns Between Portfolios S/H/WM and B/L/NM

Figure 4 presents the loadings on each factor of the extremes portfolios (S/H/WM and B/L/NM) obtained from the 60 month rolling regressions.

Table 12: Descriptive statistics for 9 portfolios classified by ME and BE/ME

	<b>BE/ME</b>		
	Low	2	High
<b>ME</b>	Average ME of each portfolio (in millions of CLP)		
<i>Small</i>	59,300	50,500	37,300
<i>2</i>	196,000	205,000	182,000
<i>Big</i>	789,000	1,010,000	949,000
	BE/ME average of each portfolio		
<i>Small</i>	0.5	1.0	2.7
<i>2</i>	0.4	0.7	1.4
<i>Big</i>	0.3	0.5	1.0
	Number of shares average in each portfolio per quarter		
<i>Small</i>	9.4	9.1	7.2
<i>2</i>	9.2	8.6	7.5
<i>Big</i>	9	8	7.6
	Monthly Average Excess Returns (%)		
<i>Small</i>	0.6	1.2	1.4
<i>2</i>	0.5	0.8	1.2
<i>Big</i>	0	0.4	0.8
	Volatility of Monthly Excess Returns (%)		
<i>Small</i>	6.9	7.5	9.9
<i>2</i>	6.2	6.6	8.2
<i>Big</i>	6.8	6.4	6.7

Table 12 shows descriptive statistics for nine dependent portfolios. These portfolios were calculated using the same procedure described in section 3, but only considering the characteristics of size and book to market equity ratio.

Table 13: Regression of excess returns of the 9 dependant portfolios against factors, controlling for portfolio classification

Variable	(1)
SMB	0.39* (9.93)
HML	0.005 (0.16)
MKT	0.99* (53.16)
intercept	0.0003 (0.23)
$R^2$	0.67
observations	1512

\*significant at a 5% level.

Table 13 presents the results of a regression of the excess returns of the dependent portfolios against the factors SMB, HML and MKT, controlling for portfolio classification.

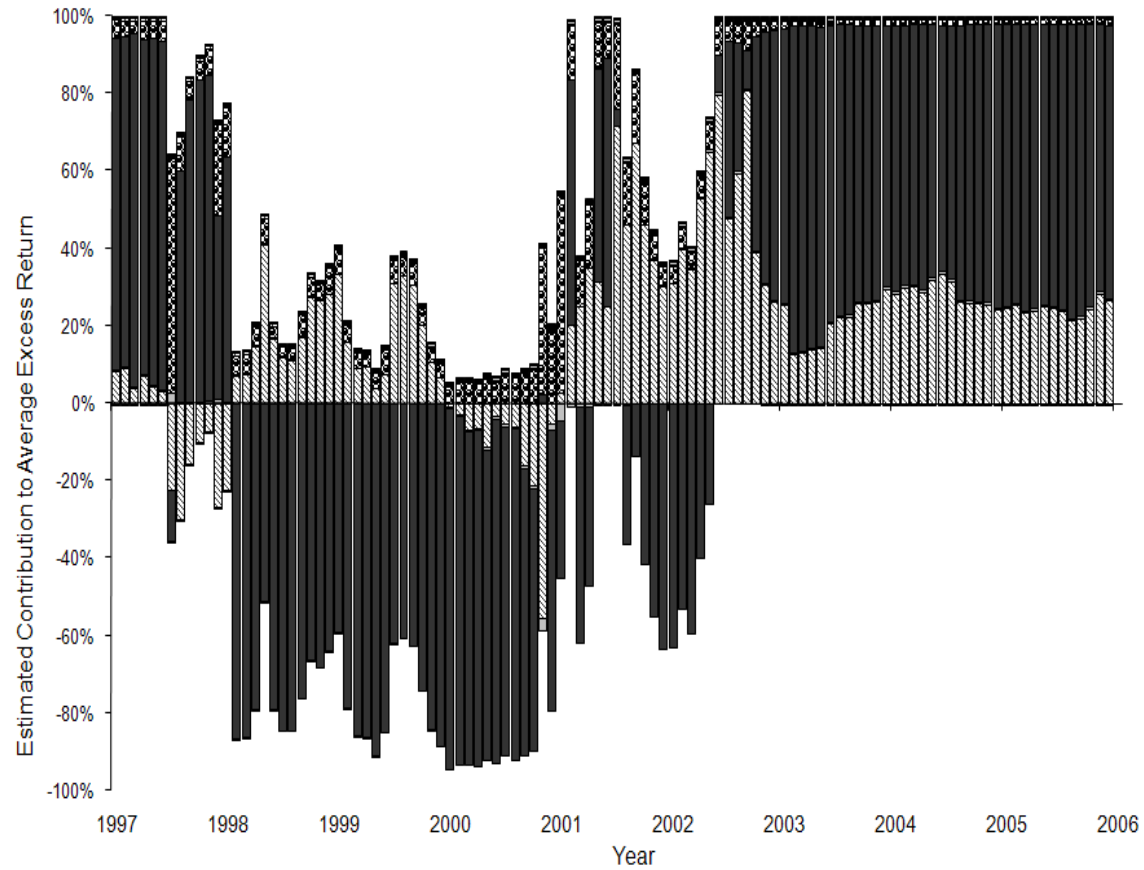


Figure 5: Average Contribution of Each Factor to the Differences in the Excess Returns Between Portfolios S/H/W/M and B/L/N/M

Figure 5 shows the 5 year moving average of the evolution of the estimated contribution of each factor to the differences in the excess returns of portfolios S/H/W/M and B/L/N/M (in time-series).

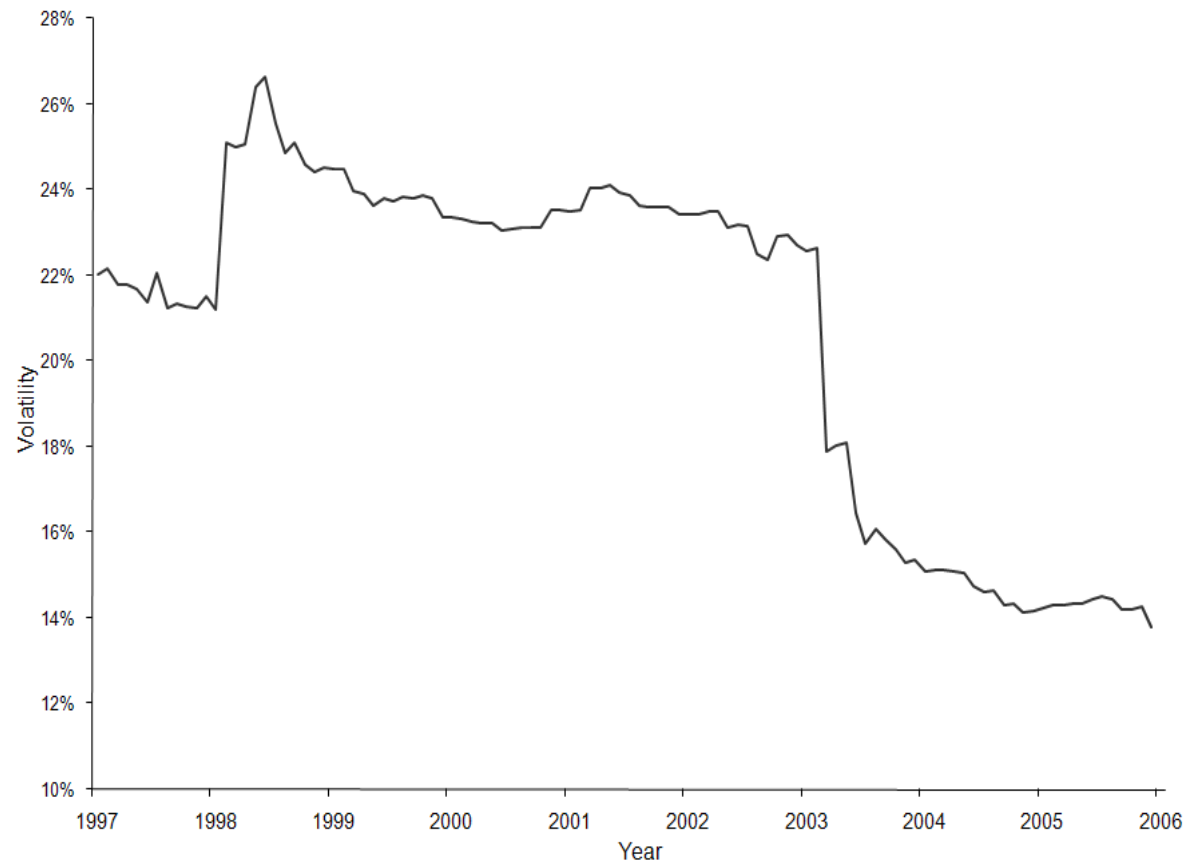


Figure 6: Annualized monthly volatility of the Chilean market

Figure 6 presents the 5 year moving average of the annualized monthly volatility of the Chilean market excess returns.

Table 14: Regressions of the 9 dependant portfolios against MKT

coef. associated to the market factor				intercept			
<b>BE/ME</b>				<b>BE/ME</b>			
<b>ME</b>	Low	2	High	<b>ME</b>	Low	2	High
<i>Small</i>	0.88*	0.91*	1.10*	<i>Small</i>	0.001	0.007**	0.009
<i>2</i>	0.90*	0.94*	1.04*	<i>2</i>	0.001	0.003	0.007
<i>Big</i>	1.03*	0.99*	1.04*	<i>Big</i>	-0.006*	-0.002	0.002

R - squared			
<b>BE/ME</b>			
<b>ME</b>	Low	2	High
<i>Small</i>	0.57	0.52	0.44
<i>2</i>	0.75	0.71	0.58
<i>Big</i>	0.82	0.85	0.86

\*significant at a 5% level  
 \*\*significant at a 10% level

Table 14 presents the results of running a regression for each dependant portfolio against the market factor (using SUR).

Table 15: Regressions of the 9 dependant portfolios against SMB and HML

coef. associated to factor SMB				intercept			
<b>BE/ME</b>				<b>BE/ME</b>			
<b>ME</b>	Low	2	High	<b>ME</b>	Low	2	High
<i>Small</i>	0.56*	0.62*	0.11	<i>Small</i>	0.004	0.007	0.008
<i>2</i>	0.18	0.13	0.03	<i>2</i>	0.004	0.005	0.003
<i>Big</i>	-0.43*	-0.48*	-0.60*	<i>Big</i>	0.001	0.003	0.005

coef. associated to factor HML				R - squared			
<b>BE/ME</b>				<b>BE/ME</b>			
<b>ME</b>	Low	2	High	<b>ME</b>	Low	2	High
<i>Small</i>	-0.11	0.25**	0.69*	<i>Small</i>	0.06	0.16	0.12
<i>2</i>	0.09	0.16	1.00*	<i>2</i>	0.02	0.03	0.32
<i>Big</i>	0.07	0.35*	0.63*	<i>Big</i>	0.04	0.05	0.12

\*significant at a 5% level  
 \*\*significant at a 10% level

Table 15 presents the results of running a regression for each dependant portfolio against SMB and HML (using SUR).

Table 16: Regressions of the 9 dependant portfolios against MKT, SMB and HML

coef. associated to factor SMB				coef. associated to the market factor			
<b>BE/ME</b>				<b>BE/ME</b>			
<b>ME</b>	Low	2	High	<b>ME</b>	Low	2	High
<i>Small</i>	0.92*	0.97*	0.51*	<i>Small</i>	0.96*	0.95*	1.09*
<i>2</i>	0.53*	0.50*	0.40*	<i>2</i>	0.94*	0.97*	0.99*
<i>Big</i>	-0.03	-0.11	-0.22*	<i>Big</i>	1.06*	0.99*	1.00*

coef. associated to factor HML				intercept			
<b>BE/ME</b>				<b>BE/ME</b>			
<b>ME</b>	Low	2	High	<b>ME</b>	Low	2	High
<i>Small</i>	-0.44*	-0.08	0.32*	<i>Small</i>	0.000	0.003	0.003
<i>2</i>	-0.24*	-0.18*	0.66*	<i>2</i>	0.000	0.001	-0.001
<i>Big</i>	-0.30*	0.01	0.28*	<i>Big</i>	-0.003	-0.001	0.001

R - squared			
<b>BE/ME</b>			
<b>ME</b>	Low	2	High
<i>Small</i>	0.71	0.70	0.53
<i>2</i>	0.80	0.76	0.82
<i>Big</i>	0.86	0.86	0.88

\* significant at a 5% level

Table 16 presents the results of running a regression for each dependant portfolio against MKT, SMB and HML (using SUR).

Table 17: F-test comparing the three factor model against the model which considers only the MKT factor

F statistic				p - value			
<b>BE/ME</b>				<b>BE/ME</b>			
<b>ME</b>	Low	2	High	<b>ME</b>	Low	2	High
<i>Small</i>	38.4	51.3	15.2	<i>Small</i>	0.00	0.00	0.00
<i>2</i>	23.6	15.5	107.8	<i>2</i>	0.00	0.00	0.00
<i>Big</i>	26.3	1.8	130.6	<i>Big</i>	0.00	0.17	0.00

Table 17 shows the result of the F test performed to compare the three-factor model with the first regression model, which considers only the market factor.

# Appendix

## The Fama-Macbeth Regressions

Fama and Macbeth (1973) developed a cross-sectional regression approach. Since then, the methodology has become commonly used in finance to test factor models and their ability to explain the cross-sectional differences in stock returns. The basic idea is the following:

1. Find a characteristic of stocks that might be related with returns and then group stocks in portfolios according to that characteristic.
2. Compute betas associated to a factor that mimics for the characteristic for each portfolio and then check if the average differences in returns are explained by those betas.

Some of the advantages of this methodology are:

1. The portfolio approach allows eliminating residual variance.
2. The time-series may be short for a particular stock.
3. The variance of the factors is maximized, which translates into a greater power of the tests.
4. The interpretation of the results is straightforward because the regressions are performed only on returns.

The following paragraphs explain the steps for carrying out the Fama-Macbeth Regressions (FM).

First, it is necessary to compute the betas (or loadings) associated to each factor<sup>8</sup>. These loadings represent the sensitivity to the factors or risk premiums. The estimation is performed through rolling time series regressions. In the case of this study, each time series regression considered 5 years of monthly returns, i.e.,  $i=60$ . The betas are estimated from equations of the form:

$$\begin{bmatrix} Z_{j,t-i} \\ \vdots \\ Z_{j,t} \end{bmatrix} = \beta_{0jt} \iota_{[ix1]} + \sum_{k=1}^K \beta_{kjt} \begin{bmatrix} X_{k,t-i} \\ \vdots \\ X_{k,t} \end{bmatrix} + \begin{bmatrix} \varepsilon_{j,t-i} \\ \vdots \\ \varepsilon_{j,t} \end{bmatrix} \quad (1)$$

Where,

$\iota$  is a vector of ones and  $K$  is the number of factors in the model.

$j = 1, \dots, J$ ; being  $J$  the number of dependant portfolios.

$Z$  are excess returns and  $X$  the factors.

$t = i + 1, \dots, T$ ; with  $T$  being the number of months in the sample.

Once the betas associated with each factor are computed, one needs to run  $T-i$  cross-sectional regressions. Each month, the excess returns of the dependent portfolios are regressed against the betas associated to each portfolio and factor:

---

<sup>8</sup>This implies to regress the dependant portfolios' excess returns against all factors at the same time.

$$\begin{bmatrix} Z_{1,t} \\ \vdots \\ Z_{J,t} \end{bmatrix} = \gamma_{0t} \iota_{[Jx1]} + \sum_{k=1}^K \gamma_{kt} \begin{bmatrix} \beta_{k1t} \\ \vdots \\ \beta_{kJt} \end{bmatrix} + \begin{bmatrix} \eta_{1,t} \\ \vdots \\ \eta_{J,t} \end{bmatrix} \quad (2)$$

Where,

$\iota$  is a vector of ones and  $J$  the number of dependant portfolios.

$Z$  are excess returns and the betas the loadings.

$t = i + 1, \dots, T$  ; with  $T$  being the number of months in the sample.

Finally, the series of  $\hat{\gamma}_{0t}$  and  $\hat{\gamma}_{kt}$  ( $\forall k$ ) are analyzed. Defining  $\gamma_0 = E(\gamma_{0t})$  and  $\gamma_k = E(\gamma_{kt})$ , if the factor model is appropriate to describe the differences in the cross-section of stock returns, then  $\gamma_0 = 0$  and  $\gamma_k > 0$  (the risk premium exists). Under the assumption that returns are normally distributed and temporally IID, the gammas will also be normally distributed and IID. In this way, the gammas can be tested through the well known t-test. Defining  $w(\hat{\gamma}_{0t})$  as the t-statistic, we have:

$$w(\hat{\gamma}_k) = \frac{\hat{\gamma}_k}{\hat{\sigma}_{\gamma_k}} \quad (3)$$

Where,

$$\hat{\gamma}_k = \frac{1}{T-i} \sum_{t=T-i}^T \hat{\gamma}_{kt} \quad (4)$$

And

$$\hat{\sigma}_{\gamma_k}^2 = \frac{1}{(T-i)(T-i-1)} \sum_{t=T-i}^T (\hat{\gamma}_{kt} - \hat{\gamma}_k)^2 \quad (5)$$

The statistic  $w(\hat{\gamma}_k) \sim t_{T-i-1}$  is distributed Student t and asymptotically is standard normal.

## The Seemingly Unrelated Regressions (SUR)

Fama y French (1993) used the time series approach of Black, Jensen and Scholes. The monthly returns of each portfolio were regressed separately (through simple regressions) against the market portfolio and the other factors that mimic for risk. Using the SUR methodology one can estimate these equations jointly, considering the problem of correlated errors across portfolios. This allows correcting the significance of the coefficients. The fact that the return of a portfolio exceeds the risk-free return on a given quantity supplies some information on the excess return of at least one of the remaining portfolios.

The SUR model is defined as:

$$\mathbf{Z}_i = \mathbf{X}_i\boldsymbol{\beta}_i + \boldsymbol{\varepsilon}_i \quad (6)$$

With  $i = 1, \dots, M$  (the number of portfolios)<sup>9</sup>.

$M$  equations and  $T$  observations are available in the sample for its estimation.

$$\boldsymbol{\varepsilon} = [\boldsymbol{\varepsilon}'_1, \boldsymbol{\varepsilon}'_2, \dots, \boldsymbol{\varepsilon}'_M]; E[\boldsymbol{\varepsilon}/\mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_M] = \mathbf{0}; E[\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}'/\mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_M] = \boldsymbol{\Omega}$$

Assuming that the errors are not correlated across observations, i.e.:

$$E[\varepsilon_{it}\varepsilon_{js}/\mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_M] = \sigma_{ij}, \text{ if } t = s \text{ and } 0 \text{ in any other case.}$$

Then,

$$E[\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}'/\mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_M] = \boldsymbol{\Omega} = \begin{bmatrix} \sigma_{11}\mathbf{I} & \sigma_{12}\mathbf{I} & \dots & \sigma_{1M}\mathbf{I} \\ \sigma_{21}\mathbf{I} & \sigma_{22}\mathbf{I} & \dots & \sigma_{2M}\mathbf{I} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{M1}\mathbf{I} & \sigma_{M2}\mathbf{I} & \dots & \sigma_{MM}\mathbf{I} \end{bmatrix} \quad (7)$$

Each separate equation is a classical regression. The parameters could be estimated consistently (if not efficiently), equation by equation by Ordinary Least Squares (OLS).

The generalized regression is applied to the conjunct model:

$$\begin{bmatrix} \mathbf{Z}_1 \\ \mathbf{Z}_2 \\ \vdots \\ \mathbf{Z}_M \end{bmatrix} = \begin{bmatrix} \mathbf{X}_1 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{X}_2 & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{X}_M \end{bmatrix} \begin{bmatrix} \boldsymbol{\beta}_1 \\ \boldsymbol{\beta}_2 \\ \vdots \\ \boldsymbol{\beta}_M \end{bmatrix} + \begin{bmatrix} \boldsymbol{\varepsilon}_1 \\ \boldsymbol{\varepsilon}_2 \\ \vdots \\ \boldsymbol{\varepsilon}_M \end{bmatrix} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \quad (8)$$

So the efficient estimator is the Generalized Least Squares estimator (GLS). For the  $t$ -th observation, the covariance matrix ( $M \times M$ ) of errors is:

---

<sup>9</sup>The boldface denotes a matrix or vector.

$$\boldsymbol{\Sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1M} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{M1} & \sigma_{M2} & \cdots & \sigma_{MM} \end{bmatrix} \quad (9)$$

Then equation 7,

$$\boldsymbol{\Omega} = \boldsymbol{\Sigma} \otimes \mathbf{I}$$

And,

$$\boldsymbol{\Omega}^{-1} = \boldsymbol{\Sigma}^{-1} \otimes \mathbf{I} \quad (10)$$

Denoting the  $ij$ -th element of  $\boldsymbol{\Sigma}^{-1}$  by  $\sigma^{ij}$ , one determines that the GLS estimator is:

$$\hat{\boldsymbol{\beta}} = [\mathbf{X}'\boldsymbol{\Omega}\mathbf{X}]^{-1}\mathbf{X}'\boldsymbol{\Omega}^{-1}\mathbf{Z} = [\mathbf{X}'(\boldsymbol{\Sigma}^{-1} \otimes \mathbf{I})\mathbf{X}]^{-1}\mathbf{X}'(\boldsymbol{\Sigma}^{-1} \otimes \mathbf{I})\mathbf{Z}$$

Expanding Kronecker Products:

$$\hat{\boldsymbol{\beta}} = \begin{bmatrix} \sigma^{11}\mathbf{X}'_1\mathbf{X}_1 & \sigma^{12}\mathbf{X}'_1\mathbf{X}_2 & \cdots & \sigma^{1M}\mathbf{X}'_1\mathbf{X}_M \\ \sigma^{21}\mathbf{X}'_2\mathbf{X}_1 & \sigma^{22}\mathbf{X}'_2\mathbf{X}_2 & \cdots & \sigma^{2M}\mathbf{X}'_2\mathbf{X}_M \\ \vdots & \vdots & \ddots & \vdots \\ \sigma^{M1}\mathbf{X}'_M\mathbf{X}_1 & \sigma^{M2}\mathbf{X}'_M\mathbf{X}_2 & \cdots & \sigma^{MM}\mathbf{X}'_M\mathbf{X}_M \end{bmatrix}^{-1} \begin{bmatrix} \sum_{j=1}^M \sigma^{1j}\mathbf{X}'_1\mathbf{y}_j \\ \sum_{j=1}^M \sigma^{2j}\mathbf{X}'_2\mathbf{y}_j \\ \vdots \\ \sum_{j=1}^M \sigma^{Mj}\mathbf{X}'_M\mathbf{y}_j \end{bmatrix}^{-1} \quad (11)$$

In the model considered in this study, in all equations the regressors are identical (factors are the same for each portfolio). In this case, the GLS estimator is the same as the OLS estimator. Because of this, the coefficients that are obtained from SUR will be identical to the ones obtained by the approach of Black, Jensen and Scholes. What changes is the significance of the variables in the model, which by being more reliable, allow a better interpretation of the results.

Then equation 11:

$$\hat{\boldsymbol{\beta}} = \begin{bmatrix} \sigma_{11}(\mathbf{X}'\mathbf{X})^{-1} & \sigma_{12}(\mathbf{X}'\mathbf{X})^{-1} & \cdots & \sigma_{1M}(\mathbf{X}'\mathbf{X})^{-1} \\ \sigma_{21}(\mathbf{X}'\mathbf{X})^{-1} & \sigma_{22}(\mathbf{X}'\mathbf{X})^{-1} & \cdots & \sigma_{2M}(\mathbf{X}'\mathbf{X})^{-1} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{M1}(\mathbf{X}'\mathbf{X})^{-1} & \sigma_{M2}(\mathbf{X}'\mathbf{X})^{-1} & \cdots & \sigma_{MM}(\mathbf{X}'\mathbf{X})^{-1} \end{bmatrix}^{-1} \begin{bmatrix} (X'X) \sum_{l=1}^M \sigma^{1l}\mathbf{b}_l \\ (X'X) \sum_{l=1}^M \sigma^{2l}\mathbf{b}_l \\ \vdots \\ (X'X) \sum_{l=1}^M \sigma^{Ml}\mathbf{b}_l \end{bmatrix} \quad (12)$$

The asymptotic covariance matrix of  $\hat{\boldsymbol{\beta}}$  is estimated:

$$\text{Asy.Cov.Est} \left[ \hat{\boldsymbol{\beta}}_i, \hat{\boldsymbol{\beta}}_j \right] = \hat{\sigma}_{ij}(\mathbf{X}'\mathbf{X})^{-1}, \quad i, j = 1, \dots, M, \quad \text{where } \hat{\Sigma}_{ij} = \hat{\sigma}_{ij} = \frac{1}{T}\mathbf{e}'_i\mathbf{e}_j.$$

## References

- Banz, Rolf W. 1981. "The Relationship Between Return and Market Value of Common Stocks." *Journal of Financial Economics*. 9:1, pp. 3-18.
- Basu, Sanjay. 1977. "Investment Performance of Common Stocks in Relation to Their Price- Earnings Ratios: A Test of the Efficient Market Hypothesis." *Journal of Finance*. 12:3, pp. 129-56.
- Bhandari, Laxmi Chand. 1988. "Debt/Equity Ratio and Expected Common Stock Returns: Empirical Evidence." *Journal of Finance*. 43:2, pp. 507-28.
- Black, Fischer. 1972. "Capital Market Equilibrium with Restricted Borrowing." *Journal of Business*. 45:3, pp. 444-54.
- Black, Fischer, Michael C. Jensen and Myron Scholes. 1972. "The Capital Asset Pricing Model: Some Empirical Tests." in *Studies in the Theory of Capital Markets*. Michael C. Jensen, ed. New York: Praeger, pp. 79-121.
- Braun, Matias and Christian Johnson. 2005. "Where does the Market Matter? Stock Prices and Investment around the World." *Working Paper*.
- Campbell, John, Jend Hilscher and Jan Szilagyi. 2006. "In Search of Distress Risk." *NBER Working Paper No. W12362*.
- Campbell, John, Andrew W. Lo and A Craig MacKinlay. 1997. "The Econometrics of Financial Markets." *Princeton University Press*.
- Carhart, Mark M. 1997. "On Persistence in Mutual Fund Performance." *Journal of Finance*. 52:1, pp. 57-82.
- Chang, James, Tarun Khanna and Krishna Palepu. 2000. "Analyst Activity Around the World." *HBS Strategy Unit Working Paper No. 01-061*.
- Cochrane, John. 2005. "Asset Pricing." *Princeton University Press*.
- DeBondt, Werner F. M. and Richard H. Thaler. 1987. "Further Evidence on Investor Over-reaction and Stock Market Seasonality." *Journal of Finance*. 42:3, pp. 557-81.
- Fama, Eugene F. and Kenneth R. French. 1992. "The Cross-Section of Expected Stock Returns." *Journal of Finance*. 47:2, pp. 427-65.
- Fama, Eugene F. and Kenneth R. French. 1993. "Common Risk Factors in the Returns on Stocks and Bonds." *Journal of Financial Economics*. 33:1, pp. 3-56.
- Fama, Eugene F. and Kenneth R. French. 1996. "Multifactor Explanations of Asset Pricing Anomalies." *Journal of Finance*. 51:1, pp. 55-84.
- Fama, Eugene F. and Kenneth R. French. 1998. "Value Versus Growth: The International Evidence." *Journal of Finance*. 53:6, pp. 1975-999.
- Fama, Eugene F. and Kenneth R. French. 2004. "The Capital Asset Pricing Model: Theory and Evidence." *Journal of Economic Perspectives*. 18:3, pp. 25-46.

- Fama, Eugene F. and James D. MacBeth. 1973. "Risk, Return and Equilibrium: Empirical Tests." *Journal of Political Economy*. 81:3, pp. 607-36.
- Fuentes, Rodrigo, Jorge Gregoire and Salvador Zurita. 2005. "Factores macroeconómicos en retornos accionarios chilenos". *El Trimestre Económico* LXXIII(1): 125-138.
- Gibbons, Michael R. 1982. "Multivariate Tests of Financial Models: A New Approach." *Journal of Financial Economics*. 10:1, pp. 3-27.
- Greene, William. 2000. "Econometric Analysis." *Prentice-Hall*.
- Jegadeesh, Narasimhan and Sheridan Titman. 1993. "Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency." *Journal of Finance*. 48:1, pp. 65-91.
- Lakonishok, Josef, Andrei Shleifer and Robert W. Vishny. 1994. "Contrarian Investment, Extrapolation, and Risk." *Journal of Finance*. 49:5, pp. 1541-578.
- Litner, John. 1965. "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets." *Review of Economics and Statistics*. 47:1, pp. 13 – 37.
- Marshall, Pablo and Eduardo Walker. 2000. "Day-Of-The-Week and Size Effects in Emerging Markets: Evidence from Chile." *Revista de Análisis Económico*. 15:2, pp. 89-108.
- Marshall, Pablo and Eduardo Walker. 2002. "Volumen, tamaño y ajuste a nueva información en el mercado accionario chileno." *Estudios de Economía*. 29, pp. 247-268.
- Pérez-Quiros, Gabriel and Allan Timmermann. 2000. "Firm Size and Cyclical Variations in Stock Returns" *Journal of Finance*. 55:3, pp. 1229-62.
- Rosenberg, Barr, Kenneth Reid and Ronald Lanstein. 1985. "Persuasive Evidence of Market Inefficiency." *Journal of Portfolio Management*. Spring, 11, pp. 9-17.
- Ross, Stephen A. 1976. "The Arbitrage Theory of Capital Asset Pricing." *Journal of Economic Theory*. 13:3, pp. 341-60.
- Sharpe, William F. 1964. "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk." *Journal of Finance*. 19:3, pp. 425-42.
- Stambaugh, Robert F. 1982. "On The Exclusion of Assets from Tests of the Two-Parameter Model: A Sensitivity Analysis." *Journal of Financial Economics*. 10:3, pp. 237-68.
- Stattman, Dennis. 1980. "Book Values and Stock Returns." *The Chicago MBA: A Journal of Selected Papers*. 4, pp. 25-45.
- Zhang, Lu. 2005. "The Value Premium." *Journal of Finance*. 60:1, pp. 67-103.