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# Investment Management and Risk Sharing with Multiple Managers

CHRISTOPHER B. BARRY and LAURA T. STARKS*

ABSTRACT

This paper addresses the investor's decision to employ multiple managers for the management of investment funds. Under conditions such that specialization of managers and diversification among managers are not motives for the use of multiple managers, the paper shows that risk sharing considerations may be sufficient. A model is developed in which the decision to use multiple managers is explicitly treated, and conditions are studied such that an increase or decrease in the number of managers would be desirable. Under some conditions, a multiple manager solution is preferred over a single manager solution.

IN HIS PRESIDENTIAL ADDRESS before the American Finance Association in December, 1980, William Sharpe explored the motivation for the employment of multiple investment managers for the management of a single portfolio (see Sharpe [8]). Sharpe considered two determinants of the multiple manager phenomenon, namely, specialization and diversification. In brief, the arguments run along these lines. If investment managers specialize and produce special insights into the likely success of certain industries or limited groups of securities, then it is reasonable to employ managers to invest funds in their areas of specialty. Thus, for example, a manager with superior skills for the analysis of, say, high technology securities would be employed to manage funds targeted for high technology securities. Also, to protect against the possibility that a particular manager might make a serious error in the management of funds, one can diversify funds among managers, hence "washing out" the danger of overall funds performance being seriously damaged by the bad fortune of a single manager.

Sharpe's arguments are reasonable and certainly account for some of the motives involved in the decision to use multiple managers. As Sharpe himself concluded, however:

> "There is of course much more to this problem. We have assumed away many important aspects of the principal-agent relationship(s) ... More research on this subject is needed." (Sharpe [8, p. 233]).

Our objective in this paper is to demonstrate that one aspect of the principal-agent relationship, risk sharing considerations, can indeed influence the decision

*Southern Methodist University and Washington University-St. Louis, respectively. The authors have benefitted from discussions with Mark Kritzman, Stephen Brown, Louis Ederington, Jim Little, Jess Yawitz, Bill Marshall, and Russell Fogler. The suggestions of an anonymous referee have improved the presentation. We retain sole responsibility for the conclusions, however. An earlier version of this paper was presented at the Financial Management Association Meetings in San Francisco, October 14, 1982.

to use multiple managers. We make these arguments not to suggest that diversification among managers and the desire to exploit comparative advantage are unimportant; rather we wish to show that risk sharing issues comprise an additional and important part of the problem. In order to clearly identify these effects, we will demonstrate that risk-sharing considerations are sufficient to produce a decision to employ multiple agents even in the absence of specialization or diversification among managers as motives for employing multiple agents. Thus, whereas Sharpe set agency considerations aside in order to consider diversification and specialization as motives, we set diversification and specialization motives aside in order to pursue agency effects.

An investor making a decision to employ a manager for the purpose of managing the investor's funds confronts a problem. That is, the manager may have perferences concerning risk that lead to a different use of the funds than the investor would prefer. The manager's choice may either be a more conservative portfolio or a more risky portfolio than the investor would prefer.

The investor has at his disposal a number of devices for helping to align the agent's decision with the investor's preferences:

1. the investor may select a manager whose stated investment policies are consistent with the investor's objectives;
2. the investor may seek to compensate the manager in such a way as to shift the manager's interest more in line with the investor's; or
3. the investor may forego the use of a manager altogether, investing his funds himself.

We will assume that the third alternative is impractical, which might be the case for any of a number of reasons. We will assume away the first alternative under the assumption that it is better to have the manager's interests coincide with the investor's interests than it is to rely on the manager to follow an advertised policy. Investment objectives are seldom stated so unambiguously as to leave no doubt as to how all invested funds will be employed. Thus, our focus will be upon the use of compensation programs that align the investor's and manager's interests.

An intuitive explanation for our conclusions may be helpful at this point. Presumably there is some method of compensating investment managers that will influence the manager's portfolio choice. If a single manager is employed, then denote by $FEE_1^*$ that compensation schedule that is in some sense optimal for the investor, perhaps in the sense that the investor's expected utility of the portfolio value net of $FEE_1^*$ is a maximum. We will show that under some circumstances it may be better to use two (or more) managers, paying each of them with some schedule $FEE_2^*$, in the sense that the investor's expected utility of aggregate portfolio value net of both fees is higher than with one manager. This can occur even if the total fee paid out is greater than with one manager as long as this higher total fee is more than offset by an improvement in the risk-return properties of the aggregate portfolio from the investor's perspective.

For example, suppose that all agents have the property that as the amount of their managed funds rises, so does their propensity to take risk.[1] An investor

[1] Actually, because of their desire to retain an account once it is in hand, the manager may behave

with a large amount of money to invest and who is highly risk averse may prefer the portfolio that managers would select using, say, half of the investor's funds rather than all of them. The question then is whether the benefits of the less risky portfolio offset the perhaps higher aggregate management fee. If the investor knew everything about the investment universe, then presumably he could force the desired behavior upon the managers by compensating them only if they selected the investor's preferred portfolio. However, it is more likely that the investor who elects to employ investment managers has less than perfect knowledge concerning the available investment opportunities.

In the next Section, we develop the basic model upon which the results are based. The subsequent Section will then provide a formal analysis of the model and an indication of the tradeoffs involved in selecting the appropriate number of managers. Section III presents conclusions and suggestions for extensions of the work.

## I. The Model

Assume that the investor is interested in the use of an investment manager for the management of the investor's total investment budget, $I$. Given the investment level $I$, the manager will select an investment portfolio $p$ from some set $P$ of alternatives. The manager is compensated by some fee schedule, $\phi(\cdot)$, that may be a function of the initial investment $I$ and/or the final value $\tilde{X}$, of the portfolio, where the "tilde" denotes that $\tilde{X}$ is random. Thus, depending on the context, $\phi(\cdot)$ may be written $\phi(I)$, $\phi(X)$, or $\phi(I, X)$.[2] The set of available fee schedules will be denoted $\Phi$; in some practical settings, $\Phi$ may be constrained by regulations.

The investor and manager are assumed to behave as if they maximize their expected values of von Neumann-Morgenstern utility functions $U_I$ and $U_M$, respectively. The manager's utility is expressed solely as a function of $\phi$. We treat the manager's utility with respect to the management of the investor's funds as being separable from the manager's other activities. Such a utility assumption is consistent with a utility function that is additive or multiplicative in the manager's activities. This assumption allows us to focus on the interaction between the investor and manager.

The investor's utility depends on $X - \phi$, the portfolio value net of management fees. One may either think of $X$ as portfolio value net of transaction costs or assume that there are no transaction costs.

Given an investment, $I$, a fee schedule, $\phi$, and a portfolio opportunity set, $P$, the manager is assumed to solve the following problem:

$$\underset{p \in P}{\text{Max}}\ EU_M[\phi(\cdot)]. \tag{1}$$

more *conservatively* than the investor would desire as the account becomes larger. The point of the discussion is merely that the conflict between investor and manager objectives can increase as the size of the account increases.

[2] The contract is restricted to be a function of outcome alone and to be comparable to existing contracts for portfolio managers. Due to monitoring costs, it is doubtful that this contract could be dominated by one which includes other factors.

The expectation operator will be defined below by the particulars of problems we consider. We assume that the manager will agree to accept the task of managing the portfolio only if the value obtained in (1) exceeds some minimum value $\bar{U}_M$. $\bar{U}_M$ might be thought of as an opportunity cost (in terms of utility) of accepting the job of managing the portfolio. Thus, the manager will accept the task only if it provides a compensation that allows him to achieve at least this minimal expected utility. $\bar{U}_M$ subsumes a whole range of complications associated with the market for investment management services.

The investor's problem is to design a compensation program (i.e., select a $\phi$) to maximize his expected utility. The investor's problem is subject to one constraint (namely, manager's expected utility must be at least $U_M$) and to the behavioral implications that the choice of $\phi$ holds for the manager's action. In the examples below, we will assume that the investor has sufficient information to fully understand the implications of $\phi$ for the portfolio choice ($p$) of the manager. Thus, the investor's problem can be expressed as[3]

$$\max_{\phi \in \Phi} EU_I(\tilde{X} - \phi) \tag{2a}$$

$$\text{subject to} \quad EU_M(\phi) \geq \bar{U}_M \tag{2b}$$

$$EU'_M(\phi) = 0. \tag{2c}$$

Expression (2a) utilizes the expectation operator of the investor, and of course the distribution of outcome expressed therein is conditioned by $p$. Expression (2b) reflects the manager's opportunity restriction. Expression (2c) reflects the first-order condition for the manager's portfolio choice, $p$.

To illustrate the problem, consider Figure 1. For a chosen fee schedule, the manager must first satisfy himself that for some $p \in P$ his utility bound constraint is satisfied. If so, he selects an optimal portfolio (optimal from his perspective). Then, the portfolio's return is realized, and the outcome is divided between the investor and manager in accordance with $\phi$.

Now consider Figure 2. The investor can calculate his expected utility as a function of the fee schedule. Thus, he can now evaluate alternative fee schedules and select an optimum.

Properties of the solution to the single-manager problem (2) are explored in detail in Starks [10]. For example, circumstances in which moral hazard is or is not a problem are considered, and properties of optimum fee schedules and portfolio selection are investigated. Our interest, however, is in conditions such that the solution to (2) is inferior to the solution to an extended version of (2) that allows multiple managers.

### *A. Multiple Managers*

First, let us consider the general nature of the problem as extended to account for multiple managers. As indicated in Figure 3, the investor now has the right to select both an optimum number of managers ($N$), among whom the investment

[3] The investor is assumed not to have the skill, knowledge, or time to choose investments from the known opportunity set.

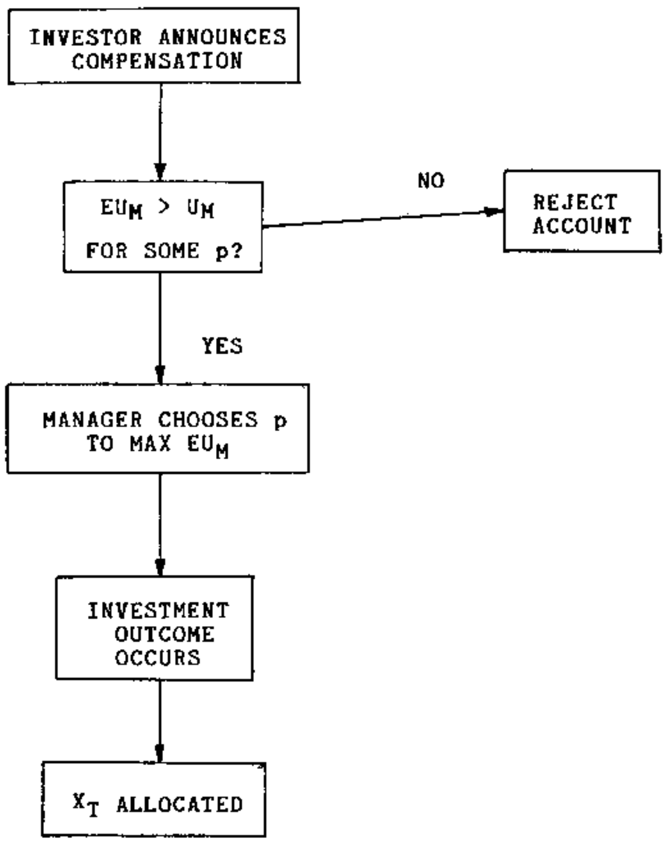


**Figure 1.** The Manager's Decision

budget can be allocated, and an optimum fee schedule. For a given $N > 1$, the optimum fee schedule will not be the solution to (2), at least not in general.

In particular, suppose that the investor is constantly risk averse but that all managers are sharply increasingly risk averse. Then for large levels of investment, a single manager would tend to hold a portfolio with very little risk. Using two managers with half of the investment allocated to each of them, there is a chance that the resulting portfolio will be preferred (from the investor's viewpoint) over the portfolio that would have been selected by only one manager. The investor's preference for the two-manager portfolio must take into account the possibility of higher total compensation of the two managers.

### *B. Multiple Managers: Further Notation and Assumptions*

In the formal results to follow, we will simplify the analysis by assuming that all managers are identical with respect to risk preferences, expectations, and allocated investment. Thus, if the investor chooses to use $N$ managers, each will invest $I/N$ in funds for the investor. While obviously the assumptions are made in part for simplicity, they also permit us to focus on risk sharing considerations

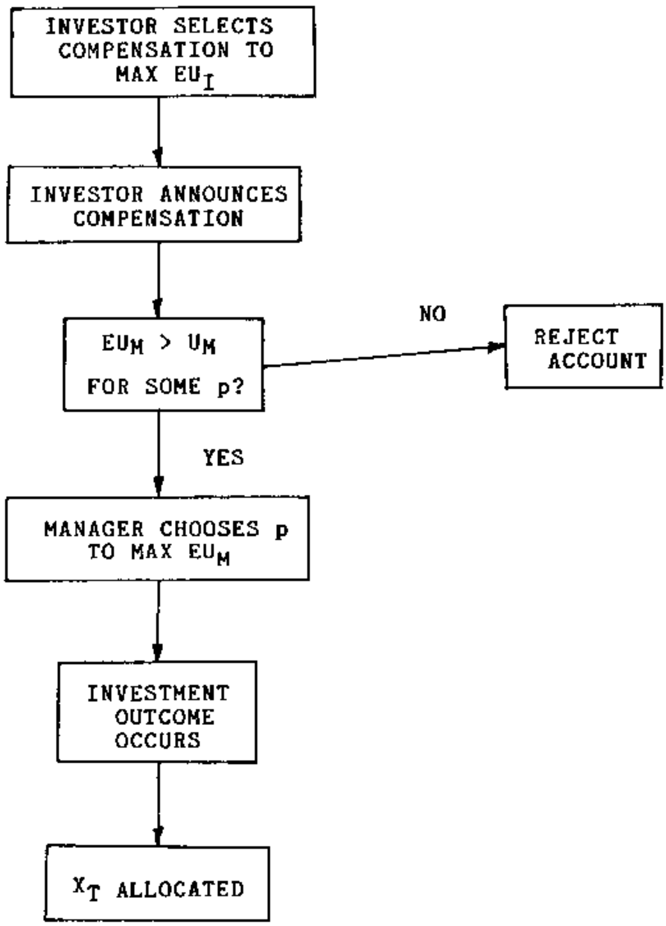


**Figure 2.** The Investor's Single Manager Decision

by themselves as motivation for employing more than one manager. Given $N$ identical managers, there appears to be no a priori reason for allocating the funds unequally except in the extreme case of allocating all funds to a single manager. In our analysis, we explicitly contrast this alternative with the equal investment alternative.

Under the multiple agent scenario, then, the investor's problem becomes:

$$\operatorname*{Max}_{\phi \in \Phi} EU_I(\tilde{X} - N \cdot \phi) \tag{3a}$$

$$\text{subject to} \quad EU_M(\phi) \geq \bar{U}_M \tag{3b}$$

$$EU'_M(\phi) = 0. \tag{3c}$$

In the next section, we investigate properties of the problem.

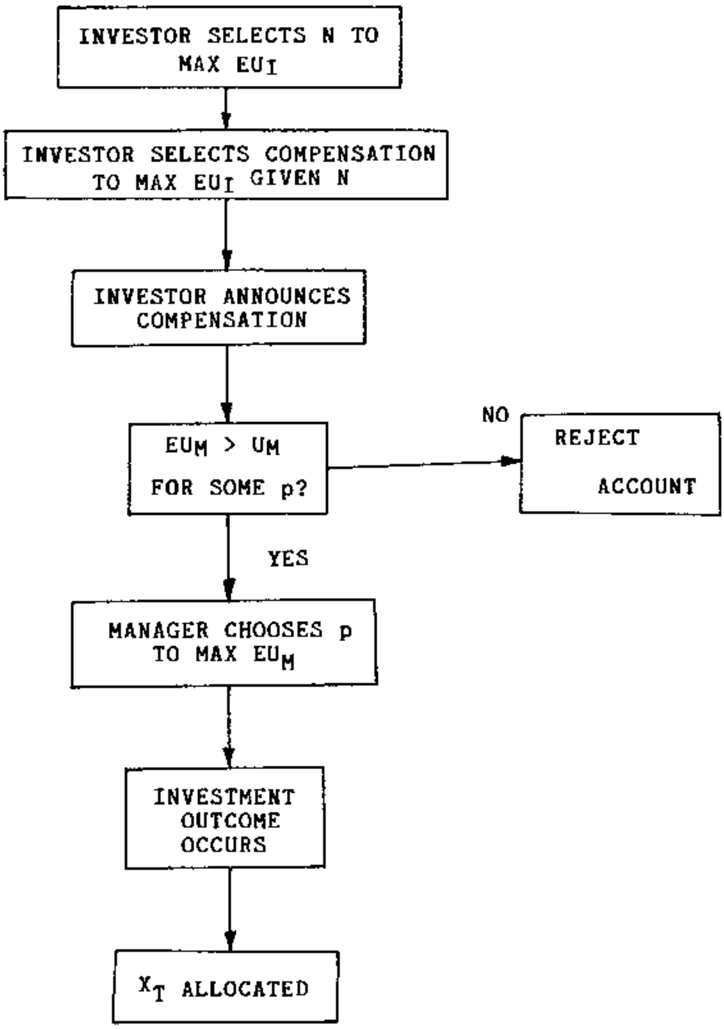


**Figure 3.** The Investor's Multiple Manager Decision

## II. Multiple Manager Optimality

The results in this paper are restricted to a single-period setting. We also assume that the investor and manager view the opportunity set identically, although the investor may not know how to construct portfolios that are on the efficient boundary of the opportunity set. Holding period returns are assumed to be normally distributed. This assumption is made in part for analytical tractability, but also because for holding periods of a month, a quarter, or a year, the normality assumption is a reasonable approximation.

Under the normality assumption, the outcome of the portfolio may be written

$$\tilde{X} = \tilde{r}I.$$

Letting $\tilde{z} = (\tilde{r} - \mu)/\sigma$, where $\mu$ and $\sigma$ are the portfolio's mean and standard deviation, respectively, of return, then the investment outcome can be written

$$\tilde{X} = (\mu + \sigma\tilde{z})I. \tag{4}$$

Thus, $\tilde{X}$ has mean $\mu I$ and variance $\sigma^2 I^2$, and $\tilde{z}$ is a standardized normal variate.

Our assumption that investors and managers are risk averse and that returns are normally distributed allows us to restrict attention to the efficient set of portfolio alternatives. In making this restriction, we are implicitly assuming that $\phi(X)$ will not be such a convex function of $X$ as to cause the manager to behave as if he were a risk taker rather than risk averse.[4]

As Roll [7] shows explicitly, the Markowitz [4] efficient boundary can be written as

$$\mu = g(\sigma) \tag{5}$$

where $g$ is linear if a riskless asset exists and strictly concave otherwise. Thus, our results assume $g'(\sigma) > 0$ and either $g''(\sigma) = 0$ or $g''(\sigma) < 0$.

In order to abstract from the moral hazard[5] problems which may exist, we assume the agent's only decision is the risk level of the portfolio, and that the investor and manager agree that the fee schedule is to be based on the ending outcome of the portfolio investment. As has been shown by Starks [10], if the fee schedule is selected in the manner assumed below, the manager's optimal risk choice will always coincide with the investor's optimal risk choice (given that he has to employ a manager).

### *A. The Single Manager Solution*

Expressions (2a), (2b), and (2c) define our basic single manager problem. The solution to this problem involves the solution to Euler's equation,[6] which provides first-order conditions for an optimum. The first-order condition is

$$U_I'(X - \phi(X)) = \lambda U_M'(\phi(X)) \tag{6}$$

for all $X$, where $\lambda$ is a Lagrange multiplier on the manager's minimum utility constraint. Since the ratio of the investor's marginal utility to the manager's marginal utility is a constant, the fee schedule obtained provides a Pareto optimal allocation of outcome.[7]

It may be shown that the optimal risk sharing rule is one that allocates

[4] If $\phi(X)$ is convex in $X$, then $U_M(\phi(X))$ *can* be a convex function of $X$ and could thus lead the manager who is risk averse in his attitude towards compensation to behave as if he were risk taking in his attitude toward investment outcome. We assume away such compensation schemes and restrict the manager's choice to the Markowitz efficient set.

[5] See Holmstrom [2] and Shavell [9] for explanations of potential moral hazard problems and conditions under which they can arise.

[6] See Weinstock [11] for an introduction to the solution techniques based on Euler's equation.

[7] See Mossin [5], Chapter two.

investment outcome according to the rule[8]

$$\phi'(X) = \frac{ARA_I}{ARA_I + ARA_M}. \tag{7}$$

where $ARA_i$ is the absolute risk aversion of individual $i$ (see Pratt [6]). Thus, for example, if $ARA_I = 1$ and $ARA_M = 20$, the manager would receive $1/21$, or 4.8%, of marginal income he generated. From (7) it is clear that as long as the manager and investor are both risk averse, $\phi'(X) \in (0, 1)$. However, if the manager is risk neutral, he will pay the investor a constant amount and assume all of the risk himself. If the investor is risk neutral, he will pay the manager a constant amount and assume all of the risk (i.e., $\phi'(X) = 0$).

### B. The Multiple Manager Solution

If $N$ managers are used, then any one manager will receive $I/N$ of the funds to invest, and the outcome pertinent to that manager will be $X/N$.[9] Given the problem expressed in (3a)–(3c), the first-order condition for an optimum fee schedule is

$$U_I'(X - N\phi(X/N)) = \lambda U_{Mi}'(\phi(X/N)). \tag{8}$$

The right-hand side is identical for all managers, so there is indeed only one required condition. Using the same approach used to derive Equation (7), it follows that in the multiple manager case optimal risk sharing dictates the following condition:

$$\phi'(X/N) = \frac{ARA_I}{N \cdot ARA_I + ARA_{Mi}}, \tag{9}$$

which is constant across managers. To illustrate, if $ARA_I = 1$, $N = 3$, and $ARA_{Mi} = 20$ for all managers, each manager would receive $1/23$, or 4.3% of marginal income he generated. As before, note that if the managers are all risk neutral, they will bear all risk. If the investor is risk neutral, he will pay the managers a fixed fee and assume all risk himself.

### C. Restriction to Linear Fee Schedules

It it interesting to note that the most common types of fee schedules in use today by investment managers are linear in outcome. Two special cases, for example are

$$\phi(X) = k_0 \tag{10}$$

and

[8] Equation (7) is obtained by taking the differential of Equation (6) with respect to $X$ and substituting for $\lambda$ from Equation (6). See Leland [3] for a development of this result in a different setting.

[9] Under our assumptions of identical managers, each manager will select the same portfolio. Thus, in essence, the investor would use multiple managers only to drive the "manager-set" to a more desirable portfolio, which would presumably be achieved by sharing the risks among many managers.

$$\phi(X) = kX. \tag{11}$$

Fee schedule (10) involves paying the manager a flat fee, regardless of the outcome, and it involves no risk sharing. For example, managers often require payment as a fixed percentage of the initial investment. Since $I$ is known in advance of $\tilde{X}$, this schedule can be thought of as a constant fee. Such schedules are encouraged, for example, by rules such as ERISA's prohibition against "joint venturing" between investment managers and the corporate owner of a pension fund. However, it is clear from the results in Equations (7) and (9) that the following theorem holds.

THEOREM 1. *Under both the single manager and multiple manager scenarios defined in this paper, a flat fee compensation scheme cannot be Pareto optimal unless the investor is indifferent to risk.*

Thus, the desirability of a lump sum fee schedule cannot be established in a principal-agent framework unless the principals (investors) are risk neutral.

If the compensation scheme is of the fixed percentage form (Equation (11)), it is interesting to know that the effect that a change in $k$ has upon the manager's choice of risk depends on the manager's relative risk aversion (see Pratt [6] for a definition). In particular, the following result can be proven (see the Appendix for the proof).

THEOREM 2. *Assume the fee schedule is given by* (11). *Then as k is increased, the manager's choice of an optimal risk level $\sigma$ will increase, remain unchanged, or decrease, as his relative risk aversion is decreasing, constant, or increasing, respectively.*

We turn now to an examination of the special case of exponential utility, which greatly facilitates the analysis.

### D. Exponential Utility

Exponential utility for both the managers and the investor has constant ARA. Hence, the optimal compensation function will be linear (see Equations (7) and (9); $\phi'(X)$ must be constant since $ARA_M$ and $ARA_I$ are constant).

Let $c$ and $d$ be the risk aversion coefficients of the investor and managers, respectively. Thus,

$$U_I(X) = -\exp\{-cX\}$$

and

$$U_M(X) = -\exp\{-dX\}.$$

Applying Equation (9), if there are $N$ managers, then the optimal compensation program for each manager will be

$$\phi(X) = k_0 + kX \tag{12}$$

for

$$k = \frac{c}{Nc + d}. \tag{13}$$

Recalling the definition of $\lambda$ implicitly contained in (8),

$$k_0 = \frac{\ln(\lambda d/Nc)}{Nc + d}. \quad (14)$$

Notice in general that $k_0$ is not restricted in sign; it may be positive, negative, or zero.

For any fee schedule of the form $k_0 + kX$, we can obtain the manager's optimal risk choice by maximizing his expected utility. The manager's mean and standard deviation of payment as a function of portfolio risk are[10]

$$\mu_M = k_0 + kIg(\sigma)$$

and

$$\sigma_M = kI\sigma,$$

respectively. Thus, the manager's optimization problem reduces to

$$\underset{\sigma}{\text{Max}} \; -\exp\{-dk_0 - dkIg(\sigma) + d^2I^2k^2\sigma^2/2\}. \quad (15)$$

(15) is optimized by merely optimizing the exponent itself, which leads to

$$\sigma^*(k) = \frac{Ng'(\sigma^*)}{dIk}. \quad (16)$$

From (16) it can be seen that the choice of risk level depends on the number of managers, the investment sum, the risk aversion of the managers and the risk-return tradeoff summarized by $g'(\sigma)$. Some direct effects to notice are:

1. As the number of managers increases, the optimal risk level increases.
2. The more favorable the risk-return tradeoff is, the higher will be the optimal risk level.

Equations (13) and (14) indicate that the manager's utility constraints ((2b) and (3b)) affect $k_0$ but not $k$. In general a tightening of the utility constraint (i.e., *increase* in $\bar{U}_M$) will increase $\lambda$ and thus will increase the size of the constant paid to the manager (or reduce the manager's transfer to the investor).

As the number of managers under contract by the investor increases, there are diverse effects. First, whether the increase is beneficial or detrimental depends on the effects the increase in $N$ has on the optimal $k$, $k_0$, and $\sigma$ and, ultimately, on whether the increased overall management fees are offset by an improvement in the risk and return combinations of the new portfolios. Although $N$ is an integer variable, it is useful to consider $\frac{dEU_I}{dN}$, which is given by

$$\frac{dEU_I}{dN} = [EU_I(N)] \cdot \left\{ ck_0 + cN\frac{dk_0}{dN} - cIg'(\sigma)\frac{d\sigma}{dN} + ckIg'(\sigma)\frac{d\sigma}{dN} + cIg(\sigma)\frac{dk}{dN} - c^2I^2\sigma^2(1-k)\frac{dk}{dN} + c^2I^2\sigma(1-k)^2\frac{d\sigma}{dN} \right\}. \quad (17)$$

[10] See any standard mathematical statistics text. Our result uses the fact that if $X \sim N(\mu, \sigma)$, then

$$E(e^{tX}) = e^{\mu t + \sigma^2 t^2/2}.$$

Since $EU_I < 0$ when utility is negative exponential, the sign of (17) will be the negative of the sign of the term in brackets ($\{\cdot\}$).

While the sign of (17) is clearly a complex function of all of the components of the problem scenario, this much can be said: $dEU_I/dN > 0$ if and only if

$$k_0 + N\frac{dk_0}{dN} + Ig(\sigma)\frac{dk}{dN} < (1-k)Ig'(\sigma)\frac{d\sigma}{dN} + \left[cI^2\sigma^2(1-k)\frac{dk}{dN} - cI^2\sigma(1-k)^2\frac{d\sigma}{dN}\right]. \quad (18)$$

The left-hand side of (18) is the increased fee paid to the overall set of managers. An additional manager is only beneficial if this increase in fees is smaller than the increment in the investor's certainty equivalent attached to incremental income. The right-hand side of (18) can be interpreted as consisting of two parts, the increased expected income to the investor $\left((1-k)Ig'(\sigma)\frac{d\sigma}{dN}\right)$ and a risk premium for the incremental income (the term in brackets).

In evaluating Relationship (18), it is useful to note these points. First, the higher the manager's reservation utility $\bar{U}_M$, the larger $k_0 + N\frac{dk_0}{dN}$; thus added managers are less desirable the higher are their demands. Second, the higher the risk-return tradeoff (i.e., the larger $g'(\sigma)$ is), the more likely it is that added managers are desirable. Finally, the optimal risk sharing ($k$) clearly affects the sign of (18); hence, the desirability of added managers depends on the absolute risk aversion of the investor and the managers.[11]

## III. Conclusions and Suggestions for Further Research

In this paper we have shown that an investor may be made better off by using multiple agents instead of only one even if there are not specialization or manager-diversification benefits attained. The results are obtained under restrictive conditions, but they clearly indicate—as Sharpe suggested—that agency considerations are an important feature of the decision to employ multiple managers.

Sharpe concluded his study with the claim that, "More research on this subject is needed." In this paper, we have only *begun* to explore one of the many avenues of research that should be conducted on this topic. Of course, a full-blown multiple agent analysis should be pursued that takes into account simultaneously, the specialization, diversification, and agency motivations for the use of multiple managers.

Some important incentive problems arise out the desire to exploit specialized skills, for example. Suppose the investor believes that certain managers have

[11] In a companion note (Barry and Starks [1]), the authors provide a set of numerical examples in which the investor's expected utility increases as the number of managers increases from one to three. The examples illustrate the main results of the paper. The reader may obtain a copy by writing to Professor Barry at the Edwin L. Cox School of Business, Southern Methodist University, Dallas, TX 75275.

specialized skills and awards those managers with investment funds to invest according to the manager's specialties. However, having acquired such an account, the manager may pursue a broad-based and diversified strategy out of fear that the higher potential for low returns in a narrow portfolio could cause the manager to lose the account!

The example in the previous paragraph suggests that a single-period optimization horizon may be inadequate to explain investor-manager behavior fully. If the manager in some sense concentrates on "retaining the account," then clearly he is concerned with income beyond the current period. Thus, he may be willing to sacrifice the potentially higher short-run returns of a risky strategy in order to ensure retaining the account and keeping moderate—but continuing—returns over a longer period. This suggests that criteria for retention can be critical elements of the principal-agent relationship existing between the investor and manager.

Another important generalization is to consider how to combine and compensate managers who differ in their attitudes towards risk. For example, a moderately risk averse investor might find it desirable to utilize the services of a very risk averse manager and a very risk prone manager.

An interesting case we have ignored is the case in which managers have more than one decision available. For example, suppose the manager can select both the risk level of the portfolio and the resources to expend in selecting the portfolio. To illustrate, the manager might expend greater computer resources by extending his analysis to include a larger number of securities. Presumably, the manager would be more prone to expend resources if he receives a greater fraction of his portfolio's returns. Starks [10] considered this problem in the single manager case and found that a moral hazard problem arose if the manager's actions are unobservable.

Other issues we have not addressed include differences in information between the investor and manager. Clearly, one motivation for the use of investment managers is that the managers may have superior investment skills over those of the investor. A particularly critical area of information is information the investor might have about the managers themselves. The design of optimal compensation schemes in our results was predicated on rather detailed knowledge about the manager; otherwise, the manager's action is not predictable by the investor. The value of being able to vary the compensation schedule may decrease and the motivation to diversify across managers may increase as the investor has greater uncertainty concerning the universe of managers.

We have left out potential "clientele effects," i.e., the tendency of investors to select managers according to publicly expressed investment objectives advertised by the managers. In fact, the market for investment management services has been largely ignored, with these exceptions: the choice of $\bar{U}_M$ proxies for the effects of competition in the management services market; and, we have considered seriously at least, the "consumer" side of the market.

With so much left out, have we accomplished anything of value? We think so. We have taken a world in which extant explanations for the use of multiple managers could not apply and, in spite of that, have found that it may be worthwhile to employ more than one investment manager. We hope that our results will kindle further interest in the incentive problems that seem critical to us in understanding relationships between investors and those who are trusted with the management of investors' funds.

## Appendix

### Proof of Theorem 2

The manager's first-order condition for maximizing, Equation (2c), results in

$$g'(\sigma) = -E[U_M'(kX)\cdot X]/E[U_M'(kX)]. \tag{A1}$$

$g'(\sigma)$ is the market's marginal return per unit of risk. The right-hand side can be interpreted as the manager's ratio of marginal expected utility of risk to marginal expected utility from return. Thus, the manager selects a risk level so that his risk-return tradeoff matches the market's.

We can restate (A1) in terms of the standard normal deviate $z = (X - g(\sigma)/\sigma)$. Let $E_z$ denote expectation with respect to $z$. Then (A1) can be expressed as

$$g'(\sigma) = -E_z[U_M'(kX)z]/E_z[U_M'(kX)]. \tag{A2}$$

(A2) must hold for all fee schedules of the form $kX$. Differentiating it w.r.t. $k$, setting the derivative equal to zero, and rearranging,

$$\frac{d\sigma}{dk} = \frac{-E_z[U_M''(kX)\cdot X\cdot(g'(\sigma) + z)]}{E_z[kI(U_M''(kX)(g'(\sigma) + z)^2 + U_M'(kX)g''(\sigma))]}. \tag{A3}$$

The denominator in (A3) is negative because, by assumption, $k > 0$, $I > 0$, $U_M' > 0$, $U_M'' < 0$, and $g''(\sigma) < 0$. Thus, we can sign $d\sigma/dk$ by signing the numerator of (A3). We do this by considering special cases of relative risk aversion (RRA).

*Constant RRA*

Define $R(kX) = -U_M''(kX)kX/U_M'(kX)$ as the manager's relative risk aversion. Assume $R(kX)$ is a constant, and denote it by $R$. Then

$$-U_M''(kX) = RU_M'(kX)/kX. \tag{A4}$$

Substituting (A4) into the numerator of (A3) yields

$$\frac{R}{k}\,E_z[U_M'(kX)(g'(\sigma) + z)]. \tag{A5}$$

The expectation in (A5) is the manager's first-order condition on $\sigma$ and must be zero. Thus, if relative risk aversion is constant, $\dfrac{d\sigma}{dk} = 0$.

*Increasing RRA*

Again we need to sign the numerator of (A3). Let $R(kX)$ be increasing. $R(kX)$ is given by

$$R(kX) = R(kI(g(\sigma) + \sigma z)). \tag{A6}$$

For $g'(\sigma) + z < 0$,

$$R(kI(g(\sigma) + \sigma z)) < R(kIg(\sigma)) \equiv R^*. \tag{A7}$$

Replacing $R$ by its definition and rearranging (using (A4)),

$$-U_M''(kX)kX < R^*U_M'(kX), \tag{A8}$$

for $g'(\sigma) + z < 0$. Multiply both sides of (A8) by $g'(\sigma) + z < 0$, producing

$$-kXU_M''(kX)(g'(\sigma) + z) > R^*U'(kX)(g'(\sigma) + z). \tag{A9}$$

Similar machinations show that (A9) holds when $g'(\sigma) + z > 0$. Taking the expectation,

$$-kE[U_M''(kX)X(g'(\sigma) + z)] > R^*E[U_M'(kX)(g'(\sigma) + z)]. \tag{A10}$$

The right-hand side of (A10) must be zero since its value leaving out $R^*$ is the manager's first-order condition on $\sigma$. Since the left-hand side of (A10) is positive, the sign of (A3) must be negative. Thus, if RRA is increasing, $d\sigma/dk < 0$.

*Decreasing RRA*

If relative risk aversion is decreasing, we can apply the same steps as above to show that $d\sigma/dk > 0$.

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