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An Equilibrium Model of Asset Trading with Sequential Information Arrival

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ABSTRACT

In an effort to better understand the dynamic market price adjustment process, this paper develops a model which describes the impact of new information on a financial market. The primary emphasis is on the price change-volume relationship in the presence of a margin requirement. We find that the margin requirement significantly affects the relation of price change to volume. Furthermore, this relationship is shown to be affected by the number of investors in the market, the degree of information dissemination, differences in interpretation of information and the implicit cost of the margin requirement.

CAPITAL ASSET PRICING AND market efficiency have become predominant themes in the financial literature of the past decade. Much of this work does not specify the effect of information on investor expectations. Indeed, according to Ross,

> Neither the arbitrage theory nor any other theory (of asset pricing) has made a serious attempt to describe the disequilibrium dynamic adjustment of ex post observations to ex ante assumption. Understanding the impact of information will be a prerequisite for such an analysis and of great interest in its own right [14, p. 211].

The study of information effects includes how information is received and processed by an agent, how revised beliefs are transformed into action by the agent, and how the market reacts to this shock. This paper develops a model describing the adjustment of an asset market to new information via changes in investors' expectations. The emphasis will be on the information's impact on asset prices and trading volume.

The sequential information arrival model developed by Copeland [2] offers a framework for the study of this dynamic adjustment process. This paper will modify the Copeland model by using portions of Mossin's [9] equilibrium analysis. The sequential information arrival process begins with the asset market in equilibrium. A single item of information then arrives at the market. In previous informational studies using equilibrium analysis, all market participants are assumed to become informed simultaneously (see [8] for a typical discussion).

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The sequential information arrival model assumes that only one trader observes the information initially. This trader interprets the news, revises his beliefs, and trades to arrive at a new optimal position. The outcome of this series of events is the generation of transaction volume and a new equilibrium price. After the market arrives at this new equilibrium, the next investor becomes informed and, after a similar sequence of events, a second temporary equilibrium is achieved. This process continues until all traders are informed and results in a series of momentary equilibria. When the last trader receives the information, the market reaches a final equilibrium. The sequential process allows one to observe the path of trades, prices, and volume. In addition this model provides a more realistic model for most information events. (See Patell and Wolfson [11] for an empirical study on the speed of information dissemination).

The model in this paper differs from other information arrival models, e.g., [1], [2], and [3], in that the market adjustment process is formulated in an equilibrium analysis derived from a market where each investor maximizes expected utility of terminal wealth under uncertainty. Sections I and II develop an equilibrium model which includes a margin requirement to restrict short sales. The price change–volume relationships predicted by this sequential information arrival model are studied in Section III. Section IV examines some existing empirical evidence that supports the relationships predicted by this model. The final section of the paper contains some concluding observations and a summary.

I. The Equilibrium Model

The market consists of M traders with identical endowments, beliefs, and preferences[1] who participate in a pure exchange economy[2] with one risky and one risk-free asset. (Once the information dissemination process begins, however, individuals do not have homogeneous beliefs). The risky asset may be thought of as the common stock of an all-equity corporation, the risk-free asset as lending among traders. All intertrader lending is done at the risk-free rate of interest, ($r - 1$), which is exogeneous to the model. The market participants are assumed to receive no information from equilibrium prices and to trade only at equilibrium prices. The further assumptions of costless information,[3] unlimited borrowing and lending, and no taxes or explicit transactions costs are also made. An implicit transaction cost is imposed by the introduction of a margin requirement. The margin requirement restricts short sales so that the impact of this restriction on price change–volume relationships can be examined. The short sales constraint is in contrast with the assumption of unlimited short sales in many equilibrium analyses or the complete exclusion of short sales as in Copeland's model. Thus,

[1] The model is not dependent on the assumptions of initial homogeneous beliefs, identical utility functions, and equal endowments. These assumptions serve to simplify the analysis.

[2] This means that the model takes as given the firm's decision as to investment and financing. This restriction obviously eliminates a large portion of the information that is commonly evaluated in the market. The assumption is made to avoid the addition of production to the analysis that would be implied by its relaxation.

[3] Under perfect capital markets, information is equally accessible to all traders. To talk about sequential information arrival is to violate this condition; however, we do retain all other implications of this assumption.

if an investor wishes to go short, he not only is denied the use of the proceeds from the sale, but he must also put up the margin requirement with the broker until he covers this short position. The special case of unrestricted short sales can be modeled by the elimination of the margin requirement.

Every transaction that occurs, whether long or short, is assumed to make use of the margin mechanism. This is, admittedly, a simplification; but under constant absolute risk aversion, all investors with identical beliefs would agree on whether to use their margin accounts. Thus, the requirement is not as limiting as it first appears. The funds borrowed from the broker by those investors taking a long position in the risky asset require the payment of the risk-free interest rate, the same as those funds borrowed from other investors. This assumption is not crucial to any of the results. The only assumption necessary to make the analysis similar to Copeland's is that there are differential transactions costs between the long and short positions, with the cost associated with a short position being greater. Allowing the investor taking a long position to borrow from the broker at the risk-free rate merely implies that he faces zero transactions costs.[4]

Each investor's wealth is defined by,

$$W_i = \bar{m}_i + \bar{Z}_i p - \bar{B}_i = m_i + Z_i p - B_i, \qquad (i = 1, \ldots, M) \tag{1}$$

where

W_i = investor i's pretrading wealth;
m_i = investor i's posttrading holding of the risk-free asset;
Z_i = investor i's posttrading holding of the risky asset;
B_i = the posttrading amount of margin borrowing of investor i;
p = the risky asset value in equilibrium;
and variables with bars denote pretrading endowments.

End-of-period wealth for an investor who assumes a long position in the risky asset is

$$\tilde{Y}_i = rm_i + Z_i\tilde{X} - rB_i, \tag{2}$$

where

$\tilde{X}$ = the end-of-period value for the risky asset. (This random variable is assumed not to be realized until after all of the trading caused by the current information event has taken place. For example, investor i's beliefs about $\tilde{X}$ may reflect a planning horizon of one year while the effects of the information currently in the market only last a short time. Investors are assumed to agree on the variance of $\tilde{X}$, S, but may have different assessments of the expectation, E_i.)

Substituting for m_i from Equation (1),

$$\tilde{Y}_i = rW_i + Z_i(\tilde{X} - rp) - B_i(r - r)$$

[4] Since the thrust of this paper is concerned with investor, not broker, behavior, another reason to assume risk-free borrowing from the broker is to avoid specifying the procedure used by the broker in setting the different rate. In this spirit, the additional assumption is made that the broker does not exhibit any optimizing behavior. He simply lends to all taking a long position and collects from all investors desiring a short position. In order to do this, the broker begins with a large endowment sufficient to satisfy the needs of all investors. Thus, the analysis is a partial equilibrium but preserves the desired emphasis on the investor.

$$= rW_t + Z_t(\tilde{X} - rp). \tag{3}$$

Notice that if the broker is allowed to charge a rate of interest different from the riskless rate, that alternative interest rate would appear in (2) and the last term in (3) would not drop out. Instead it would act as a transaction cost reducing investor wealth.

End-of-period wealth for an investor selecting a short position in the risky asset differs from that of the long investor since the broker holds the short sale proceeds as well as the margin requirement on the sale without paying interest. Thus, the end-of-period wealth for the investor choosing a negative position in the risky asset is

$$\tilde{Y}_t = rm_t + Z_t\tilde{X} - B_t. \tag{4}$$

Again, substituting for m_t from (1) gives

$$\tilde{Y}_t = rW_t + Z_t(\tilde{X} - rp) + B_t(r - 1). \tag{5}$$

In this case, $B_t = Z_t p(1 + d)$ where d is the fractional margin requirement. Substituting this into (5), and defining $K = r - (1 + d)(r - 1)$ gives

$$\tilde{Y}_t = rW_t + Z_t(\tilde{X} - Kp). \tag{6}$$

With the short position, it is immaterial that the broker is assumed to lend at the risk-free rate. What is important is that the investor must forgo the risk-free income he would earn without the margin requirement, $Z_t p(1 + d)(r - 1)$. Thus, the transaction cost takes the form of an opportunity cost.

The first order condition for the maximization of expected utility of final wealth (see Mossin [9, p. 46]), is $E[U'(\tilde{Y})(\tilde{X} - rp)] = 0$, for the investor with a long position in the risky asset and, $E[U'(\tilde{Y})(\tilde{X} - Kp)] = 0$, for the investor with a short position. Assuming all individuals have exponential utility functions, $U_t = -\exp\{-C\tilde{Y}_t\}$, and that $\tilde{X}$ is normally distributed, the optimal holding of the risky asset for the investor with a long position is,

$$Z_t = (E_t - rp)/CS \tag{7}$$

[5,6]

and for the investor with a negative position the optimal holding is,

$$Z_t = (E_t - Kp)/CS. \tag{8}$$

[5] We provide a demonstration of the derivation of Equation (7). The derivation of (8) is similar. Let final wealth be $\tilde{Y} = rW + Z(\tilde{X} - rp)$, and the utility function be $U(Y) = -\exp[-CY]$. To maximize expected utility is to $\max_z E[-\exp(-C\tilde{Y})]$, or equivalently,

$$max_z[E(Y) - (C/2)\text{var}(\tilde{Y})], \quad \text{where}$$
$$E(\tilde{Y}) = rW + Z(E - rp) \quad \text{and}$$
$$\text{var}(\tilde{Y}) = Z^2S.$$

Thus, the expression becomes

$$\max_z[rW + Z(E - rp) - (C/2)Z^2S] \quad \text{and} \quad \frac{d}{dZ} = (E - rp) - ZCS,$$

which is set equal to zero.

[6] As expected, the optimality condition is independent of investor wealth due to the constant absolute risk aversion property of the exponential utility function.

The final conditions for equilibrium in this model are the market clearing conditions (with one being redundant)

$$\sum_{i=1}^{M} Z_i = 1 \tag{9}$$

and

$$\sum_{i=1}^{M} (m_i + B_i) = 0. \tag{10}$$

Thus, the model consists of $2M + 1$ independent equations, M demand equations, (7) or (8), M budget constraints, Equation (1), and one clearing condition, Equation (9) or (10).

The investor's demand curve consists of three segments: one segment for positive holdings of the risky asset; another for zero holdings of the risky asset; and a third segment for negative risky asset holdings. Those three portions of the demand curve are defined by the first order conditions for the maximization of expected utility, $E[U'(\tilde{Y})(\tilde{X} - rp)] = 0$ and $E[U'(\tilde{Y})(\tilde{X} - Kp)] = 0$. A necessary and sufficient condition for $Z_i > 0$ is $(E_i/r) > p$, and for $Z_i < 0$ is $(E_i/K) < p$. So the investor does not hold the risky asset when $(E_i/r) < p < (E_i/K)$. Therefore, the investor's demand curve may be written

$$Z_i = (E_i - rp)/CS \qquad \text{if} \quad E_i > rp;$$

$$Z_i = 0 \qquad \text{if} \quad Kp < E_i < rp;$$

and

$$Z_i = (E_i - Kp)/CS \qquad \text{if} \quad E_i < Kp.$$

Thus, investors' demand curves are inversely proportional to equilibrium price with the slope of the negative portion of the curve being absolutely less than the positive segment. This difference in slope means that, for a given change in price, an investor with a short position in the risky asset alters his portfolio holdings less than an individual with a long position. Note that the size of this slope differential as well as the range of prices over which the investor refuses to hold the risky security depends on the relative values of K and r.

II. Sequential Information Arrival

The Sequential Information Arrival Model (SIAM) is developed within the framework of the analysis of the previous section. The SIAM begins in an equilibrium situation in which all investors are satisfied with their portfolio holdings. A single item of information then becomes available to the market. However, in contrast to many previous analyses, only one investor at a time is allowed to receive the data. Based on the news, this single market participant alters his belief concerning the expected value of the distribution for the end-of-period price of the risky security. (In this simplified model it is assumed that the investor's estimate of the variance of this distribution remains unaffected by the information). The trader's expectation moves either up or down by a fixed amount, ΔE. An investor increasing his expectation is designated an optimist, while a pessimist is an investor who decreases his assessment. After this alteration of his belief the newly informed investor is no longer satisfied with his portfolio position. He therefore enters the market and trades until a new equilibrium obtains. This results in a new market clearing price and generates trading volume. The sequence of events is repeated for each of the M traders. That is:

The financial market is in an initial equilibrium.
Step 1a: The first trader receives the information.
1b: This trader alters his beliefs and retrades.
1c: The financial market reaches equilibrium.
Step 2a: The second trader receives the information.
2b: This trader alters his beliefs and retrades.
2c: The financial market reaches equilibrium.
⋮
Step Ma: The last trader receives the information.
Mb: This trader alters his beliefs and retrades.
Mc: The financial market reaches equilibrium.

The process results in $M + 1$ equilibrium prices: an initial equilibrium prior to information arrival and M additional equilibria, one after each trader receives the news. Each of the equilibria, with the exception of the final equilibrium, is only temporary. These momentary equilibria exist only until the next investor receives the information.

Allowing for the sequence of market clearing prices the investor's budget constraint, Equation (1), and the trader's final wealth, Equations (3) or (6), may be written as:

$$W_{ij} = m_{i,j-1} + Z_{i,j-1}p_j - B_{i,j-1} = m_{ij} + Z_{ij}p_j - B_{ij}, \tag{1a}$$

$$\tilde{Y}_{ij} = rW_{ij} + Z_{ij}(\tilde{X} - rp_j), \qquad Z_{ij} \geq 0, \tag{3a}$$

and

$$\tilde{Y}_{ij} = rW_{ij} + Z_{ij}(\tilde{X} - Kp_j), \qquad Z_{ij} < 0, \tag{6a}$$

where the j subscript denotes values occurring when the jth individual becomes informed ($j = 1, \ldots, M$).

Likewise the expressions for the optimal position in the risky asset, Equations (7) and (8), become

$$Z_{ij} = (E_i - rp_j)/CS \tag{7a}$$

and

$$Z_{ij} = (E_i - Kp_j)/CS. \tag{8a}$$

Since the equilibrium price changes each time any investor receives the information, all market participants alter their portfolio positions at each trading opportunity according to their demand curve, Equation (7a) or (8a). This means that investor i is continually altering his holdings of the risky security in response to changes in the price of that security. Thus the clearing conditions, Equations (9) and (10), are imposed for each temporary equilibrium. The remainder of the paper assumes that the variables without the j subscript refer to current values. Figure 1 illustrates the investor demand curve relationships.[7]

III. Results

In order to introduce the relatively complex relationships implied by the model developed in the previous sections, we initially eliminate the margin requirement, i.e., $B_{ij} = 0, \forall\ i, j$.[8] The market consists of three types of investors: (1) n uninformed investors with expectations E_u; (2) j informed investors who are optimists with expectations E_o; and (3) k pessimists with expectations E_q. Equilibrium occurs when $nZ_u + jZ_o + kZ_q = 1$ or

$$nE_u + jE_o + kE_q - Mrp = CS. \tag{11}$$

From (11) equilibrium price is

$$p = (1/rM)(nE_u + jE_o + kE_q - CS) = 1/r[\bar{E} - CS/M] \tag{12}$$

where $\bar{E}$ is the average expectation.

The holdings of a single uninformed investor are $Z_u = (1/CS)[E_u - \bar{E}] + (1/M)$. If one of the uninformed investors becomes informed and alters his assessment of the expected future price by ΔE, the new equilibrium price can be derived by a process analogous to the derivation of Equation (12), $p' = (1/r)[\bar{E} + \Delta E/M - CS/M]$ giving a price change of

$$\Delta p = \Delta E/rM. \tag{13}$$

The volume generated by a single uninformed individual becoming informed may be written as

$$\begin{aligned} V &= [(E_u + \Delta E - rp')/CS] - [(E_u - rp)/CS] = [\Delta E - r(\Delta p)]/CS \\ &= (\Delta E/CS)[1 - (1/M)]. \end{aligned} \tag{14}$$

Equations (13) and (14) indicate that the volume and price change caused by a single investor depend only on the total number of traders, not on the composition of optimists and pessimists.

These results do not change if the assumptions of homogeneous variance estimates and identical utility functions are relaxed. It is shown in Appendix A that even when individual investors have different variance estimates and risk aversion indices, volume, and the change in price are independent of the mix of optimists and pessimists.

This independence result holds only as a single investor changes his assessment. The aggregate price change does depend on the mix of optimists and pessimists

[7] These and all other numerical results in this paper assume:

$E_i = 2.00$ for an uninformed investor; $E_i = 2.25$ for an optimistic investor;
$E_i = 1.75$ for a pessimistic investor; $r = 1.05$; $C = 0.95$; $S = 0.35$; and $d = 0.5$.

[8] For the special case in which there are unrestricted short sales, the investor's budget constraint is $W_i = \bar{m}_i + \bar{Z}_i p = m_i + Z_i p_i$ and end-of-period wealth is $\tilde{Y}_i = rW_i + Z_i(\tilde{X} - rp)$, for all investors. Thus traders with both long and short positions in the risky asset share the same demand curve, $Z_i = (E_i - rp)/CS$.

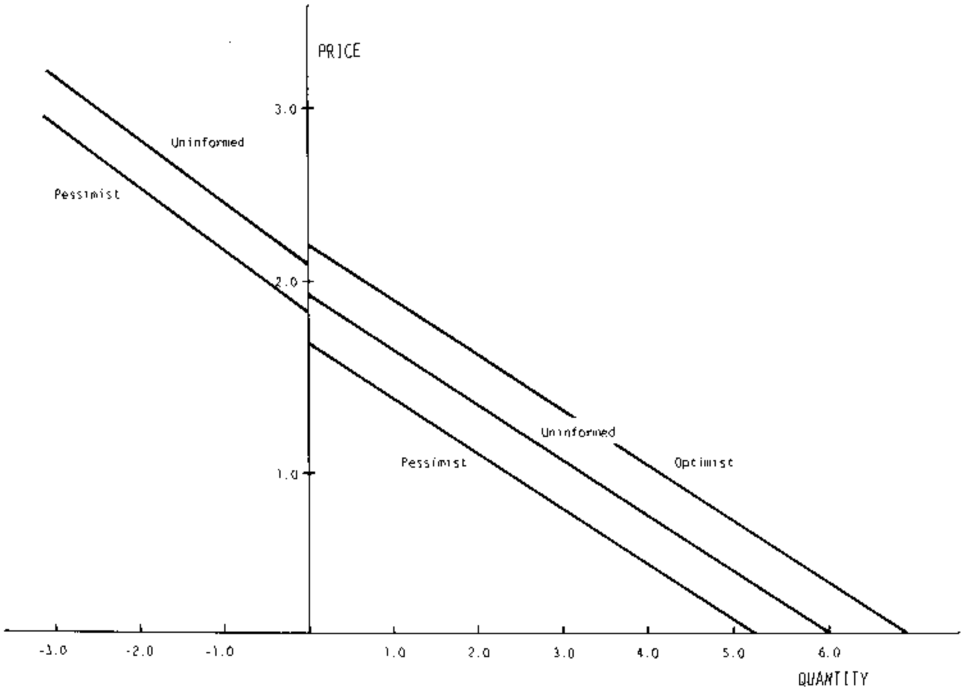

Figure 1. Investor Demand Curves

since optimists increase price and pessimists depress price. Without a margin requirement, trading by exponential utility maximizers causes the maximum *absolute* total price change to occur when traders are either all optimists or all pessimists. This is easily seen by the use of Equation (12). The initial equilibrium price is $p_0 = [E_u - CS/M]/r$ and the final equilibrium price is $p_M = [\bar{E}_M - CS/M]/r$. Price change, $|p_M - p_0|$, is determined by the difference between the uninformed expectation, E_u, and the average final expectation, $\bar{E}_M$. The maximum divergence between E_u and $\bar{E}_M$, therefore the largest $|\Delta P|$, occurs when all investors agree on the implications of the information. The minimum price change occurs with a mix of optimists and pessimists such that $\bar{E}_M = E_u$. Thus, the net price change is zero when there are an equal number of optimists and pessimists.[9]

Unlike the total change in price, total volume is independent of the fraction of optimists, since investors' demand curves (uninformed, optimistic, and pessimistic) are parallel and an equal distance apart. Thus, the volume generated by an optimist (V_o) is the same as that of a pessimist (V_p) for a given price change, and volume depends only on the number of traders, not on their composition.

The introduction of the margin requirement causes a significant complication of the market. Any analysis of equilibrium prices or volume must include a discussion of which investors have taken long, short, or zero positions in the risky asset. The various combinations of investor types (optimist, pessimist, unin-

[9] We would like to thank the editor for suggesting this simplified argument.

formed)and risky asset positions, along with their relevant boundary conditions on price, are summarized in Table I. Thus, as the sequential information arrival process unfolds, the market moves into and out of these situations depending on the ordering of optimists and pessimists and their effect on equilibrium price.

With the margin requirement it is virtually impossible to obtain any closed-form analytical results; therefore, a series of simulations were conducted. To solve the equilibrium conditions, a computer program was developed to search for the market clearing price for the risky asset. First, an initial equilibrium, with homogeneous expectations was derived. Then, as each trader received the new information and revised his beliefs, the program found the lending position and risky asset holdings of every investor. (These portfolio positions depend on the new market clearing price for the risky asset). This process yields M equilibria for each iteration of the simulation, one equilibrium as each investor becomes informed and rebalances his portfolio.

The computer program easily handles the information arrival and sequence of market situations. Therefore, it is used to demonstrate most of the results dealing with the margin model. The input parameters in this analysis were the number of traders, their initial beliefs, and the probability of a newly-informed individual revising his beliefs upward (π). The number of investors is set at 10, 20, 30, . . ., 100, and the probability of being an optimist at 0, .1, .2, . . ., 1.0. For each of the 110 simulations, the program derives the initial equilibrium with homogeneous beliefs, endowments, and preferences. Then, as each individual receives the information and revises his beliefs, a new equilibrium is established.

Figure 2 illustrates typical simulation results for the margin model. With the margin requirement, the maximum price change occurs when all traders interpret the information in an identical manner. This result occurs for the same reason as

Table I

Market Situations and Price Boundary Conditions for Each Investor-Type Under the Margin Model

	Optimist[a]	Uninformed[a]	Pessimist[a]	Boundary Conditions	Numeric Example
1.	+	+	+	$\frac{E_q}{r} > p$	$1.67 > p$
2.	+	+	0	$\min\left[\frac{E_q}{K}, \frac{E_u}{r}\right] > p > \frac{E_q}{r}$	$1.79 > p > 1.67$
3.	+	+	−	$\frac{E_u}{r} > p > \frac{E_q}{K}$	$1.90 > p > 1.79$
4.	+	0	0	$\min\left[\frac{E_q}{K}, \frac{E_o}{r}\right] > p > \frac{E_u}{r}$	$1.79 > p > 1.90$[b]
5.	+	0	−	$\min\left[\frac{E_u}{K}, \frac{E_o}{r}\right] > p > \max\left[\frac{E_q}{K}, \frac{E_u}{r}\right]$	$2.05 > p > 1.90$
6.	+	−	−	$\frac{E_o}{r} > p > \frac{E_u}{K}$	$2.15 > p > 2.05$

[a] +, 0, − signify $Z_i > 0$, $Z_i = 0$, $Z_i < 0$, respectively.

[b] Clearly this is not possible with the example selected.

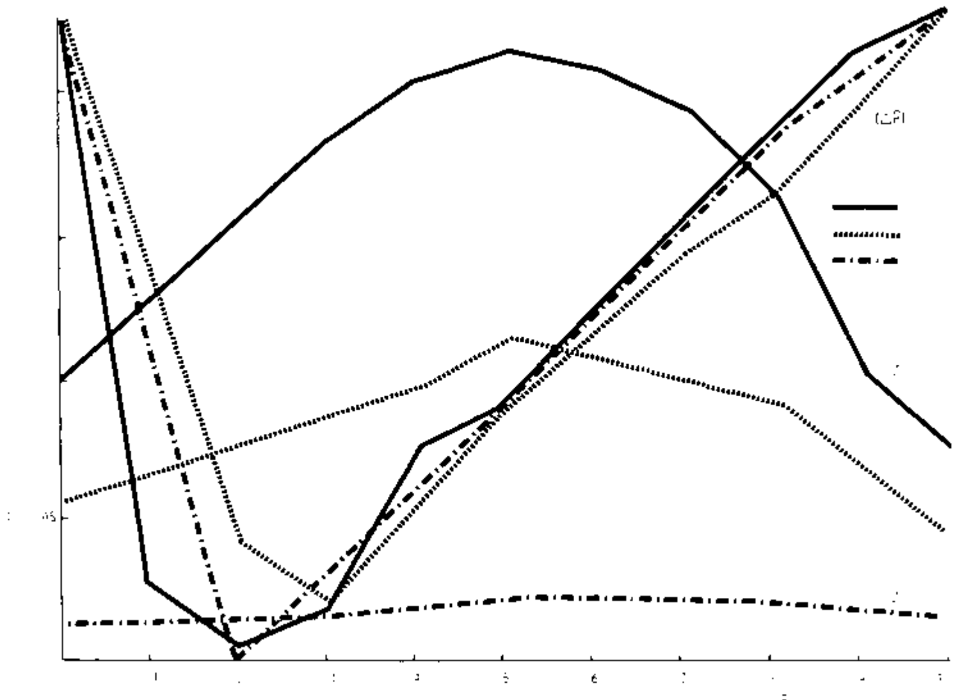

Figure 2. Price Change—Volume Relationships

it occurs in the no margin model: when all investors revise their beliefs in the same direction, all individual price changes have the same algebraic sign. Consequently, no offsetting price changes result. The addition of a margin requirement does alter the point of the minimum price change. As can be seen in Figure 2, the minimum price change in the margin model occurs at a percentage of optimists which is less than 50 percent. This is because it now costs more to take a short position than a long position. In this model, the investor who desired to take a short position has an opportunity cost, $(1 + d)(r - 1)$, which causes him to take a less extreme position than he would without a margin requirement; on the other hand, the long investor is unaffected (in the general case, less affected) by the margin. Thus, the arrival of a pessimist has less of an impact on equilibrium prices than does the arrival of an optimist and, consequently, it takes less than a one-to-one ratio of optimists to pessimists to cause minimum price change. The price change curve is then monotonically increasing at optimist levels above that point since there will be fewer offsetting price changes.

Figure 2 also shows that the addition of a margin requirement causes a significant change in the volume generated by different mixes of optimists and pessimists. Volume is now a concave function of the percentage of optimists. This volume curve may be given an intuitive explanation with the aid of Figure 3. For low prices, (those below point P), the volume generated by the arrival of an optimist is always at least as great as the volume generated by a pessimist. Since individual price changes are very small when there are a large number of investors, volume can be approximated by the horizontal distance between the uninformed

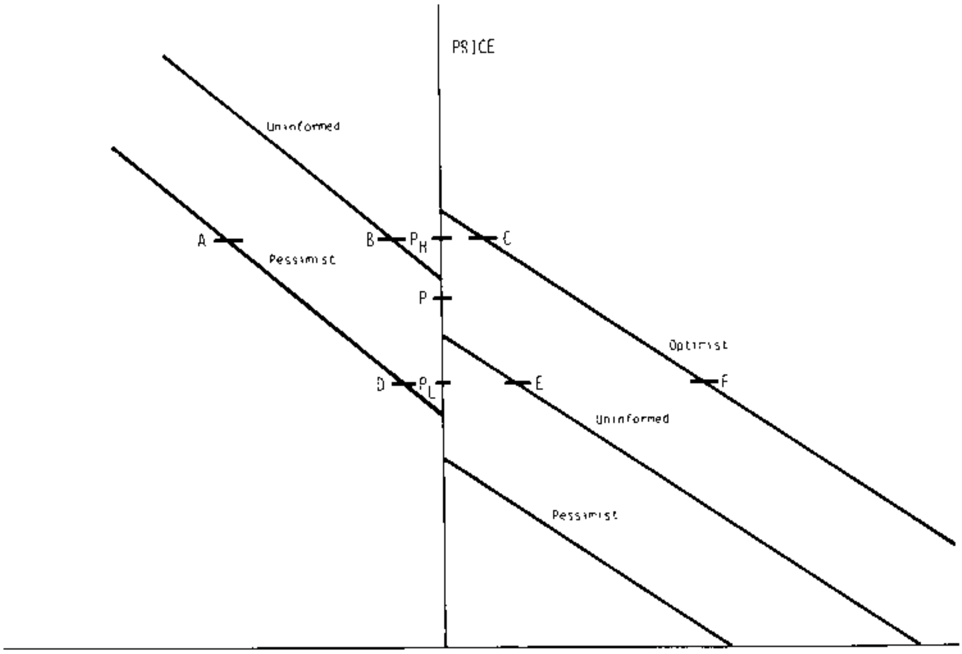

Figure 3. Individual Investor Volume at Different Price Levels

investor's demand curve and the informed (optimist or pessimist) investor's demand curve. Thus, at point P_L the volume generated by a pessimist, distance DE, is less than that generated by an optimist, EF. At prices greater than P, however, the volume of a pessimist exceeds that of an optimist. For example, at point P_H, $AB > BC$. When there are a large number of pessimists, the price tends to be depressed, so the volume of an optimist is greater than that of a pessimist. Increasing the percentage of optimists, then, increases volume, but at some point the number of optimists becomes large enough that the equilibrium price is consistently above P, and increasing the percentage of optimists now decreases volume.

Even with margin requirements, the total price change, $|\Delta P|$, is independent of the order of arrival of the optimists and pessimists, since only the initial and final equilibrium prices are important in the calculation of $|\Delta P|$. With exponential utility both of these quantities depend only on the number of optimists, uninformed traders and pessimists, their individual beliefs and risk aversions, and market parameters. The volume generated by information arrival is dependent on the ordering of optimists and pessimists because of the discontinuities and differences in slopes of investors' demand curves introduced by the margin requirement. Excluding the extreme points ($\pi = 0, 1$) of the volume-optimist curve, there are a large number of adjustment paths which may be taken by this sequential information arrival model.[10] Each path may have a different probability

[10] For a given number of optimists, say j^*, the number of price adjustment paths is

$$\binom{M}{j^*} = \frac{M!}{j^*!(M-j^*)!}.$$

and volume. The expected volume of trading activity is

$$E(V) = \sum_{i=1}^{M} [\pi(\sum_{j=1}^{6} \sum_{k=1}^{6} \phi_{jk} V_{jk}^{0}) + (1 - \pi)(\sum_{j=1}^{6} \sum_{k=1}^{J} \phi_{jk} V_{jk}^{p})],$$

where

ϕ_{jk} = the probability of moving from state j (Table I) to state k;
V_{jk}^{0} = the volume generated by the arrival of an optimist that moves the market from state j to state k; and
V_{jk}^{p} = the volume generated by the arrival of a pessimist that moves the market from state j to state k.

The expression $\sum_j \sum_k \phi_{jk} V_{jk}^{0}$ is the expected volume of an optimist and $\sum_j \sum_k \phi_{jk} V_{jk}^{p}$ is the expected volume of a pessimist.

As a first approximation to this expectation, we can develop approximate bounds on the volume as the number of optimists is altered by making some general assumptions concerning investor behavior. When the relative number of pessimists are large, the equilibrium price remains at a low level. This means that optimists and uninformed investors will assume a long position while pessimists take a short position. Using the same methodology as in the no margin case, the holdings of an uninformed investor are

$$Z_u = (1/CS)\left[E_u - r\left\{\frac{nE_u + jE_o + kE_q - CS}{(n+j)r + kK}\right\}\right].$$

Allowing the next investor receiving the information to be optimistic results in an optimist holding of

$$Z_o = (1/CS)\left[E_u + \Delta E - r\left\{\frac{nE_u + jE_o + kE_q - CS + \Delta E}{(n+j)r + kK}\right\}\right]$$

and a volume of

$$V_o = Z_o - Z_u = \frac{\Delta E}{CS}\left[1 - \frac{r}{(n+j)r + kK}\right]$$

If the next investor had been a pessimist,

$$V_p = Z_u - Z_p = (1/CS)\left[\Delta E - r\left\{\frac{nE_u + jE_o + kE_q - CS}{(n+j)r + kK}\right\} + K\left\{\frac{nE_u + jE_o + kE_q - CS - \Delta E}{(n+j)r + kK + K - r}\right\}\right].$$

To support the intuitive explanation given earlier, it must be shown that $V_o > V_p$. This proof is offered in Appendix B. Thus, when π is small, the addition of an extra optimist will increase volume.

On the other hand, if the number of pessimists is relatively small, the asset price will be forced to a high level. In general, this means that both pessimists and uninformed investors take a short position and only optimists have positive risky asset holdings. Therefore,

$$V_o = \frac{1}{CS}\left[\Delta E - r\left\{\frac{nE_u + jE_o + kE_q - CS + \Delta E}{jr + (n+k)K + r - K}\right\} + K\left\{\frac{nE_u + jE_o + kE_q - CS}{jr + (n+k)K}\right\}\right]$$

and

$$V_p = \frac{1}{CS}\left[\Delta E - K\left(\frac{\Delta E}{jr + (n+k)K}\right)\right]$$

Appendix C shows that now $V_o < V_p$. This implies that when π is large, the addition of another optimist decreases total volume.[11]

Three conditions are required in order for the proofs in Appendices B and C to hold. The first is that the points of intersection of the various demand curves have the same relative positions as in Figure 3 (i.e., $(E_q/K) < (E_u/r)$ and $(E_u/K) < (E_o/r)$. Second, the magnitude of the change in expectations is the same whether the uninformed becomes an optimist or a pessimist. Finally, the uninformed investors have the relative holdings assumed in each situation (i.e., long for Appendix B and short for Appendix C). The first condition may not hold for small values of ΔE or large values of d and r (see Figure 4). In these cases the volume of an optimist is always greater than that of a pessimist. This result is consistent with Copeland's and comes about because, under the circumstances illustrated in Figure 4, there are no short sales.

Together the results of Appendices B and C illustrate that under fairly general assumptions regarding the asset holdings of each investor type, the concave volume curve depicted in Figure 2 is typical. That is, as long as equilibrium price remains below a certain level, the *replacement* of a pessimist with an optimist increases total volume. As the fraction of optimists is increased, equilibrium price is forced upwards and the volume added through the switch of an optimist for a pessimist decreases. After the price reaches a high enough level, the continued substitution of optimists depresses volume. This means that maximum volume occurs near the point of maximum disagreement ($\pi = .5$)

[11] Another approach to the proof of Appendix C is to directly compare the horizontal distance between the demand curves. (Assuming a large number of investors so that the price change is minimal, this distance will approximate volume). The difference between the pessimistic and uninformed demand curves when both are in the negative quantity quandrant is relatively easy since these two curves are parallel. From Equation (7), the demand equations are

$$Z_q = (E_q - Kp)/CS \qquad \text{and,} \qquad Z_u = (E_u - Kp)/CS.$$

Thus, the distance between the two is $\Delta E/CS$. The optimist's demand curve and the short position of the uninformed investor's demand curve are not parallel. The demand equation for the optimist is, from Equation (8),

$$Z_o = (E_o - rp)/CS, \qquad \text{and,} \qquad Z_o - Z_u = (\Delta E/CS) - [(r - K)p/CS].$$

Since $r > K$, the quantity $(Z_o - Z_u)$ is less than $(Z_u - Z_q)$ at a given price. This means that $V_o < V_p$.

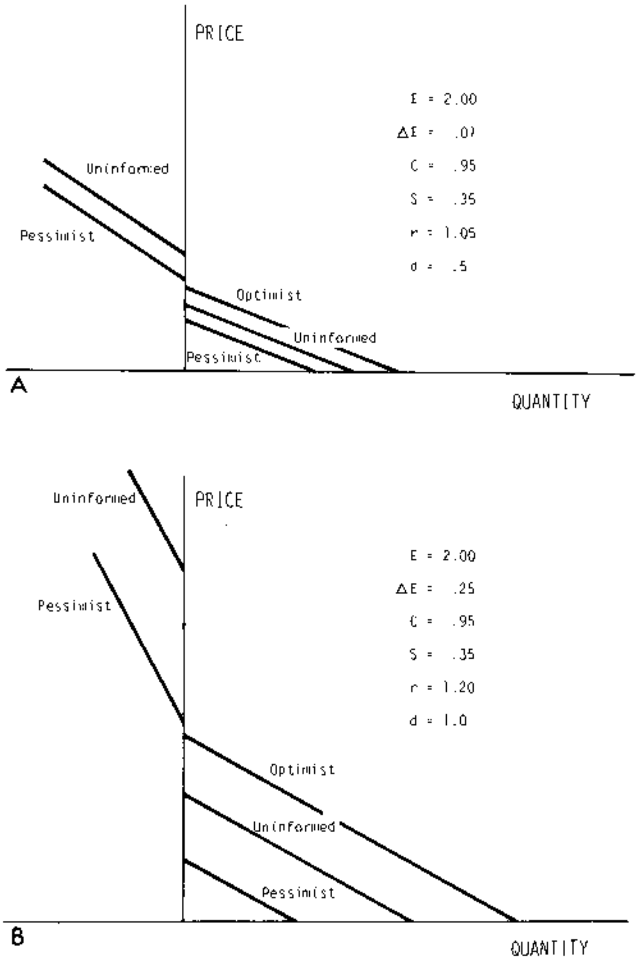

Figure 4. Limiting Cases when no Short Sales Occur

IV. Empirical Evidence

Conventional wisdom would have us believe two things about the relationship between price changes and transactions volume: first, that a positive linear relation exists between the two; and second, that volume is greater in bull markets than in bear markets. The empirical work of Crouch [4], Epps [6], Rogalski [13], and Ying [15] all tend to confirm these beliefs. Their work shows, however, that the linear relationship hypothesized above is far from perfect. Rogalski states: "The results suggest that knowledge of the behavior of volume may marginally improve conditional price forecasts based on past prices alone" [13, p. 274]. Given that Rogalski also finds price change to be a random walk, the weakness of the price change-volume relationship is evident. Crouch's results may also be characterized as weak. The best correlation coefficient between price changes and volume that he finds for an individual security is only .45 and, for the aggregate New York Stock Exchange, only .69. The generalization of the sequential information arrival model of Copeland [2] presented in this paper predicts both a nonperfect correlation between price change and volume, and bull market volumes generally in excess of bear market volumes.

The addition of a transactions cost which has a differential effect on investors desiring a long or a short position (in the risky asset) to the sequential information arrival model provides a theoretical explanation for existing empirical results. Consideration of the entire range of "fraction optimists" in Figure 2 yields a negative correlation between the absolute value of price changes and volume. By dropping the assumption that the market assesses only one item of information at a time, results are obtained that are consistent with the empirical evidence. In a real market setting, with numerous informational shocks impacting the market simultaneously and being diffused gradually, it is unlikely that investors could agree on the potential effect of the news they are currently interpreting. Thus it would be rare to observe a value of π close to zero or one.

Confining attention in Figure 2 to a range of π of .2 to .8, a correlation coefficient between price change and volume of .14 is found. The correlation coefficient predicted by each of these models would be even larger if attention is restricted to a narrower range of π. For example, if the mix of investors is restricted to a range between 20 and 60 percent optimists, the correlation coefficient is .90.[12] Thus, empirical research, especially that dealing with the movement of stock market indexes, could be expected to find widely varying correlation coefficients between price change and volume depending on such market variables as the margin requirement and the riskless rate of interest, as well as how current information is being interpreted by the market (i.e., the mix of optimists and pessimists) and individual beliefs. When restricted to the previously mentioned relevant range ($.2 \leq \pi \leq .8$), both models predict that the volume in a bull market exceeds volume in a bear market.

V. Summary and Conclusions

This paper has served to generalize a concept that may prove to be useful in reaching the goal of comprehending the disequilibrium process that adjusts beliefs to *ex post* prices. The model used by Copeland in defining the sequential information arrival process was extended by an equilibrium model that includes a margin requirement as a realistic restriction on short sales. The model illustrated that a margin requirement, like any other transaction cost, will cause investors' demand curves to contain a discontinuous segment. The costs relevant to long and short positions were shown to influence the relative slopes of the portions of the demand curve characterizing these positions.

With margin requirements, the model predicts a rather complex relationship between price change, volume, and the factors which influence these two vari-

[12] The reported correlation coefficients are for a number of traders equal to one hundred. The following table reports the coefficients for 10 and 50 traders.

	Correlation Coefficients	
	$M = 10$	$M = 50$
$.2 \leq \Pi \leq .8$	.39	.38
$.2 \leq \Pi \leq .6$	.93	.83

ables. Both variables were shown to be sensitive to the number of investors, the mix between optimists, pessimists, and uninformed, the costs of the margin requirement, and the actual level of the expectations of each class of investors.

The model presented in this paper obviously cannot be represented as an accurate picture of a disequilibrium adjustment process since it consists of a series of market equilibria. It is conceivable that the addition of another agent to the model to act as a specialist, and to match buyers and sellers at nonequilibrium prices might be a method to achieve disequilibrium trading [7]. This would tend to move the market towards equilibrium but perhaps not actually achieve equilibrium. The additional restriction that the price move in discrete amounts, i.e., eighths of a unit, may force the market to settle for a pseudoequilibrium [12]. An additional complication of the model would be to permit the market to receive more than one informational shock at a time. That is, before one item of data is perceived by all of the traders, allow another to reach the market. Even in a model with a sequence of equilibria, this would prevent a "final" equilibrium from obtaining.

There are also two forms of investor behavior lacking from this model which immediately come to mind. The first of these is the assumption that uninformed investors receive no information from the change in price. How investors receive information from market prices is a field of study in itself, but the answer to this question would have a significant impact on any model of a sequence of markets. The final point to be mentioned is that informed investors moved directly to a "consumptive" optimum. They do not speculate. It is, however, possible that a trader who perceives himself as having superior information will not be content with a consumptive optimum, especially in a world of unlimited borrowing. One might think of the traders who become informed early in this process as solving a dynamic programming problem taking into account the potential reaction of other traders as they become informed.

Appendix A

The market consists of three types of investors: (1) n uninformed investors with expectations E_u; (2) j informed investors who are optimists with expectations E_o; and (3) k informed investors who are pessimists with expectations E_q. The risk aversion index C_i and the variance estimate S_i remain constant for each investor i (but may differ among investors). All of the investors have exponential utility of wealth.

The initial equilibrium price (p) can be found by using the market clearing condition: $\sum_i Z_i = 1$

$$\sum_{i=1}^{n} \frac{E_u - rp}{C_i S_i} + \sum_{i=n+1}^{n+j} \frac{E_o - rp}{C_i S_i} + \sum_{i=n+j+1}^{M} \frac{E_q - rp}{C_i S_i} = 1$$

$$p = \frac{\sum_{i=1}^{n} \frac{E_u}{C_i S_i} + \sum_{i=n+1}^{n+j} \frac{E_o}{C_i S_i} + \sum_{i=n+j+1}^{M} \frac{E_q}{C_i S_i} - 1}{r \sum_{i=1}^{M} \frac{1}{C_i S_i}}$$

The holdings of an uninformed investor are $Z_i = [E_u - rp]/C_iS_i$. Let one of the uninformed investors (investor k) become informed and change his expectation by ΔE. His demand equation then becomes $Z_k = [E_u + \Delta E - rp]/C_kS_k$. Again using the market clearing condition, the new equilibrium price is

$$p = \frac{\sum_{i=1}^{n} \frac{E_u}{C_iS_i} + \frac{\Delta E}{C_kS_k} + \sum_{i=n+1}^{n+j} \frac{E_o}{C_iS_i} + \sum_{i=n+j+1}^{M} \frac{E_q}{C_iS_i} - 1}{r \sum_{i=1}^{M} \frac{1}{C_iS_i}}.$$

The change in price when the kth investor becomes informed is $\Delta p_k = [\Delta E/rC_kS_k \sum_{i=1}^{M} (C_iS_i)^{-1}]$ and the volume from his trade is $V_k = Z_k - Z_i = (\Delta E/C_kS_k)[1 - 1/C_kS_k \sum_{i=1}^{M} (C_iS_i)^{-1}]$. The expressions for both price change and volume are independent of the mix of optimists and pessimists.

Appendix B

Let

$$N = nE_u + jE_o + kE_q - CS,$$

$$D = (n + j)r + kK,$$

and

$$X = K - r.$$

Assuming the uninformed investor takes a long position,

$$V_o = (1/CS)[\Delta E - r(\Delta E/D)]$$

and

$$V_p = (1/CS)[\Delta E - r[(N/D)] + K[(N - \Delta E)/(D + X)].$$

Then

$$V_o - V_p = (1/CS)[r[(N - \Delta E)/D] + K[(N - \Delta E)/(D + X)]].$$

To sign $V_o - V_p$, the expression, $r[(N - \Delta E)/D] - K[(N - \Delta E)/(D + X)]$, must be signed. Suppose $V_o < V_p$, then

$$r[(N - \Delta E)/D] < K[(N - \Delta E)/(D + X)].$$

So

$$r(N - \Delta E)(D + X) < K(N - \Delta E)D$$

and

$$rD + rX < KD.$$

Substituting in for X, this expression becomes

$$r(K - r) < (K - r)D.$$

Since $K - r < 0$ by assumption,

$$r > D$$

which is clearly not true. Therefore,

$$V_o > V_p.$$

Appendix C

Let N be as defined in Appendix B,

$$D = jr + (n + k)K$$

and

$$X = r - K.$$

Assuming the uninformed investor has taken a short position,

$$V_o = (1/CS)[\Delta E - r[(N + \Delta E)/(D + X) + K(N/D)]$$

and

$$V_p = (1/CS)[\Delta E - K(\Delta E/D)].$$

Then

$$V_o - V_p = -(1/CS)[r[(N + \Delta E)/(D + X)] + K[(N + \Delta E)/D]].$$

To sign $V_o - V_p$, the expression, $-r[(N + \Delta E)/(D + X)] + K[(N + \Delta E)/D]$, must be signed (since $CS > 0$). Suppose $V_o > V_p$, then

$$K[(N + \Delta E)/D] > r[(N + \Delta E)/D].$$

So

$$K(N + \Delta E)(D + X) > r(N + \Delta E)\ D$$

and

$$KD + KX > rD.$$

Substituting in for X, this expression becomes

$$K(r - K) > (r - K)D$$

and

$$K > D$$

which is obviously not true. Therefore,

$$V_o < V_p.$$

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