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ESTIMATION RISK AND INCENTIVE CONTRACTS FOR PORTFOLIO MANAGERS*

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The fiduciary relationship between portfolio managers and the investors they represent may be viewed as a principal-agent relationship, and therefore we have used the methodology from the agency literature in economics and finance to study the impact of existing compensation arrangements on the conflicts of interest between these two groups. In this paper we employ the assumptions of the Capital Asset Pricing Model and of estimation risk concerning beta to develop a model in which portfolio managers can, through effort, choose the parameters of the beta distribution. Our model entails moral hazard because the investor cannot observe the manager's action. Given certain utility functions we show that the presence of estimation risk leads the manager to choose a lower beta portfolio than otherwise and that no first best optimal contract exists. We also show that managerial divergent behavior is related to the divergence in risk preferences between the manager and the investor. By making slightly more stringent assumptions about preferences, we show that there exist some conditions under which the manager will provide more effort but also a riskier portfolio than the investor prefers. Finally we show that the investor will prefer a manager who is less risk averse than the investor.
(RISK; PORTFOLIOS; INCENTIVES; ESTIMATION RISK)

1. Introduction

The conflict of interest between investment advisors and their clients concerns economists, policy makers, and investors, among others. Of particular interest is the way in which compensation arrangements between the two parties affect the conflict.[1] The fiduciary relationship between portfolio managers and the investors they represent may be viewed as a principal-agent relationship, and therefore we have used the methodology from the agency literature in economics and finance[2] to study the impact of existing compensation arrangements on the conflicts of interest between these two groups.

In the relationship between portfolio managers and shareholders, as in all agency relationships, problems arise due to the absence of complete information. The investor cannot costlessly observe all the actions of the portfolio manager. In particular, the investor cannot observe the returns distribution of the chosen portfolio. Since the manager is motivated to choose a portfolio which is optimal in terms of his or her own welfare, the investor wants to know whether such a choice is optimal in terms of the investor's welfare. The problem is further complicated if there is uncertainty associated with the parameters of the distribution of the portfolio returns. If the manager can lower this "estimation risk" through the provision of effort, there are two potential sources of divergent behavior on the part of the manager: the amount of effort and the portfolio characteristics.

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[1] For example, see *Wall Street Journal*, November 15, 1985, p. 7, "SEC to Let Investment Advisers Collect Performance Fees from Certain Clients" and *Wall Street Journal*, January 9, 1986, p. 1, "Business Bulletin."

[2] For example, agency theory has been used to study the insurance relationship (see Zeckhauser 1970, Spence and Zeckhauser 1971, and Shavell 1979); the employer-employee relationship (see Stiglitz 1974, 1975 and Harris and Raviv 1978); and the management-owner relationship (see Jensen and Meckling 1976).

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The above problems could possibly be resolved if the investor were able to observe portfolio outcomes over time and, as a consequence, infer the underlying stochastic process generating these outcomes. It might then be possible for the investor to eliminate the costs of past divergent behavior by requiring the manager to "settle up" *ex post.* However, *ex post* settling up is not costless, since it would most likely involve some kind of legal action. In addition, complete settling up is not possible if the manager is bankrupt. Consequently, here, as in all principal-agent relationships, the investor will prefer to avoid such problems by setting up incentive compensation contracts. These contracts are used to motivate the portfolio manager to take actions in the investor's best interest.

We develop a model which describes the portfolio choices made by the manager. We make several assumptions about both the returns distribution and what the manager can do. The driving assumption of this model is that the manager can choose the distribution of the parameters of the returns distribution of the portfolio, and that the parameters' distributions have nonzero variances. For given means of the parameters' distributions, the manager can, through the provision of effort, reduce the variance of these distributions.

We assume that expected returns follow the Capital Asset Pricing Model. In the absence of "estimation risk," the portfolio manager would choose the beta for the portfolio. When there is estimation risk, the manager chooses the mean and the variance of the distribution of beta. Expected utility for both the manager and the investor are then functions of the mean and the variance of beta; we assume that these functions are additively separable in the mean and the variance, concave in the mean, and decreasing in the variance.[3] Under these conditions, we show a moral hazard problem will always exist. However, unlike many other principal-agent models in which it is not possible to eliminate the moral hazard problem, our model includes conditions under which the agent actually provides *more* effort than the investor desires in equilibrium, but chooses a variance which is not optimal from the investor's perspective. Under these conditions we still have divergent behavior, but of a different type.

In contrast, Starks (1987) showed that without uncertainty about the parameters of the returns distribution of the portfolio, and with no effort provided by the manager, an optimal linear sharing rule may exist between the manager and the investor. However, if the manager is able to use effort to increase the expected return of the portfolio, that is, change the mean of the returns distribution without changing the variance, a linear sharing rule will not be optimal. In that case the portfolio manager will choose a level of effort that is *less* than the investor wishes in equilibrium, but will choose the variance that the investor wishes. That result depends crucially on the assumption that the manager can choose the mean and the variance of the returns distribution independently. In our model, the results depend crucially on the assumption that the manager's choices of the mean and of the variance of the portfolio are interdependent.

In the next section we formally present our assumptions and the model. §3 contains the statement of our results (proofs are in the Appendix). Conclusions are contained in §4.

2. The Model

Consider a representative investor who turns over a sum of money, K, to a representative portfolio manager for some period of time. After the investor and manager agree on how the manager is to be compensated, the manager selects a portfolio which earns

[3] For example, if utility is quadratic in wealth or returns are normally distributed, and contracts are linear in returns, then expected utility will have the stated properties.

some random return, R, over the period. At the end of the period the portfolio is liquidated, the manager is paid, and the residual goes to the investor.[4,5]

Also assume that asset returns are normally distributed and that they are modelled according to the Sharpe-Lintner-Mossin Capital Asset Pricing Model (CAPM), which implies that expected returns can be written as:[6]

$$E(R_i) = R_f + \beta_i[E(R_m) - R_f] \qquad \text{where} \tag{1}$$

R_i = the gross return on the investment portfolio i,
R_m = the gross return on the market portfolio,
R_f = the gross return on the riskfree asset, and
β_i = the systematic relationship between R_i and R_m.

Since R_i is distributed normally,

$$R_i = E(R_i) + \sigma_i z \tag{2}$$

where $z \sim N(0, 1)$, and $\sigma_i^2 = \text{VAR}[R_i]$. Substituting from (1), equation (2) becomes

$$R_i = R_f + \beta_i[E(R_m) - R_f + \sigma_m(z/r_i)] \qquad \text{where} \tag{3}$$

r_i = the correlation coefficient between the returns of portfolio i and the market portfolio.

If the investor invests the amount K in the portfolio, then the ending dollar value of the portfolio is

$$x_i = R_i K. \tag{4}$$

We further assume that the investor and the manager agree that the portfolio is to be well diversified[7] and that the appropriate benchmark of performance is the market portfolio.[8]

We assume that the manager's main functions are to estimate and select the risk (i.e., the beta) of the portfolio. Although portfolio managers make a variety of decisions, for example the selection of specific assets and the determination of the amount to invest in each of these assets, we have chosen to focus on what we consider to be the central aspect of the manager's choice process, the selection of risk. Given a valuation model such as the one we use in which the expected return of the assets are functions of their risk, the selection of the risk of the portfolio is the most important decision that the manager makes.

The driving assumption of our model is that the true value of β_i cannot be measured with certainty by the manager. However, we also assume that the manager can reduce this "estimation risk" by providing more effort. For example, the manager may search for more information about the securities in the portfolio and thereby reduce the estimation risk. (See Barry and Brown 1984, 1985 for further discussion.) Formally,

β_i is a random variable with mean b_i and variance b_i^2/a where a is a measure of the manager's effort used in estimating the true β_i.

Thus, we assume that as the mean risk level increases, the manager must expend

[4] Given these conditions we are assuming that the relationship between the investor and the manager is static rather than dynamic. The major rationale for this assumption is the tractability it supplies.

[5] In our model we ignore the ability of the investor or the manager to reduce risk through diversification. The investor is assumed to employ only one manager and the manager is assumed to work for only one investor.

[6] Although investors believe that assets are priced according to CAPM, it is not assumed that CAPM holds absolutely. There must be some type of market imperfection in order for the posited fiduciary relationship to exist. Otherwise, there would be no need for the portfolio manager, and the individual investor could select the portfolio him or herself. Note that our results would not change with a single factor arbitrage pricing model.

[7] This assumption is not terribly restrictive since Farrar (1962) and others have found that most mutual funds are well diversified. Formally, this assumption implies that $r_i = 1$ for all portfolios.

[8] Problems due to the fact that the true market portfolio may not be observable are ignored here.

more effort in order to reduce the variance of the distribution of β_i.[9] The critical assumptions we make are that the manager can reduce the variance of beta by increasing effort, and that increasing the mean of the distribution of beta also increases its variance for a fixed level of effort. While the exact specification of these assumptions is not crucial to our results, the relationship cited above is important to our results. In addition, we assume that the market risk of the portfolio, z, is independent of the estimation risk; that is, z and β_i are independent random variables.

Since the investor cannot observe the true beta of the chosen portfolio, nor the mean and variance of the distribution of beta, the manager's compensation is a function only of what is jointly observable: the ending value of the portfolio. Let $S(x)$ denote the compensation schedule.

Our main purpose in this paper is to describe the incentives created by many existing compensation contracts, not to (necessarily) define an "optimal" contract. Most existing contracts are linear functions of the portfolio returns. For example, the most common contract in current use pays the manager a percentage of the portfolio value. Another popular contract is the symmetric performance contract, which can be written as a linear function of both the portfolio and the market returns.[10] Consequently we restrict our attention to linear contracts.[11]

The manager's utility is assumed to be a function of the effort expended in reducing beta estimation risk as well as the manager's wealth W_M, which itself is a function of the compensation schedule, $S(x_i)$.[12] Specifically, the manager's utility function is additively separable in these variables, concave in the wealth variable and convex decreasing in effort. Similarly, the investor is assumed to have a utility function which is concave in wealth.

Given linear fee contracts, from equations (4) and (1) it can be seen that both the manager's and investor's wealth can be written as linear functions of beta. Thus, for expositional purposes we can talk about the individual's expected utility with respect to beta.

We also restrict our attention to certain types of utility of wealth functions, specifically those whose expectations are concave in the mean, and decreasing and concave in the variance of beta. Defining μ and σ^2 as the mean and variance of beta, respectively, we employ the following expected utility of wealth function:[13,14,15]

[9] This assumption is consistent with the traditional means of estimating beta through time series analysis. In other words, the estimation of the portfolio beta becomes more difficult as the average beta increases. For example, a portfolio with beta close to zero could be achieved with most of the investment in riskfree assets and a few risky assets. Only the risky assets need to have beta estimated. As the desired beta increases, the portfolio must include more risky assets. Thus, the greater the number of assets whose betas must be estimated, the more variance there will be in the portfolio beta.

In addition, as the desired beta increases, the opportunity set decreases. Consequently, there is less information on which to base estimates. Again, the variance of the portfolio beta will be higher. Therefore, as the desired portfolio beta increases, the manager must provide more and more effort to maintain the same variance of the estimate.

[10] See Starks (1987) for a more detailed discussion of the kinds of contracts that are used between investors and mutual fund managers, as well as a discussion of the legal restrictions placed on the kinds of contracts that can be used.

[11] In addition, some portfolio managers (including mutual fund managers) are restricted by law to linear compensation contracts.

[12] In our model, the compensation contract is assumed to be the manager's only source of wealth. Consequently, we are implicitly assuming that the manager has constant risk aversion as base wealth increases.

[13] As our concern here is not with how a particular contract is arrived at, bargaining and negotiation between the investor and the manager is ignored.

[14] In the rest of the paper we delete the subscript i referring to the portfolio. A choice of mean and variance is a choice of portfolio.

[15] Tobin (1958) demonstrated that when (i) returns are normally distributed and utility is concave in

$$E_\beta[U_\cdot(\beta)] = F_\cdot(\mu) - G_\cdot(\sigma^2) \tag{5}$$

for all μ, σ^2; where

(i) there exists a value of μ such that $F'_\cdot = 0$
(ii) $F''_\cdot < 0$
(iii) $G'_\cdot > 0$
(iv) $G''_\cdot \geq 0$
(v) $G_\cdot(0) = 0$.

Equation (5) incorporates both a utility function and a fee contract. It is the result of having taken expectations with respect to both sources of uncertainty: market risk and estimation risk.

With our previous assumption of linear contracts, there are several conceivable utility functions in wealth which would provide the above expectation. One example is the quadratic utility function. However, our results *do not* depend on a quadratic utility of wealth function. In the usual CAPM world (that is when there is no estimation risk and the variance of β is zero), if returns are normally distributed then expected utility is concave in β with an interior maximum. For our model, when $\sigma^2 = 0$, expected utility is assumed to equal $F_\cdot(\mu)$, a concave function of μ with an interior maximum. Since the distribution of returns is assumed to be normal, and the distribution of β is now degenerate at μ, we have the usual CAPM world. The additional structure that we require is that when $\sigma^2 \neq 0$, expected utility can be written as additively separable in the mean and variance of β.

Since managers maximize expected utility, they will not agree to a compensation schedule unless they can be assured the expected utility to be gained from their next best opportunity. Let Y denote the representative manager's minimum required expected utility. If the contract (compensation schedule) is accepted, the manager will choose a mean and a variance (and thus a portfolio) such that the manager's expected utility is maximized.

In the absence of uncertainty about beta, $\sigma^2 = 0$ and utility depends only upon the mean of the degenerate distribution of beta. In this case the manager would choose a value of b such that:

$$F'_M(b) = 0. \tag{6}$$

Denote this value by b_M. Similarly, in the absence of uncertainty about beta and ignoring the reservation wage of the manager, the investor's most preferred beta would solve:

$$F'_I(b) = 0. \tag{7}$$

Denote this value by b_I.

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wealth, or (ii) the utility function is "well-approximated" by a quadratic, expected utility could be completely characterized by the mean and standard deviation of returns. Expected utility will then be increasing in the mean, decreasing in the standard deviation, and will have convex indifference curves. Since (1) $r_f = 1$, the CAPM implies that the mean and standard deviation of returns are both linear in beta and (2) we assume that the contracts are linear in returns, we then know that expected utility is a strictly concave function of beta with an interior maximum. (See Fama 1968.) Any strictly concave function with an interior maximum can be well-approximated by a quadratic. In fact, when utility of wealth is quadratic in wealth, then expected utility with linear contracts will be a quadratic in beta. Thus, our assumptions are consistent with Tobin's conclusions and the CAPM: we need only assume that returns are normally distributed (as we do) or that utility is quadratic in wealth (which we do not) to show that expected utility with respect to the returns generating distribution is well-approximated by a function of beta whose expected value with respect to beta is separable in the mean and variance of beta. This latter conclusion arises from the fact that the expected utility of a quadratic function of a random variable will be separable in the mean and variance of the random variable. This gives us (5i) through (5v) as an immediate consequence.

We define the comparative risk preferences of the manager and the investor as follows.

DEFINITION. The manager will be said to be less, equally, or more risk averse as compared to the investor at (μ, σ^2) as F'_M/G'_M is greater than, equal to, or less than F'_I/G'_I, respectively.

This definition just states that the more risk averse individual require a greater increase in the mean of the beta distribution for an equivalent increase in the variance.

DEFINITION. The manager will be said to be globally less, globally equally, or globally more risk averse as compared to the investor if F'_M/G'_M is greater than, equal to, or less than F'_I/G'_I for *all* (μ, σ^2).

The results of the paper depend on the relationship between the manager's and the investor's risk preferences. In the next section we show that the relationship between the investor's most preferred values of b and a (and thus mean and variance) and the manager's choices depends upon the relative risk preferences of the manager and the investor.

3. Results

The relationship between the risk preferences of the manager and of the investor has an impact on the choices that these individuals would make in the absence of uncertainty. In particular, when $\sigma^2 = 0$, the less risk averse individual chooses a higher value for b, i.e., chooses a less risky portfolio. It is straightforward to demonstrate that if the manager is globally less, equally or more risk averse than the investor, then b_M is greater than, equal to or less than b_I respectively. This relationship is consistent with the usual definition of risk aversion in a CAPM world.

We now move on to an examination of the manager's problem. We assume that the manager's expected utility is separable in wealth and effort where $g(a)$ is the manager's disutility of effort; $g' > 0$ and $g'' > 0$. Given the assumptions of our model the manager will choose a return-risk-effort combination by choosing b and a so as to solve:

$$\operatorname*{MAX}_{b,a} F_M(b) - G_M\left(\frac{b^2}{a}\right) - g(a). \tag{7}$$

The first order conditions for the manager's problem are then:

$$F'_M(b) - G'_M\left(\frac{b^2}{a}\right)\cdot\frac{2b}{a} = 0, \qquad \text{and} \tag{8}$$

$$G'_M\left(\frac{b^2}{a}\right)\cdot\frac{b^2}{a^2} - g'(a) = 0. \tag{9}$$

Since the manager's utility function is concave in b and a, conditions (8) and (9) are both necessary and sufficient. Let (b^*, a^*) denote the solution to the manager's problem. We have implicitly assumed that the manager's expected utility is strictly greater than Y, the manager's reservation wage. If the manager's expected utility at (b^*, a^*) was just equal to Y, any moral hazard problems would be trivially eliminated.

Our first proposition states that without estimation risk the manager would prefer a higher beta portfolio; that is, the mean of the distribution of beta chosen by the manager will be strictly less than the manager's most preferred beta in the absence of estimation risk.

PROPOSITION 1. *$b^* < b_M$ when $a^* > 0$, and variance is positive.*

The intuition behind this result is straightforward. Estimation risk provides another source of risk for the manager's chosen portfolio. Since the manager's risk tolerance takes both estimation risk and systematic risk into consideration, if one is eliminated, the manager is willing to assume more of the other. Thus, if estimation risk does not exist, then the manager will take on more systematic risk. Note that this proposition is not exclusive to the manager, it also holds for the investor. Thus, *ceteris paribus*, both parties will want a riskier portfolio if there is no estimation risk.

We next consider the potential moral hazard problem between the investor and the manager. It can be eliminated if a first-best contract for the investor exists. This will be the case if the solution (b^*, a^*) to the manager's problem is also a solution to:

$$\underset{b,a}{\text{MAX}}\ F_I(b) - G_I\left(\frac{b^2}{a}\right)$$

$$\text{s.t.} \qquad F_M(b) - G_M\left(\frac{b^2}{a}\right) - g(a) \geq Y. \tag{10}$$

We call this the investor's problem and denote its solution by $(\hat{b}, \hat{a})$. The first order conditions for the investor's problem are:

$$F'_I(b) - G'_I\left(\frac{b^2}{a}\right)\cdot\frac{2b}{a} + \lambda\left[F'_M(b) - G'_M\left(\frac{b^2}{a}\right)\cdot\frac{2b}{a}\right] = 0, \tag{11}$$

$$G'_I\left(\frac{b^2}{a}\right)\cdot\frac{b^2}{a^2} + \lambda\left[G'_M\left(\frac{b^2}{a}\right)\cdot\frac{b^2}{a^2} - g'(a)\right] = 0, \tag{12}$$

$$\lambda\left[F_M(b) - G_M\left(\frac{b^2}{a}\right) - g(a) - Y\right] = 0. \tag{13}$$

Again, by concavity, these conditions are both necessary and sufficient. Since the objective function is strictly increasing in a, we know that the constraint will be binding and therefore that $\lambda > 0$. We can then rewrite (13) as:

$$F_M(b) - G_M\left(\frac{b^2}{a}\right) - g(a) - Y = 0. \tag{14}$$

Our second proposition states that given our assumptions there is no contract that eliminates the moral hazard problem inherent in the manager's portfolio selection process. This result holds because (i) the investor is unable to observe the manager's decisions and actions and (ii) the manager's preferences are such that he or she has no incentive to provide the level of effort that would be consistent with the investor's preferences. Thus, there is no way to compensate the manager to ensure that he or she makes a decision that is optimal from the investor's perspective.

PROPOSITION 2. *No first-best optimal contract exists.*

We demonstrate this proposition (in the Appendix) by showing that for all F_I, G_I, F_M and G_M satisfying our initial assumptions, $(b^*, a^*) \neq (\hat{b}, \hat{a})$.

Turning our attention to the impact of existing compensation contracts on the choices made by the manager, we note that a primary concern is the severity of the moral hazard problem inherent in these contracts. One factor which affects the problem is the relationship between the risk tolerance of the manager and that of the investor. We now consider what types of investor-manager combinations mitigate the problem.

If the manager is at least as risk averse as the investor, then a common moral hazard problem occurs in which the manager provides less effort and chooses a less risky portfolio than the investor prefers. The intuition is clear: the manager chooses less effort (smaller a) because the manager must pay for it and the investor need not. This leads the manager to trade off to a lower mean of the beta distribution as well, since that is the only other way to reduce variance.

On the other hand, if the manager is less risk averse than the investor, then conditions exist under which the manager is willing to expend more effort than the investor prefers. In fact, we show that a necessary condition for the manager to provide more than the investor's preferred effort is that the manager is globally less risk averse than the investor.

THEOREM. (1) *If the manager is globally more or globally equally risk averse than the investor, then* $b^* < \hat{b}$ *and* $a^* < \hat{a}$.

(2) *If* $\hat{a} \le a^*$, *then* $\hat{b} \le b^*$ *and the manager is globally less risk averse than the investor.*

The most interesting result can be obtained as a Corollary to the theorem when we place more structure on the manager and investor utility functions.

We assume that

$$F_I(\mu) - G_I(\sigma^2) = \alpha_{I1}\mu^2 + \alpha_{I2}\mu + \alpha_{I1}\sigma^2 + \alpha_{I3} \tag{15}$$

where $\alpha_{I1} < 0$ and $\alpha_{I2} > 0$.

$$F_M(\mu) - G_M(\sigma^2) = \alpha_{M1}\mu^2 + \alpha_{M2}\mu + \alpha_{M1}\sigma^2 + \alpha_{M3} \tag{16}$$

where $\alpha_{M1} < 0$ and $\alpha_{M2} > 0$. The $F.$ and $G.$ functions will have such a form if the expected utility with respect to market uncertainty (systematic risk) is a quadratic function of beta.

When the expected utility has the above representation, the difference in risk preferences between the manager and investor can be characterized *completely* by b_M and b_I. That is, the manager is globally less risk averse than the investor if and only if $b_M > b_I$; globally equally as risk averse if and only if $b_M = b_I$. Under this additional structure, we have the following Corollary:

COROLLARY. (1) *If* $b_M < b_I$, *then* $b^* < \hat{b}$ *and* $a^* < \hat{a}$.

(2) *For each* b_M, *there exist* $\bar{b}_I$ *and* $\bar{\bar{b}}_I$, *where* $\bar{\bar{b}}_I < \bar{b}_I < b_M$, *such that*

(a) *For* $b_I \in (\bar{b}_I, b_M)$, $b^* < \hat{b}$ *and* $a^* < \hat{a}$;

(b) *For* $b_I \in (\bar{\bar{b}}_I, \bar{b}_I]$, $b^* \ge \hat{b}$ *and* $a^* < \hat{a}$; *and*

(c) *For* $b_I \in (-\infty, \bar{\bar{b}}_I]$, $b^* > \hat{b}$ *and* $a^* \ge \hat{a}$.

In no case does the manager make the choice combination that the investor would prefer. The manager either chooses less effort and less risk than the investor would like, less effort and more risk, or more effort and more risk. The last result is a natural consequence of the assumption that, as the beta of a portfolio increases, the estimation risk increases. Thus, the manager must expend more effort to get the same variance around the chosen mean as he or she would have to for a smaller beta. Since the investor desires a smaller beta, he or she will also be satisfied with less effort put into reducing the estimation risk surrounding that beta. Note that in this last case the managerial divergent behavior is not in the choosing of too much effort, but in the choosing of too much risk. Although the investor would be satisfied with less effort, he or she is not averse to the manager putting in more effort than would be required.

What implications do this theorem and Corollary have for the investor in choosing a porfolio manager? Since the contracts in current use do not seem to protect the investor against managerial divergent behavior (considering the compensation schedule only), the investor may consider alternative strategies to mitigate this problem. For example, the investor could choose a manager with greater risk tolerance and then invest a portion of his or her funds into a risk free (unmanaged) portfolio in order to hedge the higher risk the manager has presumably undertaken. In this situation, the investor needs to know the risk preferences of the various portfolio managers in order to choose one with higher risk tolerance. With mutual fund managers such knowledge is possible due to the advertising of the portfolio's investment objective.[16] This objective has been found to be a good indicator of the risk of mutual fund portfolios.[17] The availability of this information concerning the portfolio's risk posture further supports our initial assumption that the investor's have knowledge of the manager's utility function, but not of the manager's actions.

Given our assumptions of linear contracts and quadratic expected utility functions, we can make more definitive statements concerning the investor's preference over managers. For expository purposes, take one of the simplest forms of linear contracts (and the most commonly used form), $s \cdot x$, where the manager receives a percentage of the final market value of the portfolio assets. For any fraction s, suppose the investor could search over different managers and choose one with the desired b_M. Thus, the investor could choose the manager whose b_M is "best" from the investor's perspective; let this "best" b_M be denoted by $\tilde{b}_M$. Since this action is equivalent to choosing b_M (with b_I fixed) so as to maximize the investor's expected utility (under the constraint that the manager maximizes his or her own expected utility), we can make a statement concerning the investor's choice of manager.

PROPOSITION 3. *The investor will choose a manager such that* $\tilde{b}_M > b_I$.

If the investor had the ability to choose any manager for a given contract s, the investor would choose a manager who is globally less risk averse than the investor. Since this holds for every contract s, it must also hold for the second-best optimal contract. That is, since the choice of the second-best contract is just the choice of s, and thus implicitly b_M and b_I, we can make a statement about such a contract.

PROPOSITION 4. *The investor prefers a manager and compensation contract combination in which the manager is globally less risk averse than the investor.*

The intuition behind these propositions is that if the manager is enough less risk averse than the investor, the manager may be willing to supply extra effort in order to make a "more risky" portfolio acceptable to the investor.[18] These results are consistent with Ross' (1979) partial equilibrium analysis of the market for agents in which he shows that less risk averse agents can outbid more risk averse agents in contracting with any risk averse principal.[19]

One aspect of this final result that should be noted is that the difference between the investor's and the manager's preferences over beta depends in part upon the manager's aversion to effort, which is captured in the $g(\cdot)$ function. If the manager is sufficiently less risk averse than the investor, the manager will provide more than the investor's first best level of effort, and choose a portfolio with more systematic risk than the investor's first best choice.

4. Conclusions

In this paper we have shown the impact of estimation risk on the portfolio manager-investor problem when the investor does not have complete information concerning the manager's actions. In our model under the assumptions of the (1) Capital Asset Pricing Model, and (2) estimation risk concerning beta, portfolio managers are able to choose the parameters of the beta distribution. Given this model the managers are

[16] This relationship between fund objective and mutual fund risk has been found in a number of studies. See, for example, Jensen (1969) or Radcliffe (1987, p. 643).

[17] In fact, the Investment Company Act of 1940 requires that mutual fund managers make this disclosure.

[18] Notice that since the investor knows the manager's preferences, the investor then has the ability to reduce his or her own risk exposure by investing more funds in risk-free assets.

[19] However, in Ross' model there is no aversion to effort by the agent.

shown to choose portfolios with less systematic risk than they would if no estimation risk existed. We also show that no linear contract in our world can be first best. That is, the manager will not choose the same amount of effort or risk as the investor would prefer. Since the investor cannot observe the manager's actions, a moral hazard problem exists.

Previous literature in agency theory has generally proved that when a moral hazard problem exists, the agent provides less effort than the principal would prefer. However, our results show that the direction of the moral hazard problem depends on the relationship between the risk tolerance of the manager and that of the investor. That is, there exist some situations in which the manager may provide more effort than the investor would prefer. However, the manager also chooses a riskier portfolio than the investor would prefer. We show that given the most common compensation contract in use today, the investor prefers a manager whose risk choice is higher than the investor's own. Thus, the relationship between these risk tolerances must be an important factor in the selection of a portfolio manager. By taking this relationship into consideration, the investor may be able to mitigate the inherent moral hazard problem.

Although our results are developed under specific assumptions regarding the manager's and investor's utility functions, these assumptions are not as restrictive as they may at first appear. The important assumption is that expected utility is concave in beta. There are a number of reasonable utility functions that are consistent with this assumption. Thus, we believe our results are quite generalizable.[20]

[20] The authors wish to thank the anonymous referees for their comments and suggestions. All errors, however, remain our own.

Appendix

PROOF OF PROPOSITION 1. Since $G'_M > 0$ for positive variance, we have (from the first order conditions (9) and (10)) that $F'_M(b^*) > 0$. Using $F''_M < 0$, we get $b^* < b_M$.

PROOF OF PROPOSITION 2. A contract would be first best if there existed a (b, a) pair that satisfied (8)–(14). Substituting (9) into (12) we have:

$$G'_I\left(\frac{b^2}{a}\right)\cdot\frac{b^2}{a^2} = 0.$$

Since $0 < a < \infty$ (from (9)), either $G'_I(b^2/a) = 0$ or $b^2/a^2 = 0$. If $b^2/a^2 = 0$, then $b = 0$; the mean and the variance of beta are both zero. There is no longer any estimation risk; nor is there any systematic risk. In that case investors do not need managers, since they could just put all of their investment in the risk-free asset. If $b \neq 0$, then $b^2/a > 0$. That gives us $G'_I(b^2/a) > 0$. Therefore, no (b, a) pair exists satisfying (8)–(14).

PROOF OF THEOREM. We first demonstrate some preliminary results. Let $(b_1(a), a)$ solve:

$$F'_M(b_1(a)) - G'_M\left(\frac{b_1^2(a)}{a}\right)\cdot\frac{2b_1(a)}{a} = 0 \qquad (8')$$

and $(b_2(a), a)$ solve:

$$G'_M\left(\frac{b_2^2(a)}{a}\right)\cdot\frac{b_2^2(a)}{a^2} - g'(a) = 0. \qquad (9')$$

Clearly, $b^* = b_1(a^*) = b_2(a^*)$.

It is straightforward to verify that:

$$\frac{db_1}{da} > 0, \qquad \frac{db_2}{da} > 0, \qquad \frac{db_1(a^*)}{da} - \frac{db_2(a^*)}{da} < 0,$$

$$\lim_{a\to 0} b_1(a) = \lim_{a\to 0} b_2(a) = 0, \qquad \lim_{a\to\infty} b_1(a) = b_M,$$

and verification is left to the reader. This give us:

$b_1(a) > b_2(a)$ iff $a < a^*$ and

$b_1(a) < b_2(a)$ iff $a > a^*$.

The investor's first order conditions (where "hats" indicate functions evaluated at $(\hat{b}, \hat{a})$) are:

$$\hat{F}'_I - \hat{G}'_I \cdot \frac{2\hat{b}}{\hat{a}} + \lambda\left[\hat{F}'_M - \hat{G}'_M \cdot \frac{2\hat{b}}{\hat{a}}\right] = 0, \tag{11'}$$

$$\hat{G}'_I \cdot \frac{\hat{b}^2}{\hat{a}^2} + \lambda\left[\hat{G}'_M \cdot \frac{\hat{b}^2}{\hat{a}^2} - \hat{g}'\right] = 0, \tag{12'}$$

$$\hat{F}_M - \hat{G}_M - \hat{g} - Y = 0. \tag{14'}$$

Combining (11') and (12') gives:

$$\hat{g}'\left[\hat{F}'_I - \hat{G}'_I \cdot \frac{2\hat{b}}{\hat{a}}\right] = \frac{\hat{b}^2}{\hat{a}^2}\left[\hat{F}'_I \hat{G}'_M - \hat{G}'_I \hat{F}'_M\right].$$

The term on the right-hand side in brackets is greater than zero, equal to zero, or less than zero as the manager is globally more, globally equally, or globally less risk averse as compared to the investor, respectively.

Since $G'_I > 0$ and $\lambda > 0$, we have that:

$$\hat{G}'_M \cdot \frac{\hat{b}^2}{\hat{a}^2} - \hat{g}' < 0.$$

This gives us the result that $\hat{b} < b_2(\hat{a})$. By assumption of the Theorem: $\hat{F}'_I \hat{G}'_M - \hat{G}'_I \hat{F}'_M \geq 0$. Using $g' > 0$, we get: $\hat{F}'_I - \hat{G}'_I \cdot (2\hat{b}/\hat{a}) \geq 0$. Combining this with (11') and $\lambda > 0$, yields: $\hat{F}'_M - \hat{G}'_M \cdot (2\hat{b}/\hat{a}) \leq 0$. Therefore, $\hat{b} > b_1(\hat{a})$.

Combining $\hat{b} < b_2(\hat{a})$ and $\hat{b} > b_1(\hat{a})$, and using the established properties of $b_1(\cdot)$ and $b_2(\cdot)$, we have our desired result: $\hat{a} > a^*$. This also gives $\hat{b} > b^*$, since both $b_1(\cdot)$ and $b_2(\cdot)$ are increasing in a.

(2) If $\hat{a} \leq a^*$, and b_2 is increasing in a, we get $\hat{b} < b_2(\hat{a}) \leq b_2(a^*)$, or $\hat{b} < b^*$. The rest is just the contraposition of part 1.

PROOF OF THE COROLLARY. (1) Immediate from the Theorem.

(2) Using the newly defined F. and G. functions, we can rewrite the investor's first order conditions as:

$$-2\hat{b}\left(1 + \frac{1}{\hat{a}}\right) + 2b_I + \gamma\left[-2\hat{b}\left(1 + \frac{1}{\hat{a}}\right) + 2b_M\right] = 0, \tag{11''}$$

$$\frac{\hat{b}^2}{\hat{a}^2} + \gamma\left[\frac{\hat{b}^2}{\hat{a}^2} - \bar{g}'(\hat{a})\right] = 0, \tag{12''}$$

$$-\left(\hat{b}^2 + \frac{\hat{b}^2}{\hat{a}}\right) + 2b_M\hat{b} + \bar{\alpha}_{M3} - \bar{g}(\hat{a}) - \bar{Y} = 0, \tag{14''}$$

where $\bar{g} = g/-\alpha_{M1}$, $\bar{\alpha}_{M3} = \alpha_{M3}/-\alpha_{M1}$ and $\bar{Y} = Y/-\alpha_{M1}$.

Combining (11") and (12") we get

$$\hat{b}^2(b_I - b_M) = \bar{g}'(\hat{a}) \cdot \hat{a}^2\left[-\hat{b}\left(1 + \frac{1}{\hat{a}}\right) + b_I\right].$$

We wish to fix b_M (and thus b^* and a^*) and see what happens to $\hat{b}$ and $\hat{a}$ as we change the investor's risk parameter b_I. Totally differentiating (11"), (12") and (14"), and using Cramer's rule to solve, we get

$$\frac{d\hat{a}}{db_I} = \frac{-2((\hat{b}^2/\hat{a}^2) - \bar{g}'(\hat{a}))(-2\hat{b}(1 + (1/\hat{a})) + 2b_I)}{|H|},$$

$$\frac{d\hat{b}}{db_I} = \frac{2((\hat{b}^2/\hat{a}^2) - \bar{g}'(\hat{a}))^2}{|H|},$$

where H is the Hessian matrix of second partial derivatives. Since $|H| > 0$ from concavity, we have $d\hat{b}/db_I > 0$. However, the sign of $d\hat{a}/db_I$ depends on the sign of $-2b(1 + (1/a)) + 2b_I$, which in turn depends on the difference $b_I - b_M$. Using our combined first order conditions and conditions (11") and (12"), we can easily show that if

(a) $b_I > b_M$, then $d\hat{a}/db_I < 0$.

(b) $b_I = b_M$, then $d\hat{a}/db_I = 0$.

(c) $b_I < b_M$, then $d\hat{a}/db_I > 0$.

When $b_I = 0$, the investor wants only risk free assets and does not need the manager. Therefore,

$$\lim_{b_I \to 0} \hat{b}(b_I) = 0, \qquad \lim_{b_I \to 0} \hat{a}(b_I) = 0.$$

We thus know that $\hat{a}(\cdot)$ is a continuous function of b_I that reaches a maximum at $b_I = b_M$. We have already demonstrated that when $b_I < b_M$, $\hat{a}(b_I) > a^* > 0$. Therefore, there exists $\bar{\bar{b}}_I < b_M$ such that $\hat{a}(\bar{\bar{b}}_I) = a^*$ by the intermediate value theorem, and $\hat{a}(b_I) < a^*$ for all $b_I < \bar{\bar{b}}_I$. If $\hat{a} < a^*$, then $\hat{b} < b^*$, which gives us (2)(c) of the Corollary.

Again using the continuity of $\hat{b}(\cdot)$, we know that there exists $\bar{b}_I$, where $\bar{\bar{b}}_I < \bar{b}_I < b_M$, such that $\hat{b}(\bar{b}_I) = b^*$. Additionally, $\hat{b}(b_I) < b^*$, $\hat{a}(b_I) > a^*$ for all b_I such that $\bar{\bar{b}}_I < \bar{b}_I < b_I < b_M$. This gives us (2)(b) of the Corollary. Using the continuity of $\hat{b}(\cdot)$ and $\hat{a}(\cdot)$ again, we know that $\hat{b}(b_I) > b^*$ and $\hat{a}(b_I) > a^*$ for all b_I such that $\bar{b}_I < b_I < b_M$. This is (2)(a) of the Corollary.

PROOF OF PROPOSITION 4. Recall that the manager chooses b and a by solving the first order conditions:

$$-2b\left(1 + \frac{1}{a}\right) + 2b_M = 0, \qquad \frac{b^2}{a^2} - g'(a) = 0.$$

The investor chooses b_M (a manager) by solving the constrained maximization problem:

$$\underset{b_M,a,b}{\text{MAX}} - b^2\left(1 + \frac{1}{a}\right) + 2b_I \cdot b + \bar{\alpha}_{I3},$$

$$\text{s.t.} \quad -2b\left(1 + \frac{1}{a}\right) + 2b_M = 0,$$

$$\frac{b^2}{a^2} - g'(a) = 0.$$

Using the first constraint to solve for b in terms of b_M and a, and substituting into the second constraint and the objective function, we can rewrite the investor's problem as:

$$\text{MAX} - \frac{b_M^2 a^2}{(1+a)^2} \cdot \frac{(1+a)}{a} + 2b_I \frac{b_M a}{(1+a)} + \bar{\alpha}_{I3},$$

$$\text{s.t.} \quad \frac{b_M^2}{(1+a)^2} - g'(a) = 0.$$

The Lagrangian for this problem is then:

$$L(b_M, a, \lambda) = \frac{a}{(1+a)} [b_M(2b_I - b_M)] + \lambda\left[\frac{b_M^2}{(1+a)^2} - g'(a)\right].$$

The first order conditions for the investor's problem are then:

$$\frac{\partial L}{\partial b_M} = \frac{a}{(1+a)} [2b_I - 2b_M] + \lambda\left[\frac{2b_M}{(1+a)^2}\right] = 0,$$

$$\frac{\partial L}{\partial a} = \frac{1}{(1+a)^2} [b_M(2b_I - b_M)] + \lambda\left[-\frac{2b_M^2}{(1+a)^3} - g''(a)\right] = 0,$$

$$\frac{\partial L}{\partial \lambda} = \frac{b_M^2}{(1+a)^2} - g'(a) = 0.$$

Let $(\tilde{b}_M, \tilde{a})$ denote the solution to the rewritten problem. If $\lambda \le 0$, then $b_I - \tilde{b}_M \ge 0$ and $2b_I - \tilde{b}_M \le 0$, or $b_I \ge \tilde{b}_M \ge 2b_I$, which is a contradiction. Therefore, $\lambda > 0$ and $\tilde{b}_M > b_I$, which is our desired result.

PROOF OF PROPOSITION 5. Immediate from Proposition 4 and the definition of a second-best optimal contract.

Bibliography

BARRY, C. AND S. BROWN, "Differential Information and the Small Firm Effect," *J. Financial Economics*, 13 (June 1984), 283–294.

——— AND ———, "Differential Information and Security Market Equilibrium," *J. Financial and Quantitative Anal.*, 20 (December 1985), 407–422.

FAMA, E., "Risk, Return and Equilibrium: Some Clarifying Comments," *J. Finance*, (March 1978), 29–40.

FARRAR, D., *The Investment Decision Under Uncertainty*, Prentice-Hall, Englewood Cliffs, NJ, 1962.

HARRIS, M. AND A. RAVIV, "Some Results on Incentive Contracts with Applications to Education and Employment, Health Insurance, and Law Enforcement," *Amer. Economic Rev.*, (March 1978), 20–30.

JENSEN, M., "Risk, the Pricing of Capital Assets, and the Evaluation of Investment Portfolios," *J. Business*, 42 (April 1969), 167–247.

——— AND W. MECKLING, "Theory of the Firm: Managerial Behavior, Agency Costs, and Ownership Structure," *J. Financial Economics*, 3 (October 1976), 305–360.

RADCLIFFE, R., *Investment Concepts, Analysis and Strategy*, Second Ed., Scott Foresman and Co., 1987.

ROSS, S., "Equilibrium and Agency-Inadmissable Agents in the Public Agency Problem," *Amer. Economic Review*, 69 (May 1979), 308–312.

SHAVELL, S., "Risk Sharing and Incentives in the Principal and Agent Relationship," *Bell J. Economics*, 10 (Spring 1979), 55–73.

SPENCE, M. AND R. ZECKHAUSER, "Insurance, Information, and Individual Action," *Amer. Economic Rev.*, 61 (May 1971), 380–387.

STARKS, L., "Performance Incentive Fees: An Agency Theoretic Approach," *J. Financial and Quantitative Anal.*, (March 1987).

STIGLITZ, J., "Risk Sharing and Incentives in Sharecropping," *Rev. Economic Studies*, 2 (April 1974), 219–256.

TOBIN, J., "Liquidity Preferences as Behavior Towards Risk," *Rev. Economic Studies*, (February 1958), 65–86.

ZECKHAUSER, R., "Medical Insurance: A Case Study of the Tradeoff Between Risk Spreading and Appropriate Incentives," *J. Economic Theory*, 2 (March 1970), 10–26.