

Enterprise Risk Management Through Strategic Allocation of Capital

By

Jing Ai*

Department of Financial Economics and Institutions
Shidler College of Business
The University of Hawaii at Manoa
Honolulu, HI 96822
jing.ai@hawaii.edu
Tel: (808) 956-9519
Fax: (808) 956-9887

Patrick L. Brockett

Gus S. Wortham Chaired Professor in Risk Management and Insurance
Red McCombs School of Business
Department of Information, Risk, and Operations Management
Global Fellow IC² Institute
The University of Texas at Austin
Austin, TX 78712
brockett@mail.utexas.edu
Tel: (512) 471-3322
Fax: (512) 471-0587

William W. Cooper

Foster Parker Professor of Finance and Management (Emeritus) and
Nadja Kozmetsky Scott Centennial Fellow of the IC² Institute
Department of Information, Risk, and Operations Management
Red McCombs School of Business
The University of Texas at Austin
Austin, TX 78712
cooperw@mail.utexas.edu
Tel: (512) 471-3322
Fax: (512) 471-0587

Linda L. Golden

Marlene and Morton Meyerson Centennial Professor in Business
Red McCombs School of Business
Department of Marketing
Global Fellow IC² Institute
The University of Texas at Austin
Austin, TX 78712
linda.golden@mcombs.utexas.edu
Tel: (512) 471-1126
Fax: (512) 471-1034

* indicates corresponding author

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Abstract

This paper presents a conceptual framework for operationalizing strategic enterprise risk management (ERM) in a general firm. We employ a risk-constrained optimization approach to study the capital allocation decisions under ERM: Given the decision maker's risk appetite, the problem of holistically managing enterprise-wide hazard, financial, operational, and real project risks is treated by maximizing the expected total return on capital, while trading off risks simultaneously in Value-at-Risk type of constraints. This approach explicitly quantifies the concepts of risk appetite and risk prioritization, in light of the firm's default and financial distress avoidance reflected in its target credit rating. Our framework also allows the firm to consider a multi-period planning horizon so that changing business environments can be accounted for. We illustrate the implementation of the framework through a numerical example. As an initial conceptual advancement, our formulation is capable of facilitating more general ERM modeling within a consistent strategic framework, where idiosyncratic variations of firms and different modeling assumptions can be accommodated. Managerial implications are also discussed.

Key Words: enterprise risk management (ERM); integrated risk management; Value-at-Risk (VaR); portfolio choice; capital allocation; risk appetite; risk prioritization; operational risk

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1. Motivation for the Research

Enterprise risk management (ERM), considered a breakthrough idea by Harvard Business Review (Buchanan 2004), emphasizes that all risks should be treated holistically from an enterprise-wide perspective (as opposed to individually in “silos”). This concept has been widely embraced by industry, regulatory forces (e.g., Sarbanes Oxley Act of 2002, Basel Capital Accord II), rating agencies (e.g., Standard & Poor’s 2005, 2006a, 2008) and the academic literature (cf., Ai and Brockett 2007). Yet, the implementation of this goal is illusive.

This paper offers a mathematical approach to operationalizing the integration of ERM within the firm to achieve its holistic strategic goals across time periods. The overall purpose of this paper is to provide both a conceptual framework and mathematical tools to make the implementation of ERM more concrete. There are significant challenges to the implementation of ERM (Gate 2006) as risk considerations have yet to be fully integrated into business decision making (Deloitte 2008). We attempt to provide an approach to overcoming some of these challenges.

In order to do this, we first have to adopt a perspective on what ERM is as it is still an emerging and evolving concept. The prevailing formulation of ERM, that we adopt as our perspective, was offered by COSO (2004).¹ This formulation includes the crucial concept of risk appetite: risk appetite is a corporation’s willingness and ability to undertake risks to achieve its strategic objectives that governs business decision-making. It also recognizes inter-relations between risks and the prioritization of risks (i.e., rank order of risk types according to importance), which is critical to holistic integration, a goal of our mathematical approach. This holistic integration is an important characteristic of the stated end-goal for ERM: to gain competitive advantage and create value (Economic Intelligence Unit 2007).

¹ No single definition or perspective on ERM is accepted by everyone. The specifics of what ERM is remains controversial. We adopted the COSO (2004) perspective on what ERM is because it can be described reasonably as the prevailing one. Other current models, however, define ERM in terms of risk aggregation, which is important for determining the amount of economic capital needed for financial institutions (e.g., Rosenberg and Schuermann 2006). While risk aggregation perspective models capture inter-relationships between risks, they do not explicitly address risk appetite, prioritization, operational decisions, trade-offs among different risk categories, etc., making these models less applicable to a general firm’s business decision-making.

2. ERM Decision Framework

2.1 The Setting

We present our ERM framework in a general setting to incorporate fundamental aspects of ERM. For simplicity, we consider a two-stage dynamic risk/return optimization problem with a two-period planning horizon, wherein the firm plans for the entire horizon (i.e., both periods) in stage 1 and can later change period 2 decisions upon receiving new information in stage 2.² This dynamic formulation allows the firm to adapt to changing internal and external environments, which is one of the key drivers for ERM development (Protiviti 2007). The proposed ERM framework allows for managerial inputs (e.g., risk appetite, risk limits, and risk prioritization) at the onset.

Under our framework, the firm's strategic goals are encompassed within an objective function, and risk concerns are controlled in impact and likelihood by using constraints.³ We address two broad types of investment opportunities: short-term investment that can be reassessed each period or long-term investment which cannot be changed for the entire two-period planning horizon. These investment opportunities can include real manufacturing projects (e.g., production projects for a manufacturing firm or lines of insurance for an insurance company) and liquid or illiquid financial assets. In addition to uncertainty in project and financial investment returns, the firm also faces operational and hazard risks (fire, theft, etc). By convention, hazard risks are managed by the selection of insurance coverage levels. Operational risks depend upon the duration and the nature of the investment.

2.1.1 Model notations

Our dynamic model allows decision makers the option to invest capital at the beginning of period 2 based on newly arriving information, so we provide a distinct notation for the returns, decision variables, and other parameters based on information at the beginning of period 1 (time 0) and the beginning of period 2 (time 1).

² We present our framework as a two-stage model, however it can be extended to a multi-stage model for a multi-period planning horizon (see Section 2.5.3 for more discussions).

³ This paper uses Value-at-Risk (VaR) constraints which allow the computations to proceed using well-developed mathematical programming methods. Other risk measures could be used in the constraint set with computations performed using heuristic methods such as the genetic algorithm, as will be discussed subsequently in Section 2.5.1.

(1) We let subscripts S and L denote respectively a one-period (or short-term) investment and a two-period (or long-term) investment, with additional subscripts added to designate multiple investment opportunities when necessary and superscripts to indicate the time period.

(2) We let \mathcal{F}_0 and \mathcal{F}_1 denote information sigma-algebras of information available at time 0 and time 1. Let $R_S^{(i)}$ denote total return (i.e., 1+ rate of return) on a short-term investment in period i ($i=1, 2$), and $r_S^{(i)} = E[R_S^{(i)} | \mathcal{F}_0]$ denote the (stochastic) best assessment of return during period i given information at time 0. Similarly, $r_L = E[R_L | \mathcal{F}_0]$ denotes total return on a long-term investment over two periods as assessed at time 0, i.e., the long-term investment consumes resources during period 1 and returns r_L at the end of period 2.

(3) At time 1, returns of the short-term investments in period 1 are realized, and new strategic information which has arisen is incorporated into the new information sigma-algebra \mathcal{F}_1 . At the onset of period 2, we let $\bar{r}_S^{(2)} = E[R_S^{(2)} | \mathcal{F}_1]$ denote revised (stochastic) assessment of returns on short-term investments in period 2 given information at time 1. Due to new environmental or market information, $\bar{r}_S^{(2)}$ may not equal $r_S^{(2)}$, so strategic decisions may evolve. Similarly we define $\bar{r}_L = E[R_L | \mathcal{F}_1]$ for long-term investments.

(4) Let decision variables $w_S^{(1)}$ and $w_S^{(2)}$ denote proportions of capital planned at time 0 to be committed to short-term investments in period 1 and period 2 respectively and w_L is the proportion committed to long-term investment at time 0. Here $w_S^{(2)}$ defines a strategic plan at time 0 intended to be implemented for period 2 but which may change at the end of period 1 in light of new information.

(5) At time 1 (stage 2 of the model), the firm updates period 2 decisions. Let $\bar{w}_S^{(2)}$ be updated decisions at time 1 for proportions of capital committed to short-term investments in period 2 given \mathcal{F}_1 .

(6) The firm has capital $C^{(0)}$ and $C^{(1)}$ to allocate at time 0 and time 1, respectively. Capital available at time 1, $C^{(1)}$, depends on realizations of returns and losses during period 1, so at time 0 $C^{(0)}$ is known and $C^{(1)}$ is stochastic (denoted as $\tilde{C}^{(1)}$), but $C^{(1)}$ becomes known for the stage 2 optimization.

(7) At time 2 (end of period 2), the firm has capital $C^{(2)}$. The goal of stage i ($i = 1, 2$) optimization is to maximize $E[\tilde{C}^{(2)} | \mathcal{F}_{i-1}]$ subject to period i constraints using conditional probabilities given \mathcal{F}_{i-1} .

2.1.2. Model assumptions

We consider four categories of risks: project risk, financial risk, operational risk, and physical hazard risk. Financial and project risks are speculative, whereas operational and hazard risks are pure risks. Conventional risk management for hazard risk involves insurance arrangements, so we incorporate the level of insurance coverage as a decision variable. To facilitate this, we make three assumptions on hazard risk.

(1) We measure the direct financial impact of hazard risk (loss of capital due to natural hazards and liability) as a proportion of the firm's capital C . Indirect loss caused by hazard risk (such as income loss from business interruptions) is assumed to be proportional to direct hazard loss. Let the unit hazard loss (both direct and indirect losses) be characterized as a random variable h (and thus total hazard loss is hC). More specifically, based on \mathcal{F}_0 , unit hazard risk in period 1 and period 2 is respectively $h^{(1)}$ and $h^{(2)}$ with expected value $\mu^{(1)}$ and $\mu^{(2)}$. At time 1 based on \mathcal{F}_1 , unit hazard risk in period 2 is assessed as $\bar{h}^{(2)}$ with expected value $\bar{\mu}^{(2)}$.

(2) Insurance premiums include an additional loading factor d (for the insurance company's profit, expenses, etc.) on expected losses. At time 0, the firm is charged insurance loading $d^{(1)}$ for period 1 and we assume there will be projected loading in period 2, $d^{(2)}$. At time 1, the firm is actually charged $\bar{d}^{(2)}$.

(3) At time 0, the firm plans to insure a percentage $u^{(i)}$ of hazard risk for period i ($i = 1, 2$). At time 1, the firm updates decisions and insures a proportion $\bar{u}^{(2)}$ in period 2.

2.2 The Objective Function

The strategic objective we consider is to maximize the total expected end-of-horizon wealth through investment in real projects and financial assets along with consideration for insurance purchase. The total return will be in the form $\Pi = Cwr$, where w and r are vectors of appropriate dimensions if multiple investment opportunities are considered. The insurance premium expense is deducted directly from the total expected return and, thus, we obtain the general form of objective function

$$\text{Max}_{w,u} CwE(r) - ud\mu C.$$

The second term reflects extra expenses incurred due to insurance loading. However, insurance purchase ($u > 0$) can be justified in a constrained model since it may help the firm satisfy risk considerations. As shown below, the specifications of the objective function for each stage are similar conceptually but different notationally.

2.2.1 The first-stage objective function

At time 0, the firm plans operational decisions for the entire planning horizon based on information \mathcal{F} . Decisions for period 1 will be implemented immediately while decisions for period 2 are strategic plans and can change at time 1 in light of new information. The strategic goal is to maximize expected returns over the entire planning horizon, i.e.,

$$\text{Max}_{w_S^{(1)}, w_S^{(2)}, w_L, u^{(1)}, u^{(2)}} C^{(0)} E \left\{ \left[w_S^{(1)} r_S^{(1)} - u^{(1)} d^{(1)} \mu^{(1)} \right] \times \left[w_S^{(2)} r_S^{(2)} - u^{(2)} d^{(2)} \mu^{(2)} \right] + w_L r_L \right\}. \quad (\text{Obj})$$

2.2.2 The second-stage objective function

At time 1, the firm receives new information set \mathcal{F} . Here the superscript * indicates the value obtained for the investment decisions as a result of stage 1 optimization, and \wedge indicates realizations of the period 1 random variables now known at time 1. In stage 2 optimization, the firm is not required to stick with the original investment and risk management plans for period 2 ($w_S^{*(2)}, u^{*(2)}$), if new information leads to

other more profitable choices. The objective is still to maximize end-of-horizon (time 2) total expected return,⁴ i.e.,

$$\underset{\bar{w}_S^{(2)}, \bar{u}^{(2)}}{\text{Max}} C^{(0)} E \left\{ \left[w_S^{*(1)} \hat{r}_S^{(1)} - u^{*(1)} d^{(1)} \mu^{(1)} \right] \times \left[\bar{w}_S^{(2)} \bar{r}_S^{(2)} - \bar{u}^{(2)} \bar{d}^{(2)} \bar{\mu}^{(2)} \right] + w_L^* \bar{r}_L \right\}. \quad (\text{Obj}')$$

2.3 The Constraints

While the strategic goal of maximizing end-of-horizon wealth is specified in the objective function, the constraints address risks and other considerations in pursuit of this goal. Two types of constraints are used: stochastic constraints similar to Value-at-Risk (VaR) measure and deterministic constraints.

We use VaR measure of risk because it is a very commonly used method for specifying risk in financial applications. In fact, according to Tieset and Troussard (2005), “Although the methodologies for measuring risk are still evolving, Value-at-Risk (VaR) is the most widely used method for estimating aggregate risk” (Tieset and Troussard 2005, pp. 63). It is also a risk measure often required by regulatory agencies (for example, cf., Basel II 2004). VaR has been shown to be a coherent risk measure for elliptical multivariate distributions (Embrechts et al. 2002). In addition, we use VaR because it allows us to address in a single measure the topics of risk appetite(s) and risk prioritization which are fundamental to ERM, as detailed below in Section 2.3.1. An overview of quantile-based risk measures in the finance and insurance context, including VaR, is given in Dowd and Blake (2006).

We allow each of the four risk types (i.e., project, financial, operational, and hazard risk) to be considered in a separate constraint to provide for different risk appetites and risk limits for each risk. We capture prioritization among risks (such as determined by traditional risk maps) by the relative magnitude of individual risk appetites. Managerial expertise and preference are utilized at this juncture. We also add a solvency constraint encompassing all risk types to comply with the firm’s overall risk appetite that governs all business activities. Other considerations (e.g., budgetary or regulatory considerations) are captured in deterministic constraints.

⁴ At this point the firm could alternatively choose to do another two-period projection of allocations involving new assessments of investment opportunities, risks, etc., as done in stage 1, with other potential two-period projects commencing at time 1 and continuing to time 3.

We discuss in detail the formulation of each constraint for period 1 in stage 1, and summarize the constraints for period 2 in each stage since they have similar structures.

2.3.1 Stochastic constraints for period 1 (in stage 1)

In the interest of making the ERM optimization more explicit, we shall henceforth consider two types of short-term investments: real projects which occupy a single period and liquid financial investments. We also consider long-term real projects (such as research and development) as well as illiquid financial investments which consume resources at the beginning of period 1 but do not pay off until the end of period 2. In the vector notation used in the objective function, we partition $w_S^{(1)}$ into $(w_{SP}^{(1)}, w_{SA}^{(1)})$ for short-term real projects and liquid financial assets respectively. Likewise, w_L is partitioned into (w_{LP}, w_{LA}) , and similar notations are used for the return vectors.

Project risk:

Project risk refers to possible unfavorable outcomes from real project investments. “Project” is broadly defined to include all activities associated with the real business part of the firm. To control project risk, the firm selects projects so that a pre-specified minimum level of total return (risk limit or hurdle rate) can be secured with specified confidence. This is governed by the firm’s risk appetite toward project risk, which is a managerial input concerning other firm characteristics (e.g., the credit rating target of the firm). Risk appetite for project risk is denoted by α_1 . Discussion related to the managerial selection of parameters such as α_1 is given in Section 3 following the model formulation.

Let $r_{P0}^{(1)}$ be the (deterministic) minimum total return requirement per unit of capital invested in real projects. Mathematically, we formulate the project risk constraint as

$$P\left[C^{(0)}w_{SP}^{(1)}r_{SP}^{(1)} \leq C^{(0)}\left(w_{SP}^{(1)}e + w_{LP}e\right)r_{P0}^{(1)}\right] \leq \alpha_1, \quad (\text{Cons 1-1})$$

where e is a vector of 1’s of appropriate dimension.

To satisfy (Cons 1-1), the firm must balance investments in different short-term and long-term projects to ensure that project risk in period 1 is within the firm’s project risk appetite α_1 .

Financial risk:

Financial risk is prototypically risk from financial investment, and can have characteristics different from real projects (e.g., tradability, liquidity, and hedging methods). In ERM, hedging opportunities may already exist among financial assets, between financial assets and real projects, or among a broader range of business components/units even before any targeted risk management, i.e., so-called “natural hedges.” Our model takes advantage of these natural hedge opportunities by allowing different business component activities to interact through constraints and the dependence structure.

Other than natural hedges, we do not impose a separate hedging policy for financial risk, so the constraint on financial risk is formulated similar to the project risk constraint.⁵ The firm may have a different risk appetite (α_2) for financial risk. Specifically, the financial risk constraint is

$$P\left[C^{(0)}w_{SA}^{(1)}r_{SA}^{(1)} \leq C^{(0)}\left(w_{SA}^{(1)}e + w_{LA}e\right)r_{A0}^{(1)}\right] \leq \alpha_2, \quad (\text{Cons 1-2})$$

where $r_{A0}^{(1)}$ is the minimum required return per unit of capital invested in the portfolio of financial assets.

Operational risk:

Operational risk is a nascent risk category recently ranked as the most important risk domain by U.S. corporate executives (Towers Perrin 2006). It spans a variety of internal and external business activities (cf., Basel Committee 2004). Currently there is little data available even at the industry level for most industries (cf., Guillen et al. 2007). Unlike hazard risk and financial risk, there is not yet any commonly accepted approach to quantify operational risk, or to set the risk constraint accordingly.

To construct a stochastic constraint on operational risk we follow the spirit of the Standardized Approach from the Basel Capital Accord II (Basel Committee 2004)⁶ wherein the operational risk

⁵ A hedging policy could be incorporated in our framework, should the firm wish to hedge financial risk (cf., Caldentey and Haugh 2006). Similar to the way we address hazard risk in our model, one could introduce a hedge ratio parameter for financial risk as a decision variable and incorporate it into the objective function. The stochastic constraints then involve “after-hedging returns” required to exceed the hurdle rate with the specified probability (1-risk appetite). We do not consider this extension here, but instead assume that the returns on assets are “after-hedging” or without hedging.

⁶ Basel II proposed three approaches: Basic Indicator Approach, Standardized Approach, and Advanced Measurement Approach (AMA). AMA is more sophisticated, but requires estimating loss distributions from operational loss databases, which are currently available only for a few particular industries like financial services at

corresponding to a business component of the firm is a multiple of the underlying risk exposure in this component. We use the Standardized Approach from Basel II because it is a well-known uniformly applicable approach and no other currently available approach will provide such general applicability. Following this approach, a proportion, $\gamma_P \geq 0$, of total project return is estimated to be attributable to operational risk for projects. Similarly, a proportion, $\gamma_A \geq 0$, corresponds to operational risk in financial investments. Applying these factors to respective total returns yields an estimate of operational risk from projects and financial asset investments. The γ 's could differ to partly reflect different relative risk exposure of various business components. Note that the firm can also similarly impose different γ 's on different sub-classes of short-term and long-term investments.

While returns from long-term projects are not realized until period 2, they are subject to operational risk in both periods. Thus, we use the total return from the long-term projects $w_L r_L$ to measure the time 0 assessment of the operational risk exposure for long-term projects in period 1 (and in period 2). Accordingly, the constraint for operational risk in period 1 is

$$P\left[\gamma_P^{(1)} C^{(0)} \left(w_{SP}^{(1)} r_{SP}^{(1)} + w_{LP} r_{LP}\right) + \gamma_A^{(1)} C^{(0)} \left(w_{SA}^{(1)} r_{SA}^{(1)} + w_{LA} r_{LA}\right) \geq C^{(0)} l_{op}^{(1)}\right] \leq \alpha_3, \quad (\text{Cons 1-3})$$

where $\gamma_P^{(1)}$ and $\gamma_A^{(1)}$ are, respectively, the operational risk factors for projects and financial assets and $l_{op}^{(1)}$ is the specified risk limit. Thus, (Cons 1-3) requires that the likelihood of exceeding a given operational risk limit to be restricted within the firm's operational risk appetite α_3 .

Hazard risk:

The firm insures a proportion $u^{(1)}$ of hazard risk in period 1. Assume that the firm is willing to retain a maximum hazard loss of $l_h^{(1)}$ per unit of capital (risk limit), subject to risk appetite α_4 . The hazard risk constraint is formulated as

$$P\left[\left(1 - u^{(1)}\right) h^{(1)} C^{(0)} \geq l_h^{(1)} C^{(0)}\right] \leq \alpha_4. \quad (\text{Cons 1-4})$$

best (e.g., Guillén et al. 2007). Our formulation is to be generally applicable, so the Standardized Approach is more useful.

Overall risk:

In addition to individual risk type constraints, we impose a solvency constraint based on the firm's overall risk appetite α_5 . This constraint requires that returns in period 1, after any losses, are sufficient to repay any financial obligation $c^{(0)}$ due at the end of period 1 (debt or fixed financial obligations including rent etc.) with probability $(1 - \alpha_5)$:

$$P \left[\begin{array}{l} (1 - \gamma_P^{(1)})C^{(0)}w_{SP}^{(1)}r_{SP}^{(1)} - \gamma_P^{(1)}C^{(0)}w_{LP}r_{LP} + (1 - \gamma_A^{(1)})C^{(0)}w_{SA}^{(1)}r_{SA}^{(1)} - \gamma_A^{(1)}C^{(0)}w_{LA}r_{LA} \\ - (1 - u^{(1)})h^{(1)}C^{(0)} - u^{(1)}(1 + d^{(1)})\mu^{(1)}C^{(0)} \leq c^{(0)} \end{array} \right] \leq \alpha_5. \quad (\text{Cons 1-5})$$

The parameter $c^{(0)}$ is exogenous and known to the firm at time 0.

2.3.2 Deterministic constraints for period 1 (in stage 1)

A few deterministic constraints are in place to capture regulatory and other considerations.

Budget constraint:

Total capital in period 1 is allocated to invest in projects and financial assets and to pay for the insurance premium for hazard risk. Thus, the budget constraint is

$$w_{SP}^{(1)}e + w_{LP}e + w_{SA}^{(1)}e + w_{LA}^{(1)}e + u^{(1)}(1 + d^{(1)})\mu^{(1)} \leq 1. \quad (\text{Cons 1-6})$$

Strategic/Regulatory constraint:

It may be desirable to have a minimum amount of capital invested in real projects for strategic purposes in each period (e.g., to keep market presence, to keep current employment staff in force for subsequent time periods, etc.) or to comply with regulatory requirements (e.g., on maximum investment in financial assets for industries such as the insurance industry). The associated constraint is then

$$w_{SP}^{(1)}e + w_{LP}e \geq \gamma_S^{(1)}, \quad (\text{Cons 1-7})$$

where $\gamma_S^{(1)}$ is a pre-specified minimum percentage invested in real projects.

$$\text{Range constraints:} \quad w_{SP}^{(1)} \geq 0, w_{SA}^{(1)} \geq 0, w_{LP} \geq 0, w_{LA} \geq 0, 0 \leq u^{(1)} \leq 1. \quad (\text{Cons 1-8})$$

2.3.3 Stochastic constraints and deterministic constraints for period 2 in each stage

The constraints for period 2 in stage 1 and stage 2 are constructed similar to those detailed for period 1, but with different capital to allocate to different investment opportunities. In stage 1, the firm plans the allocation of the expected capital for period 2 ($E[\tilde{C}^{(1)} | I_0]$) with investment return characteristics assessed based on the information set \mathcal{F}_0 . In stage 2, the known amount of capital $C^{(1)}$ (determined by realized returns and implemented optimal decisions in period 1) is allocated based on information set \mathcal{F}_1 . Notations are changed to reflect the different time period and/or updated information. More specifically, (Cons 1-1) to (Cons 1-8) have period 2 analogues (Cons 2-1) to (Cons 2-8) using the information set \mathcal{F}_0 , with time-period superscript “(2)” replacing superscript “(1)” and note also that the long-term investment decisions made in period 1 are now generating returns in period 2. The second stage optimization for period 2 have analogues (Cons 2’-1) to (Cons 2’-8) using the information set \mathcal{F}_1 , with a bar symbol indicating updated decisions, return assessments, and other parameters. These period 2 constraints (for each stage) are explicitly given in the Appendix.

2.4 The Optimizing ERM Framework

Our two-stage ERM decision framework optimizes the firm’s global strategic goal of maximizing end-of-horizon expected returns in the objective function in each stage while addressing risk considerations in the constraints for each period in each stage. Our formulation is summarized as below.

2.4.1 The first stage⁷

$$\underset{w_S^{(1)}, w_S^{(2)}, w_L, \bar{w}^{(1)}, \bar{w}^{(2)}}{Max} \quad (Obj)$$

s.t. Period 1 constraints: (Cons 1-1) to (Cons 1-8)

Period 2 constraints: (Cons 2-1) to (Cons 2-8)

2.4.2 The second stage

$$\underset{\bar{w}_S^{(2)}, \bar{w}^{(2)}}{Max} \quad (Obj')$$

s.t. Period 2 constraints: (Cons 2’-1) to (Cons 2’-8)

⁷ Please refer to Section 2.2, 2.3, and the Appendix for the specifications of objective functions and constraints.

2.5 Discussions and Extensions of the Dynamic ERM Framework

2.5.1 Distributional assumptions of the model and solution techniques

Since the VaR type constraints that arise in our ERM optimization involve linear combinations of random variables within the probability operator, the distribution of such linear combinations is important in order to calculate the constraint. In general, if we know the distribution function F of the random variable X within the probability operator of the constraint, a constraint like $P [X \leq r] \leq \alpha$ can be expressed equivalently as $r \leq F^{-1}(\alpha)$, a deterministic constraint, making the optimization a linear objective function subject to deterministic constraints. The constraint set will generally form a compact set (often convex) allowing for computation of the optimal values (cf., Mas-Collel, Whinston, and Green 2005). The problem, of course, is that when X is a linear combination of random variables for which we may know marginal distributions, the joint distribution of the components that the combination X depends on may not be known in closed form.

In a very general setting, each individual risk can be expressed in terms of its marginal distribution and copula methods (cf., Embrechts et al. 2002) can be used to model the multivariate dependence structure to construct the joint distribution. The probabilistic constraints appearing in our ERM formulation could then be calculated from the copula. For example, Rosenberg and Schuermann (2006) use copula methods to aggregate risks for VaR computations in a financial institution portfolio context.

For certain joint distributions, such as multivariate normal, elliptically symmetric or skewed distributions, the computation can be carried out by developing the “deterministic equivalents” (cf., Cooper, Lelas, and Sullivan 2006, who also address a method for performing computations in spite of non-convexity). Computations in more general situations involving log-concave distributions is shown by Prekopa (1971) to determine a convex constraint set and standard techniques can be employed. In the most general situation, for the formulation developed in this paper, once an appropriate joint distribution has been created (by copulas or other methods) a genetic algorithm can be used to derive the solution (c.f., Poojari and Varghese 2008). The availability of these techniques makes sure that our model is

implementable under different distributional assumptions driven by practical considerations. It also allows for variations and extensions to our current framework.

2.5.2 Incorporation of risk management strategies

Aside from hazard risk, we do not impose specific risk management (RM) strategies in our current formulation, leaving it open to various RM choices by the firm. The selected RM activities will normally alter the risk distributions in the constraints (e.g., truncate the range of possible values) and entailed costs can be included in the objective function as was done for hazard risk. In addition, the effect of RM may be reflected in the parameterization as well, e.g., proper RM activities can result in lower factor loadings (γ_P and γ_A) for operational risk.

2.5.3 Extension to a multi-stage model

The model herein is a two-stage model, chosen to keep the presentation simple. However, it can be extended to a multi-stage model following the same rationale. In an N -stage model, the firm has an N -period planning horizon. In stage 1 at time 0, planning decisions are made for each period in the horizon, based on information \mathcal{I}_0 . Decisions for period 1 are implemented immediately. In stage 2, new information, \mathcal{I}_1 , becomes available, including realizations from period 1. The firm then updates and implements its decisions for period 2 based on \mathcal{I}_1 . More generally, in stage n ($n=2, \dots, N$), the firm uses information \mathcal{I}_{n-1} to update and implement its decisions for period n . Here the long-term investments can last 2 to N periods. For an n -period long-term investment, capital is committed at the beginning of period i ($i=1, \dots, N-n+1$) and return is not realized until the project is finished. At the end of period N , all returns are realized and all optimized decisions are implemented. Risk constraints for each period of the planning horizon in each stage are constructed similarly as in the two-stage model.

Other extensions can be accommodated such as allowing additional injections of capital for the long-term investment during its duration by deducting them in each period from that period's total capital and making an adjustment to the constraints. The model can also be similarly adjusted to allow long-term investments to generate returns in multiple periods.

3. Choice of Risk Appetite and Other Parameters

We use two types of parameters in our ERM framework: exogenous parameters and decision parameters. Exogenous parameters are given to the decision makers (e.g., investment return parameters). Decision parameters, on the other hand, are within managerial discretion (e.g., risk appetite parameters) and are used as inputs to the decision making optimization model. We now give guidance on the managerial choice of decision parameters.

3.1 Risk Appetite

Recognition and incorporation of risk appetite into the strategic decision process is an important feature of ERM. For this process, a firm articulates the choice of risk appetite parameters in line with its strategic goals and risk culture. Previously these parameters are taken as given and incorporated in stochastic constraints. We now elaborate on how one might determine quantitatively firm-specific risk appetite parameters.

3.1.1 Overall risk appetite

The choice of overall risk appetite is guided by a major performance target of the firm, namely, the credit rating (cf., Nocco and Stulz 2006). Credit rating determines a firm's ability to raise capital and the associated cost of capital (West 1973), and can affect contract provisions with other constituencies. Credit rating concerns also influence corporate policies and business strategies such as capital investment (Sufi 2007). Thus it can serve as a proxy for the strategic goal governing risk appetite(s). In addition, the target credit rating is a readily identified parameter and can be a starting point for analysis.

Specifically, target credit rating translates into an implied target probability of default and a target probability of financial distress.⁸ To maintain its target credit rating, a firm should restrict the default probability to be within the level implied by that credit rating, i.e., the solvency constraint in our model.

This level of default probability defines the overall risk appetite, α_s . This establishment adopts the

⁸ Default probabilities associated with various credit ratings can be obtained using, for example, the rating transition matrix in Moody's default and recovery rates of corporate bond issuers: 1920-2005, March 2006. Financial distress probability can also be similarly derived if using bond rating to determine the financial distress threshold (cf., Nocco and Stulz 2006).

solvency concern that Standard & Poor's identifies as one of the main themes for risk appetite determination in their rating process (Standard & Poor's 2006b).

3.1.2 Risk appetite for individual risk types

In our ERM operationalization, we have standardized the description of risk appetites among individual risk types through the use of parameters α_i 's. This allows us to capture risk prioritization by the relative magnitude (i.e., rank order) of individual risk appetite parameters (α_i 's) in accordance with *a priori* managerial prioritization (obtained by traditional ERM risk maps, for example). Consistent with the "target credit rating" theme for the overall risk appetite, one way to obtain the exact values is to allow α_i 's to vary around the probability of financial distress implied by the target credit rating. The rationale is that each risk puts pressure on the financial soundness of the firm, so even for less prioritized risk types (i.e., ones the firm is more willing to take), the firm may want to limit their impact to avoid financial distress to the extent of the target probability. While the credit rating target provides a reasonable general guideline for risk appetite articulation, other themes can be used as well to determine these parameters depending on the firm's overall strategic goals (cf., AIRMIC 2009).

3.2 Other Parameters

The ERM framework involves other parameters supplied by the firm, including risk limits ($r_{P0}, r_{A0}, l_{op}, l_h$), operational risk loading factors (γ_P, γ_A), and minimum real project investment requirement γ_S . Risk limits can be chosen correspondingly to reflect each risk's impact on financial distress, determined using firm specific historical data, industry level data, or managerial assessment. Risk limits may also come from regulatory requirements or benchmarking, especially for financial firms (cf., Basak et al. 2006).

The operational risk loading factors should be based on quantitative analysis using firm historical data (or industry data if available) and/or qualitative analysis of each business component's exposure to operational risk. Parameter γ_S describes the minimal proportion of capital invested in real projects and may be based on a firm's overall strategies and balance between different constituencies, or based on

regulatory requirements. Finally, there are additional range constraints: $0 \leq \alpha_i \leq 0.5, i = 1, \dots, 5$, and other non-negativity parameter constraints.

4. Numerical Illustration

We now give a numerical illustration, showcasing the choice of baseline parameters and the implementation of our ERM decision framework. We examine the firm's decisions in both stages and exhibit the dynamic nature of the model (i.e., how the firm's time 0 assessment for period 2 can affect its decisions for period 1). Since firms can have different strategies and risk culture leading to varied choice of parameters, we also use a set of prototype analyses to examine how firms can incorporate these different considerations and explore the implications.

4.1 Distributional Assumptions Used in the Numerical Illustration

As discussed previously in Section 2.5.1, the model proposed in this paper can accommodate numerous distributional assumptions. The most efficient computational techniques, however, may differ depending upon the form of the distribution used. In this example we use the multivariate normal formulation because of its computational ease, as the purpose of the illustration is to demonstrate the implementation of our ERM framework. We do not want to distract from this purpose with lengthy presentation of the computations necessary for various other distributions (see, for example, Prekopa 1971, Cooper, Lelas, and Sullivan 2006, and Poojari and Varghese 2008, for efficient computations under alternative distributional assumptions).

(1) The returns on all projects and financial assets assessed at time 0 ($r_S^{(1)}, r_S^{(2)}, r_L$) follow a multivariate normal distribution, with the vector of expected returns $(E(r_S)^{(1)} : E(r_S)^{(2)} : E(r_L))$ and a positive semi definite variance-covariance matrix $\Sigma^{(r)}$. Similarly, the returns on projects and financial assets assessed at time 1 ($\bar{r}_S^{(2)}, \bar{r}_L$) are random variables assumed to follow a multivariate normal distribution, with the vector of expected returns $(E(\bar{r}_S)^{(2)} : E(\bar{r}_L))$ and a positive semi definite variance-covariance matrix $\Sigma^{(\bar{r})}$.

Note that different distributional parameters are allowed for time 0- and time 1- assessment of period 2 investment returns, since new information may have arrived during period 1 for the firm to use in the assessment. Note also that the use of the total covariance matrix $\Sigma^{(r)}$ and $\Sigma^{(\bar{r})}$ allows for correlations across investments and operational risks. It does not treat these risks individually as “silos,” but rather allows for the possibility of natural hedges between these risk types as is desirable in ERM strategies.

(2) We assume time 0 assessment of hazard risk per unit capital for period i ($i=1, 2$), $h^{(i)}$, follows a normal distribution, $h^{(i)} \sim N(\mu^{(i)}, \sigma^{(i)2})$, $\mu^{(i)} > 0$, $\sigma^{(i)} > 0$. The time 1 assessment for unit hazard risk in period 2 is $\bar{h}^{(2)}$, where $\bar{h}^{(2)} \sim N(\bar{\mu}^{(2)}, \bar{\sigma}^{(2)2})$. We assume that hazard risk is independent of project and asset returns since accidental or “acts of God” losses are usually uncorrelated with investment returns.

4.2 A Numerical Illustration for the Dynamic Model

In our numerical illustration, the problem is to make ERM-oriented optimal operational decisions for a two-year planning horizon. Since we do not have access to a specific firm’s proprietary data, we use both real market-level data and simulated firm-level data to implement it. We additionally investigate possible alternative parameterizations in Section 4.4 as these may impact risk management and operational decisions.

For specificity, we consider an investment portfolio with three real projects (two one-year short-term projects, and one two-year long-term project) and two short-term financial assets. The financial assets are an index fund (S&P Depository Receipts) that tracks the S&P 500 index and the three-month Treasury bill. We use available historical data (02:1993 – 12:2006) to obtain expected returns and variances for one-year holding period resulting in an estimated expected return and variance of 12% and 3% for the index fund, and 3.8% and 0.025% for the Treasury bill.

The three real projects considered have the following different risk/return characteristics: a risky long-term research and development (R&D) strategic project (expected return 60%, variance 15%), a short-term routine manufacturing project #1 (expected return 8%, variance 0.1%), and another short-term adventurous manufacturing project #2 (expected return 20%, variance 4%). A small negative correlation

(cf., Breen et al. 1989) is imposed between the financial assets, and a small positive correlation is imposed among the real projects. Correlations between the index fund and the projects are positive, while correlations between the Treasury bill and projects are negative. We impose higher correlation within the investment classes than across the investment classes. In our baseline model presented in this subsection, we assume that time 0 assessments of returns in period 1 and period 2 are the same for short-term investments.

As per the model, the firm updates assessments of period 2 investments at time 1 based on newly available information. For the numerical illustration, we assume that at time 1 it is assessed that the economy is in recession and the return characteristics are revised accordingly. Namely, in stage 2 of the model, expected returns for real projects and the index fund are reduced, volatilities of all returns are increased, and there is a higher correlation between the index fund and the Treasury bill. Table 1 presents time 0 and time 1 (in parentheses) assessments of investment opportunities. Stage 2 optimization requires inputs from the realizations of random return variables in period 1, which, for illustration purposes, are taken to be the expected returns assessed at time 0.

Table 1 Time 0 assessment of investment opportunities for period 1 and period 2, with the subsequent time 1 assessment for period 2 given in parentheses

Expected Return and Variance					
	Manufacturing Project 1	Manufacturing Project 2	R&D Project ⁹	Index Fund	3 Month T-Bill
Expected return	0.08 (0.06)	0.2 (0.18)	0.6 (0.3)	0.12 (0.08)	0.038 (0.04)
Variance	0.001 (0.002)	0.04 (0.06)	0.15 (0.2)	0.03 (0.05)	0.00025 (0.0003)
Correlation of the investments					
	Manufacturing Project 1	Manufacturing Project 2	R&D Project	Index Fund	3 Month T-Bill
Manufacturing Project 1	1	0.1	0.1	0.05	-0.05
Manufacturing Project 2		1	0.08	0.05	-0.05
R&D Project			1	0.05	-0.05
Index Fund				1	-0.1 (-0.2)
T-Bill					1

⁹Note that the R&D project is a long-term project. Since we use a single table (Table 1) to describe time 0 assessments for both period 1 and period 2, we include the R&D project here. However, the R&D project only generates return in period 2 and it is the two-period return.

Variance Covariance Matrix

	Manufacturing Project 1	Manufacturing Project 2	R&D Project	Index Fund	3 Month T-Bill
Manufacturing Project 1	0.001 (0.002)	0.000632 (0.001095)	0.001225 (0.002)	0.000274 (0.0005)	-2.5E-05 (-3.9E-05)
Manufacturing Project 2		0.04 (0.06)	0.006197 (0.008764)	0.001732 (0.002739)	-0.00016 (-0.00021)
R&D Project			0.15 (0.2)	0.003354 (0.005)	-0.00031 (-0.00039)
Index Fund				0.03 (0.05)	-0.00027 (-0.00077)
T-Bill					0.00025 (0.0003)

Table 2 presents the other parameter values for the baseline setting. Decision parameters are not revised between periods in stage 1 since the firm generally uses separate decision processes to select them in alignment with the strategic objective which does not tend to change over the planning horizon (e.g., target credit rating). For simplicity, we also assume hazard risk exposure and other parameters remain constant over periods in stage 1. Additionally, we assume that the same set of parameter values are employed (for period 2) in stage 2, except for the insurance loading d , which is assumed to increase in the recession period.¹⁰

Table 2 Parameter value

Risk Appetite					Risk Limits			
α_1	α_2	α_3	α_4	α_5	r_{A0}	r_{P0}	l_{op}	l_h
0.05	0.05	0.05	0.05	0.0008	0	0	0.2	0.01
Hazard Risk					Op Risk Factors	Strategic Factor	Borrowed Capital	
μ	σ	d	$\bar{d}^{(2)}$		γ_P	γ_A	γ_S	c
0.01	0.1	0.2	0.3		0.2	0.1	0.5	0.7

In selecting the parameters for Table 2, we used real data, experience, and reasonable assumptions. When a firm internalizes our framework to create their own model, these parameters will be managerial inputs. We assume that the firm has a target credit rating of “A” (Moody’s), corresponding to a default probability of 0.08%, which is set to be the overall risk appetite.¹¹ We also assume that downgrading to

¹⁰ However, note that despite our simplifying assumptions, our model allows the choice of parameter values to differ for different periods or stages.

¹¹ This number is extracted from the average one-year rating transition matrix (1920-2005) in Moody’s Default and Recovery Rates of Corporate Bond Issuers, 1920-2005, March 2006.

“Baa” represents the financial distress threshold, leading to about a 5% financial distress probability.¹² In our baseline setting, all individual risk appetite parameters are currently chosen to be 5% (i.e., no risk prioritization in the baseline setting).

Since there is hardly any publicly available data on operational losses for general industries, the operational risk loading factors γ_P and γ_A are chosen relative to the guidelines provided in Basel II (cf., Basel II 2004). Finally, for the numerical illustration we assume the required capital obligations for each period to be proportional to the total capital in each period, i.e., $c^{(0)}=cC^{(0)}$ for period 1 and $c^{(1)}=cC^{(1)}$ for period 2, although they could alternatively be specified as fixed dollar amounts.

Under the simplifying Multivariate Normal assumption, we solve the optimization problem in this example using the techniques outlined in Charnes and Cooper (1963). The optimal values under the baseline setting are given in Column 2 in Table 3 below. From Table 3, we can see that the majority of the capital goes to the two manufacturing projects and the Treasury bill due to the risk constraints. The firm revises its decisions toward the safer choices for period 2 in stage 2 in light of an updated (now assessed as recessionary) information set at time 1. Optimally the firm insures about 95% of hazard risk. Optimal expected return for the horizon is 18% at time 0 and reduces to 13.6% in stage 2 after recessionary information is incorporated.

Table 3 Optimal operational decisions for each stage

Decision Variable	Baseline Optimal Decision (Period 2 technology change not anticipated at time 0)	New Optimal Decision (Period 2 technology change anticipated at time 0)
Stage 1		
Investment in manufacturing project 1 in period 1	0.5801	0.6223
Investment in manufacturing project 2 in period 1	0.1237	0.0917
Investment in R&D project in period 1	0.0181	0.0217
Investment in the index fund in period 1	0.0466	0.0442
Investment in the Treasury bill in period 1	0.2190	0.2077
The proportion of hazard risk insured in period 1	0.9479	0.9479
Investment in manufacturing project 1 in period 2	0.2409	0.4169

¹² Similar to Nocco and Stulz (2006), we use bond rating as a criterion for financial distress. If “Baa” is the financial distress threshold, the financial distress probability is the sum of the probability of downgrading from “A” to “Baa” and below. These probabilities are calculated based on Moody’s transition matrix.

Investment in manufacturing project 2 in period 2	0.3363	0.2386
Investment in the index fund in period 2	0.0720	0.0583
Investment in the Treasury bill in period 2	0.3383	0.2738
The proportion of hazard risk insured in period 2	0.9479	0.9479
Optimal Return (for the planning horizon)	1.1797	1.1418
Stage 2		
Investment in manufacturing project 1 in period 2	0.4996	0.4600
Investment in manufacturing project 2 in period 2	0.1682	0.1853
Investment in the index fund in period 2	0.0365	0.0391
Investment in the Treasury bill in period 2	0.2822	0.3021
The proportion of hazard risk insured in period 2	0.9479	0.9479
Optimal Return (for the planning horizon)	1.1356	1.1343

4.3 Demonstration of the Dynamic Nature of Our Modeling Approach

In our dynamic ERM framework, the firm initially makes plans for both periods at time 0 in stage 1, taking into account investment opportunities and risks for both periods. While keeping all stage 2 assessments and all other assessments for both periods in stage 1 the same as in the baseline setting, we now illustrate the dynamics of the model by investigating how period 1 decisions would have changed if the firm anticipated at time 0 less favorable investment opportunities for manufacturing projects in period 2 (e.g., due to technology change) (note that in the baseline setting, time 0 assessment of these investment opportunities are assumed to be the same for both periods). Table 4 presents the description of this new set of investment opportunities for period 2 assessed at time 0. Because of the interactivity of risks, stage 1 decisions for both periods will change, and since stage 2 decisions are based on outcomes of period 1 decisions, they change as well. The optimal results under this new setting are shown in Table 3 (Column 3) above.

Table 4 Time 0 assessment of investment opportunities for period 2 (technology change anticipated)

Expected Return and Variance					
	Manufacturing Project 1	Manufacturing Project 2	R&D Project	Index Fund	3 Month T-Bill
Expected return	0.05	0.15	0.6	0.12	0.038
Variance	0.002	0.05	0.15	0.03	0.00025

Correlation of the investments

	Manufacturing Project 1	Manufacturing Project 2	R&D Project	Index Fund	3 Month T-Bill
Manufacturing Project 1	1	0.1	0.1	0.05	-0.05
Manufacturing Project 2		1	0.08	0.05	-0.05
R&D Project			1	0.05	-0.05
Index Fund				1	-0.1
T-Bill					1

Variance Covariance Matrix

	Manufacturing Project 1	Manufacturing Project 2	R&D Project	Index Fund	3 Month T-Bill
Manufacturing Project 1	0.002	0.001	0.001732	0.000387	-3.5E-05
Manufacturing Project 2		0.05	0.006928	0.001936	-0.00018
R&D Project			0.15	0.003354	-0.00031
Index Fund				0.03	-0.00027
T-Bill					0.00025

From the side-by-side comparison of optimal decisions under the two settings in Table 3, we see that with the new time 0 assessment of period 2 investment opportunities, the firm invests more in the long-term project, which is more attractive now since the returns from manufacturing projects have worsen. The firm also becomes more conservative when planning for period 2. This interaction of the two periods manifests the dynamic nature of our framework.

4.4 Comparative Impacts of Different Parameterizations

We now assess the impact of the decision parameters. Such parameters are not arbitrarily determined, but firm-specific strategic choices. In doing these analyses, we explore how the firm might incorporate alternative considerations (e.g., risk prioritization), and examine the implications of parameter choice on ERM decisions. To conserve space we only present a re-analysis of stage 2 of the model. Table 5 below presents a summary of the impact of each decision parameter as detailed in the rest of this section. In general, analyses below have illustrated that our formulation leads to intuitive results including how the strategic decisions reflect risk considerations, the interaction among different risk categories in an ERM setting, the increased efficiency of holistic decision making rather than that in “silos,” the important role of risk management in achieving strategic goals, etc. These help demonstrate the operationalizability of our ERM framework.

Table 5 Optimal stage 2 decisions with varying parameter value choices under alternative considerations

	Manu Project 1	Manu Project 2	Index Fund	T-Bill	Insurance	Optimal Return
Baseline	0.4996	0.1682	0.0365	0.2822	0.9479	1.1356
Panel A: Alternative Choices of Risk Appetite Parameters						
(1) Appetite I: High project, Low hazard ^a	0.3145	0.2698	0.0461	0.3558	0.9625	1.1468
(2) Appetite II: Low Financial, Low hazard ^b	0.5009	0.1683	0.0163	0.3006	0.9625	1.1345
(3) Appetite III: High hazard ^c	0.4991	0.1681	0.0367	0.2832	0.9035	1.1364
(4) Appetite IV: Low operational, Low hazard ^d	0.4523	0.1625	0.0426	0.3288	0.9625	1.1337
Panel B: Alternative Choices of Risk Limits Parameters						
(1) Limit I: Low project ^e	0.1476	0.0609	0.0892	0.6888	0.9479	1.1146
(2) Limit II: High operational ^f	0.7787	0.1964	0.0013	0.0100	0.9479	1.1441
Panel C: Alternative Choices of Other Parameters						
(1) Other I: Low OP factor ^g	0.7891	0.1973	0	0	0.9479	1.1444
(2) Other II: Low OP factor and tight solvency ^h	0.4115	0.0885	0.0291	0.4573	0.9496	1.1217
(3) Other III: High strategic ⁱ	0.7389	0.0111	0.0201	0.2164	0.9479	1.1168

Note: all parameters are the same as in the baseline setting except as noted below:

^a $\alpha_1 = 0.1, \alpha_4 = 0.01$; ^b $\alpha_2 = 0.01, \alpha_4 = 0.01$; ^c $\alpha_4 = 0.2$; ^d $\alpha_3 = 0.01, \alpha_4 = 0.01$; ^e $r_{p0} = 0.05$; ^f $l_{op} = 0.3$; ^g $\gamma_p = 0.1$, $\gamma_A = 0.05$; ^h $\gamma_p = 0.1, \gamma_A = 0.05, c = 0.88$; ⁱ $\gamma_S = 0.75$

4.4.1 The impact of different risk prioritizations

Under our formulation, risk prioritization is obtained by varying the individual risk appetite parameters (a larger α for low priority risks). We discuss its impact on optimal operational decisions below.

(1) *Risk appetite I: high appetite for project risk, prioritize hazard risk ($\alpha_1 = 0.1$ and $\alpha_4 = 0.01$)*

A firm may have a higher risk appetite than the baseline for project risk since project risk is its core competence, and taking necessary risk yields competitive advantage. Also, if the firm does not have expertise in managing hazard risk, it may want to prioritize hazard risk (i.e., a lower risk appetite), especially since this risk can be better managed through insurance. From Table 5, we see that the result of

these changed risk appetites is a 61% increase in the optimal investment in the riskier manufacturing project 2. By requiring tighter control of the remaining hazard risk (i.e., lower α_4), a larger proportion is insured. The total expected return also increases.

(2) *Risk appetite II: prioritize financial risk and hazard risk ($\alpha_2 = 0.01$ and $\alpha_4 = 0.01$)*

In addition to hazard risk, a general firm may find financial risk less attractive. From Table 5 we see that adjusting the risk appetite parameters to provide for smaller financial risk appetite leads to less investment in the “riskier” index fund and a lower expected total return.

(3) *Risk appetite III: high appetite (low priority) for hazard risk ($\alpha_4 = 0.2$)*

A larger firm may have a relatively high risk appetite for hazard risk and choose to retain a higher proportion of it as insurance is costly. Savings on insurance loadings allow more capital to be invested and hence this trade-off leads to higher optimal expected returns. The implication is that maximally insuring hazard risk may not be the best strategy, especially when the firm foresees good alternative investment opportunities, and/or has a higher capacity for risk retention.

(4) *Risk appetite IV : prioritize operational risk and hazard risk ($\alpha_3 = 0.01$ and $\alpha_4 = 0.01$)*

We show that the recognition of operational risk is important in a firm’s operating decisions. Higher prioritization (or lower appetite) can arise because operational risk is less well understood. While the firm maintains the same appetite for project risk, investment in both projects are reduced, since these real projects have larger exposure to operational risk than financial investments ($\gamma_P > \gamma_A$). Operational risk highlights the interactions among different risk categories as is characteristic of ERM.

4.4.2 Risk limits

Now we discuss the effects of changing risk thresholds or limits. Risk limits can be set based on the assessment of each risk’s impact on financial distress or can be derived from managerial assessment of historical data and operating experience.

(1) *Risk limit I: lower risk limit for project risk ($r_{p0} = 0.05$)*

A lower permissible risk limit (higher hurdle rate) results in greatly reduced investment in both projects. Within the context of this scenario, it is important to note that the previous strategic constraint that “investment in projects must be at least 0.5” must be loosened in order to find a feasible solution for the ERM framework in this setting. This does not necessarily mean that the firm cannot use individual risk limits, but rather shows that all parameters need to be simultaneously carefully chosen. After adjusting the strategic constraint we find that more capital goes to financial assets, and the total expected return is reduced.

(2) Risk limit II: higher risk limit for operational risk ($l_{op} = 0.3$)

Similar to having a high risk appetite for operational risk, relaxing limits for operational risk results in more capital invested in projects because the high project loading on operational risk had previously limited investment in projects.

(3) Risk limit III: Lower risk limit for financial risk ($r_{A0} = 0.05$)

When the risk limit for financial risk is set at $r_{A0} = 0.05$ no feasible solution exists. In this case (as previously), the decision parameters need to be re-assessed by managers, or the firm may need to search for alternative investment opportunities if they are available. This highlights an ERM benefit – if managed in silos, individual decisions may lead (through interactivity with other risks) to decisions incompatible with global corporate strategic goals, however infeasibility in the ERM model can highlight such incompatibilities and bring those to the attention of the decision makers.

4.4.3 Other parameters

(1) Other factor I: low operational risk factors ($\gamma_P=0.1$ and $\gamma_A=0.05$)

The parameters γ_P and γ_A form the basis of our quantification of operational risk. The factors developed from Basel Capital Accord II can be used as a guideline, however in practice they could be made firm-specific. The presented operational risk loadings γ_P and γ_A could also be lowered by undertaking risk management actions to reduce operational risk exposure (e.g., refining organizational supervisory

structure, employee training, etc.). This in turn results in all resources going to the more profitable real project investments and thus increases the total expected return.

(2) *Other factor II: low operational risk factors ($\gamma_P=0.1$ and $\gamma_A=0.05$), tight solvency constraint ($c=0.88$)*

Operational risk factors are important in the presence of a tighter solvency constraint. If the firm raises the proportion of borrowed capital c to, for example, 0.88, no feasible solution can be found within the original baseline parameterization ($\gamma_P=0.2$ and $\gamma_A=0.1$). However, if one could lower these two factors to 0.1 and 0.05 (e.g., through risk management), then the solvency risk constraint can be satisfied.¹³ This finding has important risk management implications: risk management is an important tool for managers to reconcile potentially conflicting risk considerations and is critical in situations where strategic objectives cannot be achieved without risk management support (e.g., in a heavily leveraged firm).

(3) *Other factor III: high strategic or regulation parameter ($\gamma_S = 0.75$)*

The parameter γ_S reflects the firm's strategic consideration (e.g., to retain a competitive market presence), or it may be a response to regulatory requirements (e.g., a firm may only be allowed to invest a certain proportion of capital in certain assets or an insurance firm can be prohibited from exiting a market). Here we find more capital is invested in the "safer" project and the optimal return drops significantly.

(4) *Other factor IV: Insurance loading (d)*

When a solvency constraint is not imposed, the magnitude of the loading d does not affect the proportion of hazard risk optimally insured; however, if the solvency constraint is added, the amount of hazard risk insured may change.

In summary, note that the maximum total expected return is obtained by *simultaneously* considering all constraints integrated in the ERM framework. If risks were handled in silos, the decisions may fail to honor one or more risk constraints and thus may not be optimized to achieve the goals of the firm.

5. Conclusion

¹³ Numerical experiments suggest adjusting these two factors is an easy way to satisfy stringent overall constraints among adjustment of other parameters.

This paper fills a gap in the ERM literature by presenting a single quantitative model for operationalizing ERM decision making. Using an optimization approach, we introduce a mathematically formulated dynamic framework which allows firms to align risk appetite for multiple projects, financial investments, operational and uninsured hazard exposure, with their overall strategic goals over time. More specifically, the model optimizes these strategic goals while constraining risks via delineation of acceptable risk levels allowing for various risk appetites, similar in spirit to the traditional two-dimensional risk analysis approach familiar to decision makers. The commonly used VaR risk measure is adopted to formulate risk constraints so as to be able to incorporate fundamental characteristics of ERM, such as risk appetite and risk prioritization. Our approach to ERM operationalization is dynamic in that it depicts the firm as first making strategic plans for the entire multi-period planning horizon, and updating its operational decisions every period as new information becomes available. This is important to the development of ERM in general as firms are always making decisions in a non-stationary environment.

The model is illustrated with a numerical implementation that shows how a hypothetical firm can make optimal capital allocation decisions while managing across business units, risk types, and decision parameters (e.g., risk appetite and risk prioritization). We have also provided discussions of managerial choice of decision parameters and its implications for the ERM operational decisions. With these, the paper provides implementation guidance to firms seeking to operationalize ERM.

While we provide a formulation that uses the frequently employed Value-at-Risk (VaR) measure, other risk measures, such as variance, semi-variance, and expected shortfall, can be used in place of VaR to formulate the constraint set and still yield a computationally feasible optimization problem within our model. The different solution techniques entailed by various model specifications are explicitly prescribed in the paper.¹⁴

This paper also makes a contribution to optimal capital allocation/portfolio selection literature under VaR constraints (cf., Campbell, Huisman, and Koedijk 2001 and Danielsson, Jorgensen, de Vries, and

¹⁴ Prekopa (1971), Cooper, Lelas, and Sullivan (2006), and Poojari and Varghese (2008) provide guidance relevant to the computations involved in using other types of risk measures in our formulation.

Yang 2008). We could find no paper in this literature that addresses optimal allocation when several VaR constraints are imposed at the individual levels, as well as at the aggregate level, as is done in our paper. We go beyond portfolio selection models (such as Markowitz 1952), because the paper considers multiple risk types, is dynamic, uses a different risk metric formulation, allows a choice of desirable risk levels and risk appetite, incorporates constraints, and has the ability to treat different types of risks differently. The combination of these characteristics in our formulation is unique in the literature.

While our formulation is a contribution to ERM operationalization and related literature, it underscores numerous opportunities for future research. Possible extensions to our formulation include 1) other implementation of operational risk, 2) forming constraints based on aggregated distributions derived from empirical (or simulated) marginal distributions using a copula model for dependence (cf., Rosenberg and Shuermann 2006), 3) adding/modifying risk types and measures, 4) explicitly incorporating risk management strategies for all risk types, 5) switching to other business objectives rather than maximizing end-of-horizon wealth, and 6) different themes and processes for the choice of decision parameters. Our formulation is sufficiently flexible to be adapted to other strategic considerations of interest to decision makers.

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Appendix

Period 2 constraints resemble those for period 1 (e.g., Cons 2-1 corresponds to Cons 1-1), except for the treatment of long-term investment components (capital committed in period 1 and returns generated in period 2) in the relevant risk constraints. Superscripts are also changed from “(1)” to “(2)” consequently to reflect the time periods. They are explicitly shown below.

Constraints for period 2 in stage 1:

Using $\tilde{C}^{(1)} = C^{(0)}(w_S^{(1)}r_S^{(1)} - u^{(1)}d^{(1)}\mu^{(1)})$ and $\tilde{E}^{(1)} = E[\tilde{C}^{(1)} | I_0] = C^{(0)}E(w_S^{(1)}r_S^{(1)} - u^{(1)}d^{(1)}\mu^{(1)})$, we have

$$P[\tilde{E}^{(1)}w_{SP}^{(2)}r_{SP}^{(2)} + C^{(0)}w_{LP}r_{LP} \leq \tilde{E}^{(1)}w_{SP}^{(2)}er_{p0}^{(2)}] \leq \alpha_1, \quad (\text{Cons 2-1})$$

$$P[\tilde{E}^{(1)}w_{SA}^{(2)}r_{SA}^{(2)} + C^{(0)}w_{LA}r_{LA} \leq \tilde{E}^{(1)}w_{SA}^{(2)}er_{A0}^{(2)}] \leq \alpha_2, \quad (\text{Cons 2-2})$$

$$P\left[\begin{array}{l} \gamma_P^{(2)}\tilde{E}^{(1)}w_{SP}^{(2)}r_{SP}^{(2)} + \gamma_P^{(2)}C^{(0)}w_{LP}r_{LP} \\ + \gamma_A^{(2)}\tilde{E}^{(1)}w_{SA}^{(2)}r_{SA}^{(2)} + \gamma_A^{(2)}C^{(0)}w_{LA}r_{LA} \geq \tilde{E}^{(1)}I_{op}^{(2)} \end{array}\right] \leq \alpha_3, \quad (\text{Cons 2-3})$$

$$P\{(1 - u^{(2)})h^{(2)}\tilde{E}^{(1)} \geq I_h^{(2)}\tilde{E}^{(1)}\} \leq \alpha_4, \quad (\text{Cons 2-4})$$

$$P\left[\begin{array}{l} (1 - \gamma_P^{(2)})\tilde{E}^{(1)}w_{SP}^{(2)}r_{SP}^{(2)} + (1 - \gamma_P^{(2)})C^{(0)}w_{LP}r_{LP} + (1 - \gamma_A^{(2)})\tilde{E}^{(1)}w_{SA}^{(2)}r_{SA}^{(2)} \\ + (1 - \gamma_A^{(2)})C^{(0)}w_{LA}r_{LA} - (1 - u^{(2)})h^{(2)}\tilde{E}^{(1)} - u^{(2)}(1 + d^{(2)})\mu^{(2)}\tilde{E}^{(1)} \leq c^{(1)} \end{array}\right] \leq \alpha_5, \quad (\text{Cons 2-5})$$

$$w_{SP}^{(2)}e + w_{SA}^{(2)}e + u^{(2)}(1 + d^{(2)})\mu^{(2)} \leq 1; w_{SP}^{(2)}e \geq \gamma_S^{(2)}; w_{SP}^{(2)} \geq 0, w_{SA}^{(2)} \geq 0, 0 \leq u^{(2)} \leq 1. \quad (\text{Cons 2-6,-7,-8})$$

Stage 2 constraints (for period 2) are similar to period 2 constraints in stage 1 (e.g., Cons 2'-1 corresponds to Cons 2-1), except expected capital $\tilde{E}^{(1)}$ is now replaced by actual capital $C^{(1)}$ and updated information \mathcal{F} is used to form probability distributions. “Bar” is used to indicate updated information \mathcal{F} and updated decisions. They are explicitly shown below.

Constraints for period 2 in stage 2:

Using $C^{(1)} = C^{(0)}[w_S^{(2)*}\hat{r}_S^{(1)} - u^{(1)*}d^{(1)}\mu^{(1)}]$, we have

$$P[C^{(1)}\bar{w}_{SP}^{(2)}\bar{r}_{SP}^{(2)} + C^{(0)}w_{LP}^* \bar{r}_{LP} \leq C^{(1)}\bar{w}_{SP}^{(2)}er_{p0}^{(2)}] \leq \alpha_1, \quad (\text{Cons 2'-1})$$

$$P\left[C^{(1)}\bar{w}_{SA}^{(2)}\bar{r}_{SA}^{(2)} + C^{(0)}w_{LA}^*\bar{r}_{LA} \leq C^{(1)}\bar{w}_{SA}^{(2)}er_{A0}^{(2)}\right] \leq \alpha_2, \quad (\text{Cons 2'-2})$$

$$P\left[\begin{aligned} &\bar{\gamma}_P^{(2)}C^{(1)}\bar{w}_{SP}^{(2)}\bar{r}_{SP}^{(2)} + \bar{\gamma}_P^{(2)}C^{(0)}w_{LP}^*\bar{r}_{LP} \\ &+ \bar{\gamma}_A^{(2)}C^{(1)}\bar{w}_{SA}^{(2)}\bar{r}_{SA}^{(2)} + \bar{\gamma}_A^{(2)}C^{(0)}w_{LA}^*\bar{r}_{LA} \leq C^{(1)}\bar{l}_{op}^{(2)} \end{aligned}\right] \leq \alpha_3, \quad (\text{Cons 2'-3})$$

$$P\left[(1 - \bar{u}^{(2)})\bar{h}^{(2)}C^{(1)} \geq \bar{l}_h^{(2)}C^{(1)}\right] \leq \alpha_4, \quad (\text{Cons 2'-4})$$

$$P\left[\begin{aligned} &(1 - \bar{\gamma}_P^{(2)})C^{(1)}\bar{w}_{SP}^{(2)}\bar{r}_{SP}^{(2)} + (1 - \bar{\gamma}_P^{(2)})C^{(0)}w_{LP}^*\bar{r}_{LP} + (1 - \bar{\gamma}_A^{(2)})C^{(1)}\bar{w}_{SA}^{(2)}\bar{r}_{SA}^{(2)} \\ &+ (1 - \bar{\gamma}_A^{(2)})C^{(0)}w_{LA}^*\bar{r}_{LA} - (1 - \bar{u}^{(2)})\bar{h}^{(2)}C^{(1)} - \bar{u}^{(2)}(1 + \bar{d}^{(2)})\bar{\mu}^{(2)}C^{(1)} \leq c^{(1)} \end{aligned}\right] \leq \alpha_5, \quad (\text{Cons 2'-5})$$

$$\bar{w}_{SP}^{(2)}e + \bar{w}_{SA}^{(2)}e + \bar{u}^{(2)}(1 + \bar{d}^{(2)})\bar{\mu}^{(2)} \leq 1; \bar{w}_{SP}^{(2)}e \geq \bar{\gamma}_S^{(2)}; \bar{w}_{SP}^{(2)} \geq 0, \bar{w}_{SA}^{(2)} \geq 0, 0 \leq \bar{u}^{(2)} \leq 1. \quad (\text{Cons 2'-6,-7,-8})$$