CEO Horizon, Optimal Duration and the Escalation of Short-Termism*

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January 29, 2017

Abstract

This paper studies optimal CEO contracts when managers can manipulate their performance measure, sometimes at the expense of firm value. Optimal contracts defer compensation. The manager's incentives vest over time at an increasing rate and compensation becomes increasingly sensitive to short-term performance. This process generates an endogenous *CEO horizon problem* whereby managers intensify performance manipulation in the final years in office. Contracts are designed to foster effort while minimizing the adverse effects of manipulation. We characterize the optimal mix of short and long-term compensation along the manager's tenure, the optimal vesting period of incentive pay, and the resulting dynamics of managerial short-termism over the CEO's tenure.

1 Introduction

Short-termism seems prevalent among managers. Graham, Harvey, and Rajgopal (2005) find that 78% of U.S. CEOs are willing to sacrifice long-term value to beat market expectations.

*We thank Jeremy Bertomeu, Tim Baldenius, Peter Cziraki, Alex Edmans (discussant), Simon Gervais, Ilan Guttman, Zhiguo He, Adriano Rampini, Florin Sabac and Vish Vishwanathan for helpful comments, and seminar participants at Alberta (Canada), Duke University (Fuqua), Bocconi, NYU, SFS Finance Cavalcade, Tilburg, and FIRS Conference for helpful feedback.

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For example, Dechow and Sloan (1991) argue that, by the end of tenure, CEOs tend to cut *R&D* investments that –though profitable– have negative implications for the firm's reported earnings. Managerial short-termism has been an important concern for many years but it has taken a particular prominent role in recent years after the Enron scandal and the financial crisis in 2008.

To understand this phenomenon, the theoretical literature has adopted two approaches. One approach studies CEO behavior taking managerial incentives as given, thus being silent about optimal incentives (see e.g., Stein (1989)). But the complexity of CEO contracts in the real world (which include accounting-based bonuses, stock options, restricted stock, deferred compensation plans, clawbacks etc.) suggest that shareholders are aware of CEO's potential manipulations and design compensation to mitigate the consequences of such manipulations. An alternative approach studies optimal compensation contracts that are designed to fully remove CEO manipulations. In this class of models, manipulation is not observed on the equilibrium path (Edmans, Gabaix, Sadzik, and Sannikov (2012)). This approach is particularly helpful in settings in which CEO manipulations are too costly to the firm or easy to rule out, but cannot explain why manipulation seem so frequent in practice, and why real world contracts may tolerate them or even induce it (see Bergstresser and Philippon (2006)).

To understand this problem, we study optimal compensation contracts when CEOs exert hidden effort but can also manipulate their performance measure to increase their compensation, sometimes at the expense of firm value. Building on Holmstrom and Milgrom (1987) we consider a setting in which a risk averse CEO, who can save privately and consume continuously, exerts two costly actions: effort and manipulation. Both actions increase the manager's performance measure in the short run, but manipulation also has negative consequences for firm value. In the spirit of Stein (1989), we assume these consequences are not perfectly/immediately captured by the performance measure, but take time to be verified, potentially creating an externality when the CEO tenure is shorter than the firm's life.

Our paper makes two contributions to the literature. First, on the normative side, we study the contract that maximizes firm value in the presence of a realistic friction, namely performance manipulation. We characterize the mix of long and short-term incentives, the optimal duration of CEO pay throughout CEO tenure, and the ideal design of clawbacks and post-retirement compensation. Second, on the positive side, we are able to make predictions about the evolution of CEO manipulations along CEO tenure, and establish the existence of an endogenous CEO horizon problem.

Of course, a dynamic model is not necessary to explain the existence of manipulation; a static multitasking model such as that in Holmstrom and Milgrom (1991) would suffice for this purpose. Our main objective in this article, is to study the timing of manipulation: how it evolves over the CEO tenure and whether optimal contracts may generate an horizon effect in which the CEO distorts performance at the end of their tenure. Previous literature has shown that in dynamic settings one can implement positive effort and zero manipulation at the same time (unlike in static settings), by appropriately balancing the mix of short and long-run incentives. However, in our setting, inducing zero manipulation is not optimal: tolerating some manipulation is desirable because, in return, it allows the firm to elicit higher levels of effort than a manipulation-free contract. Furthermore, to fully discourage manipulations, the firm would have to provide the manager with a large post-retirement compensation package that ties his wealth to the firm's post-retirement performance. Such post-retirement compensation is costly to the firm, because it imposes risk on the manager during a time when effort does not need to be incentivized, and managers must be compensated for bearing this extra risk (see Dehaan et al. (2013)).

In the absence of the possibility of manipulation, short-term incentives, measured as the contract's pay-performance sensitivity (*PPS*) are constant over time, as in Holmstrom and Milgrom (1987). Unfortunately, the simplicity of this contract is broken under the possibility of manipulation. A constant *PPS* is no longer optimal for it induces excessive manipulation, particularly in the final years in office. Indeed, offering the manager a stationary contract would lead him to aggressively shift performance across periods, boosting current performance at the expense of firm value. To mitigate this behavior, an optimal contract calls for lower levels of short-term compensation and higher levels of long-term compensation, measured, roughly, as the present value of the contract's future *slopes*.

The optimal contract includes a post-retirement package that has the flavor of a clawback because it ties the manager's continuation value to the performance observed for some time after his retirement. This contracting tool is helpful but has limited power when the CEO is risk averse: even when the firm has the ability to tie forever –and to any degree – the manager's wealth to the firm's performance, the contract generally induces some manipulation. Though it would be possible to defer compensation long enough to deter manipulation altogether, firms might not do it given its cost. A key insight in this paper is that firms find more beneficial to defer compensation while the CEO is on the job than after he retires. This result has the implication that long term incentives are higher at the beginning of the CEO's tenure and decay toward the end. As a consequence, CEO manipulation increases towards the end, generating a CEO horizon effect.

We think that our model may help rationalize several facts documented in the executive compensation literature. First, the amount of CEO manipulation depends on the CEO horizon: it tends to accelerate prior to retirement being smaller for CEO's with longer horizons, who are more likely to pursue long term projects (Dechow and Sloan, 1991; Gonzalez-Uribe and Groen-Xu, 2015). Second, CEO pay duration is negatively associated to CEO manipulation, as measured by earning-increasing accruals. Also pay duration is shorter for older executives and executives with longer tenure (Gopalan et al., 2014). Third, equity vesting is correlated with short term behavior as measured by cuts in long-term investment (Edmans et al., 2013; Ladika and Sautner, 2016).

Empirically, CEO contracts defer compensation by including stock options and restricted stock, rather than just cash. This observation suggests that the duration of CEO pay is significantly higher than zero. Moreover the maturity of compensation changes throughout CEO tenure (Gopalan et al. (2014)). Both features of CEO contracts may speak to the possibility of performance manipulation. Indeed, in the model the possibility of manipulation impacts both the optimal pay duration and its evolution along CEO tenure. Specifically, the model predicts that pay duration decreases over time. At the beginning of tenure, pay duration is relatively high; a significant portion of the CEO pay is deferred. This is a double edge sword because these incentives eventually vest thereby boosting the manager's short-run incentives. If such process is not accompanied by similar increases in deferred compensation, pay maturity goes down thus stimulating not only the CEO effort but also his incentives to engage in manipulations. This effect is particularly acute in the final years in office, which are characterized by both a sharp decline in the maturity of CEO pay and by increasing manipulation intensity. Such phenomenon, whereby manipulation escalates over time, commonly referred to as the *CEO horizon problem*, is caused by the progressive vesting of CEO incentives and the relatively low pay duration that precedes retirement (Dechow and Sloan, 1991; Gibbons and Murphy, 1992).

The model predicts a subtle relation between corporate governance, as measured by the cost borne by the CEO from manipulating performance measures, the level of manipulation, and pay duration. Better governance results in stronger short-term incentives but, as a result, may also lead to higher levels of manipulation. This suggests that ignoring the endogeneity of CEO incentives may lead to wrongly conclude that short-term incentives cause the CEO's manipulation. A positive association between short-term incentives and performance manipulation may be the sign of healthy corporate governance.

Under the possibility of manipulation, optimal CEO contracts are non-linear, unlike in Holmstrom and Milgrom (1987). Following Edmans et al. (2012) we characterize the optimal contract within the subclass of contracts that implement deterministic sequences of effort and manipulation. Under such deterministic contracts, long-term incentives and effort are intertwined. Long-term incentives can be reduced only via increasing the current slope of short-term compensation, which necessarily distort the level of effort. This is why, in general, the optimal contract implements stochastic incentives. The benefit of providing stochastic incentives, which are history dependent and lead to stochastic effort and manipulation, resides precisely in allowing the principal to control the evolution of long-term incentives independently of the CEO's effort. We find that at the beginning of the CEO tenure, the sensitivity of long-term incentives to performance is positive; positive shocks increase de use of long term incentives. On the contrary, towards the end of tenure such sensitivity becomes negative; positive shocks accelerate vesting thereby reducing long term incentives. We find that long term incentives are mean reverting and they follow a target level. If due to their stochastic nature, the long-term incentives overshoot relative to the target level, the sensitivity of long-term incentives with respect to shocks becomes negative, in order to drive the long-term incentives back down.

Related Literature Beginning with Narayanan (1985), Dye (1988) and Stein (1989), a large literature in accounting, economics and finance studies the causes and consequences of performance manipulation in corporate settings. Most of the literature studying managerial short-termism has either taken incentives as exogenously given (Stein (1989); Fischer and Verrecchia (2000); Guttman et al. (2006), Kartik, Ottaviani, and Squintani (2007)) or it has restricted attention to static or two period settings with linear contracts which are unsuited to study the dynamics of short-termism and its relation to optimal long-term incentives (Baker, 1992; Goldman and Slezak, 2006; Dutta and Fan, 2014; Peng and Röell, 2014). A related strand of the literature examines optimal compensation contracts in the presence of moral hazard and adverse selection (Beyer et al. (2014); Maggi and Rodriguez-Clare (1995); Crocker and Slemrod (2007)).

A more recent literature, studies dynamic contracts under the possibility of manipulation (Edmans, Gabaix, Sadzik, and Sannikov, 2012; Varas, 2013; Zhu, 2013). This literature restricts attention to contracts that prevent manipulation altogether. Because we are interested in making predictions about the evolution of short-termism we consider more general contracts that implement optimal levels of manipulation.

On the technical side, we borrow heavily from Holmstrom and Milgrom (1987), Williams (2011), He, Wei, and Yu (2014) and Sannikov (2014). He et al. (2014) study dynamic contracts with moral hazard and learning. We follow He et al. (2014) and use a version of the model in Holmstrom and Milgrom (1987) that incorporates private savings. By introducing private savings, He et al. (2014) develop a surprisingly tractable model to analyze dynamic contracting problems with persistent private information. We model the inter-temporal effect of the CEO's action in a similar way as Sannikov (2014). Long-term incentives are required in Sannikov (2014) because effort today increases the CEO's productivity tomorrow. By contrast, the multi-tasking nature of our setting introduces a trade-off between short and long-term incentives. While short-term incentives induce effort, long-term incentives mitigate manipulation aimed at increasing today's performance at the expense of future cash flows.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 studies incentive compatibility. Section characterizes the optimal deterministic incentives. Section 6 considers the optimal non-deterministic contract. Section 7 discusses the empirical implications of the model, and finally Section 8 concludes.

2 Setting

We consider a continuous time agency problem in which the CEO can manipulate his performance measure. The CEO exerts two costly actions, effort a_t and manipulation m_t. The principal observes neither action. There is however a noisy performance measure. Let $\{B_t\}_{t\geq 0}$ be a standard Brownian motion in a probability space with measure $\mathbf{P}$, and let $\{\mathcal{F}_t\}$ be the filtration generated by B. For any $\mathcal{F}_t$ -adapted effort, a_t, and manipulation, m_t, processes, the firm's cash flow process is

$$dX_t = (a_t + m_t - \theta M_t)dt + \sigma dB_t,$$

where

$$M_t = \int_0^t e^{-\kappa(t-s)} m_s ds$$

is the stock of manipulation accumulated since the start of the manager's tenure until time t. The stock of manipulation depreciates at rate κ; thus its negative consequences are longer lived when κ is smaller. At each point, the marginal effect of the stock of manipulation on the firm cash flows is θ. When κ grows large or θ vanishes, manipulation does not have future cash-flow consequences being qualitatively equivalent to effort.

This way of modeling short-termism is similar to Stein (1989), and it captures the idea that manipulation is an action that increases short term cash flows but reduces future cash flows, eventually destroying value. Some examples of this type of manipulation arise when managers cut investment in advertising or *R&D*; offer excessive discounts to meet earnings expectations; or risk the firm's reputation by reducing product quality/safety. More generally, we can think of manipulation as any potentially unproductive action taken by the manager with the purpose of boosting the firm's short-run performance. Notice that unlike effort, manipulation is inherently dynamic: it increases today's performance but decreases the firm's performance in future periods. In essence, manipulation is a bad investment. So we can think of manipulation as a mechanism the manager might use to *borrow* performance from the future to boost current performance.[1]

Following Holmstrom and Milgrom (1987) we assume the CEO has exponential preferences given by

$$u(c, a, m) = -e^{-\gamma\left(c - h(a) - g(m)\right)}/\gamma$$

where $h(a) \equiv a^2/2$ and $g(m) \equiv gm^2/2$.

We follow the literature on costly state falsification (Lacker and Weinberg, 1989; Crocker

[1] Prior literature (see Dutta and Fan (2014)) has modeled the reversal of manipulation as taking place in the second period of a two-period setting. Though not fundamental, one of the benefits of our specification is its flexibility to accommodate different reversal speeds. In our model the effect of manipulation vanishes gradually based on κ. As we shall see, this parameter is a key determinant of the manager manipulation patterns.

and Morgan, 1998; Kartik, 2009) and assume that manipulation is costly to the manager. The cost of manipulation $g(m)$ captures the different personal costs the CEO bears from manipulating performance. This cost includes the effort required to find ways of distorting cash-flows, litigation risk, fines imposed by the SEC, or the natural dislike the CEO experiences from behaving in an unethical way.[2] We allow the cost parameter g to be zero, so our analysis also incorporates the case in which manipulation is costless.

The CEO is infinitely-lived but works for the firm for a finite period, $t \in [0, T]$. We refer to T as the manager's retirement date and $T - t$ as his horizon at time t. We also assume the contract can stipulate compensation beyond time T, until time $T + \tau$ for some $\tau \geq 0$. In other words, the manager's compensation can be made contingent on the outcome observed after his retirement. This possibility captures the principal's ability to implement clawbacks: a longer τ represents institutional settings where clawbacks are easier to enforce, and can be active for longer periods.

We make the assumption of a fix retirement date T for tractability. One can think of T, as the CEO retirement date or as an approximation to some predictable separation date.[3] The CEO's expected utility given a consumption flow $\{c_t\}_{t\geq 0}$ is

$$U(c, a, m) = E^{(a,m)} \left[\int_0^T e^{-rt} u(c_t, a_t, m_t) dt + \int_T^{T+\tau} e^{-rt} u^R(c_t) dt + e^{-r(T+\tau)} \frac{u^R(c_{T+\tau})}{r} \right],$$

where $u^R(c) \equiv u(c, 0, 0)$ is the flow utility after retirement. Notice that though the contractual relationship is finite, the CEO is infinitely lived, and the principal effectively controls the CEO's compensation over the CEO entire life, even after $T + \tau$. As it will be clear, this assumption allow us to study the effect of changes in the contracting environment while holding the length of the CEO's life expectancy constant. This way we cancel effects on the evolution of incentives driven by the shortening of the period available to the principal to compensate the CEO. Edmans et al. (2012) show that with a finite life, *PPS* rises at the end because there are fewer remaining periods to pay the agent his promised compensation. The same would be true in our setting if we assume the CEO life ends at $T + \tau$. Following He et al. (2014), we assume the CEO can save privately at the common interest rate r.

[2] This last interpretation is consistent with introspection and existent experimental evidence (Gneezy, 2005).

[3] Cziraki and Xu (2014) find that CEO terminations are concentrated around contracts' end dates.

The principal designs the contract to maximize firm value. A contract is a consumption process $\{c_t\}_{t\geq 0}$, and an effort-manipulation pair $\{(a_t, m_t)\}_{t\in[0,T]}$ adapted to the filtration generated by the performance measure X_t, which is denoted by $\{\mathcal{F}_t^X\}_{t\geq 0}$. Formally, the principal chooses the contract to maximize the present value of the firm's discounted cash flows, as given by

$$V(c, a, m) = E^{(a,m)}\left[\int_0^\infty e^{-rt}\big(dX_t - c_t\mathbf{1}_{\{t\leq T+\tau\}}dt\big) - e^{-r(T+\tau)}\frac{c_{T+\tau}}{r}\right].$$

The firm value is thus equal to the discounted stream of cash flows, net of the manager compensation, which includes a terminal bonus granting the manager a consumption flow $c_{T+\tau}$ from time $T + \tau$ onward.

Some parametric restrictions are in order. We assume the negative long-run effect of manipulation dominates the instant benefit, so manipulation destroys value, so manipulation is a negative NPV project. The following lemma is useful.

Lemma 1.

$$\frac{d}{dm_t}E_t\left[\int_t^\infty e^{-r(s-t)}dX_t\right] = 1 - \frac{\theta}{r+\kappa}$$

Hence, when θ is large relative to $r + \kappa$ manipulation is detrimental to firm value –yet potentially attractive to the manager. In other words, manipulation destroys firm value when either the stock of manipulation M_t does not depreciate too fast (i.e., $r + \kappa$ is large) or the marginal effect of manipulation on firm cash flows (i.e., θ is large). We can formally state our parametric assumption.

Assumption 1. Manipulation destroys firm value, or $\theta \geq r + \kappa$.

Before characterizing the contract, we note that the firm's expected cash flows can be expressed as a function of the manager's effort net of manipulation costs.

Lemma 2. *Given effort a and manipulation m processes, the present value of the firm cash-flows boils down to*

$$E\left[\int_0^\infty e^{-rt}a_t dt - \int_0^\infty e^{-rt}\lambda m_t dt\right],$$

where $\lambda = \frac{\theta}{r+\kappa} - 1$.

So λ captures the value destroying effect of manipulation on the firm cash flows. When $\lambda = 0$ the CEO manipulation has no cash flow effect; it only shifts income across periods, being similar to accrual earnings management. By contrast, when $\lambda > 0$ manipulation not only shifts performance across periods, but also has real effects.

We can now write the principal's expected payoff as

$$V(c,a,m) = E\left[\int_0^T e^{-rt}(a_t - \lambda m_t)dt - \int_0^{T+\tau} e^{-rt}c_t dt - e^{-r(T+\tau)}\frac{c_{T+\tau}}{r}\right].$$

By mildly generalizing the performance measure –in the way discussed in Section 7– the model can accommodate both the case of real earning management –i.e, when the CEO boosts performance by changing the firm's investment policy–, and the case of accounting manipulations –when the CEO boosts short term performance by manipulating accrual accounting policies.

3 Incentive Compatibility: the CEO's Problem

As a first step toward characterizing the optimal contract, we study incentive compatibility namely we characterize the conditions that a contract must satisfy to induce the CEO take a certain consumption, effort, and manipulation path. Given any arbitrary contract, the CEO solves the following optimization problem

$$\sup_{\hat{c},\hat{a},\hat{m}} U(\hat{c},\hat{a},\hat{m}) \tag{1}$$

subject to the inter-temporal budget constraint

$$dS_t = (rS_t - \hat{c}_t + c_t)dt, \;\; S_0 = 0. \tag{2}$$

We rely on the first order approach to derive incentive compatibility conditions by looking at the necessary first order conditions of the CEO's optimization problem given an arbitrary contract. As is the case for static principal-agent models, we must verify that the necessary conditions are sufficient. We relegate this to the appendix. The next proposition presents the CEO's incentive compatibility constraints.

Proposition 1. *A necessary condition and sufficient condition for the strategy* (a_t, m_t) *to be optimal given a contract* (c_t, ζ_t, β_t) *is that for any* $t \in [0, T]$

$$r\gamma h'(a_t) = \beta_t \tag{3}$$

$$g'(m_t) = \phi \frac{p_t}{W_t} + \frac{\beta_t}{r\gamma} \text{ if } m_t > 0 \tag{4}$$

$$g'(m_t) \geq \phi \frac{p_t}{W_t} + \frac{\beta_t}{r\gamma} \text{ if } m_t = 0, \tag{5}$$

where $\phi \equiv \frac{\theta}{r\gamma}$ *and* $(W_t, p_t)_{t \geq 0}$ *solves the backward stochastic differential equation*

$$dW_t = -\beta_t W_t \sigma dB_t, \ W_{T+\tau} = \frac{u^R(c_{T+\tau})}{r} \tag{6}$$

$$dp_t = \big[(r+\kappa)p_t + \beta_t W_t\big] dt + \zeta_t W_t \sigma dB_t, \ p_{T+\tau} = 0. \tag{7}$$

Finally, the private savings condition requires that for any $t \in [0, T+\tau]$ *consumption satisfies*

$$rW_t = u(c_t, a_t, m_t). \tag{8}$$

Equation (3) is the standard incentive compatibility constraint in continuous time principal-agent models. The marginal cost of effort must equal its marginal benefit, represented by the contract's pay-performance sensitivity β_t. Equation (4) is new; it says that the marginal cost of manipulation must equal the marginal benefit of manipulation, as represented by the extra compensation the manager gets today (β_t) net of the decrease in future compensation resulting from the lower future cash-flows caused by manipulation (p_t). Integrating equation (7) forward in time, yields

$$p_t = -E_t \left[\int_t^{T+\tau} e^{-(r+\kappa)(s-t)} \beta_s W_s ds \right]. \tag{9}$$

When choosing manipulation, the CEO trades off the current benefit in compensation given by $-\beta_t W_t$ with the future negative cash flow effect, appropriately discounted by the interest rate and the speed of mean reversion in cash flows. This relation shows why long term-incentives are crucial for discouraging manipulation: the presence of long term incentives means that compensation will go down in the future when manipulation reverses. The speed and intensity of this reversal determines how much effort the principal can induce without triggering manipulation.

Equation (8) is driven by the private savings assumption, and the absence of wealth effects, given our CARA utility specification. Due to consumption smoothing, the CEO's flow utility is the same as the annualized present value of future utility. This has the further implication that the continuation value W_t is a martingale.

We derive the conditions in Proposition 1 by analyzing the CEO's optimization problem given an arbitrary contract (c_t, a_t, m_t). The reader who is not interested in the details of this derivation can safely skip the remainder of this section. Given a contract recommending actions (c_t, a_t, m_t), and any strategy $(\hat{c}, \hat{a}, \hat{m})$ for the CEO, we denote the deviation from the contract by $\Delta c \equiv \hat{c} - c$, $\Delta a \equiv \hat{a} - a$ and $\Delta m \equiv \hat{m} - m$. The contract is incentive compatible if and only if $\Delta c = \Delta a = \Delta m = 0$ is optimal for the CEO. To simplify the notation, let $u_t \equiv u(c_t, a_t, m_t)$ be the CEO's utility flow if he follows the contract's recommendation and $u_t^\Delta \equiv u(c_t + \Delta c_t, a_t + \Delta a_t, m_t + \Delta m_t)$ be the utility flow from a deviation.

In general, a contract is a complicated function of the realized path of the performance measure X. This history dependence makes the analysis of the CEO's optimization problem involved. We follow Williams (2011) by making a change of measure that allows us to fix the probability measure of the expectation in (1). Let $\mathbf{P}$ be the probability measure under recommendation $\{(a, m)\}_{t\in[0,T]}$ and let $\mathbf{P}^\Delta$ be the probability measure induced by the deviation $\{(\hat{a}, \hat{m})\}_{t\in[0,T]}$. For any such deviation, define the exponential martingale

$$\xi_t := \exp\left(-\frac{1}{2}\int_0^t \eta_s^2 ds + \int_0^t \eta_s dB_s\right),$$

where

$$\eta_t := \frac{1}{\sigma}(\Delta a_t + \Delta m_t - \theta \Delta M_t).$$

By Girsanov's theorem, the Radon-Nikodym derivative between $\mathbf{P}^\Delta$ and $\mathbf{P}$ is given by $d\mathbf{P}^\Delta/d\mathbf{P} = \xi_{T+\tau}$ where

$$E\left(\xi_{T+\tau} \mid \mathcal{F}_t\right) = \xi_t.$$

Using iterated expectations, we can write the CEO's expected payoff given a deviation as

$$U(\hat{c}, \hat{a}, \hat{m}) = E^{(a,m)} \left[\int_0^{T+\tau} e^{-rt} \xi_t u_t^{\Delta} dt + e^{-r(T+\tau)} \xi_{T+\tau} \frac{u_{T+\tau}^{\Delta}}{r} \right]. \tag{10}$$

The change of variables in equation (10) is useful because it allows us to fix the expectation operator by introducing the new state variable ξ_t in the CEO's optimization problem. Using the representation in Equation (10) we can write the CEO's problem as

$$\sup_{\Delta c, \Delta a, \Delta m} U(c + \Delta c, a + \Delta a, m + \Delta m)$$

subject to

$$\begin{aligned} d\xi_t &= -\frac{\xi_t}{\sigma}(\Delta a_t + \Delta m_t - \theta \Delta M_t) dB_t \\ d\Delta M_t &= (\Delta m_t - \kappa \Delta M_t) dt \\ dS_t &= (rS_t - \Delta c_t) dt. \end{aligned}$$

This problem is a stochastic control problem that can be studied using the stochastic maximum principle – a generalization of Pontryagin's maximum principle to stochastic control problems (Yong and Zhou, 1999). As is common in optimal control theory, we begin by defining the (current value) Hamiltonian

$$\mathcal{H} = \xi u^{\Delta} + q^M (\Delta m - \kappa \Delta M) + q^S (rS - \Delta c) - \nu^{\xi} \frac{\xi}{\sigma} (\Delta a + \Delta m - \theta \Delta M), \tag{11}$$

The first step is to maximize the Hamiltonian with respect to the control variables. Because the Hamiltonian is jointly concave in $(\Delta c, \Delta a, \Delta m)$, it suffices to consider the first order conditions evaluated at $\Delta a = \Delta m = \Delta c = 0$. This yields

$$\begin{aligned} u_{a_t} &= \frac{\nu_t^{\xi}}{\sigma} \\ u_{m_t} &= \frac{\nu_t^{\xi}}{\sigma} - q_t^M \\ u_{c_t} &= q_t^S \end{aligned}$$

along with the respective complementary slackness conditions for the non-negativity constraint of m_t. The three adjoint variables (q_t^S, q_t^M, q_t^ξ) follow stochastic differential equations which are the stochastic analog to the differential equations in optimal control theory:

$$\begin{aligned}
dq_t^S &= rq_t^S dt - \frac{\partial \mathcal{H}_t}{\partial S_t} dt + \nu_t^S dB_t \\
&= \nu_t^S dB_t \\
dq_t^M &= rq_t^M dt - \frac{\partial \mathcal{H}_t}{\partial \Delta M_t} dt + \nu_t^M dB_t \\
&= \left((r+\kappa) q_t^M + \theta \frac{\nu_t^\xi}{\sigma} \right) dt + \nu_t^M dB_t \\
dq_t^\xi &= rq_t^\xi dt - \frac{\partial \mathcal{H}_t}{\partial \xi_t} dt + \nu_t^\xi dB_t \\
&= (rq_t^\xi - u_t) dt + \nu_t^\xi dB_t.
\end{aligned}$$

Furthermore, the adjoint equations must satisfy the transversality conditions $q_{T+\tau}^\xi = \frac{u_{T+\tau}}{r}$, $q_{T+\tau}^M = 0$, and $q_{T+\tau}^S = \frac{u_{c,T+\tau}}{r}$.

Theses equations are standard in the theory of stochastic optimal control but their economic intuition will become clear later.

We solve for q_t^ξ by integrating its SDE and using the transversality condition to get

$$q_t^\xi = E_t \left[\int_t^{T+\tau} e^{-r(s-t)} u_s ds + e^{-r(T+\tau)} \frac{u_{T+\tau}}{r} \right].$$

This shows that the adjoint variable q_t^ξ corresponds to the CEO's continuation value commonly used in dynamic contracting problems. Following the more familiar notation in the contracting literature, we denote the continuation value by $W_t \equiv q_t^\xi$ and we let $\nu_t^\xi \equiv -\beta_t W_t \sigma$ be the sensitivity of the continuation value to shocks, appropriately scaled by the CEO's continuation value

$$dW_t = (rW_t - u_t) dt - \beta_t W_t \sigma dB_t. \tag{12}$$

We can think of β_t as the contract pay-performance-sensitivity (PPS) namely the sensitivity of the CEO's continuation value with respect to one dollar of unexpected performance.[4] We

[4]In the case of contracts implementing a deterministic sequence of effort and manipulation, β_t is propor-

refer to β_t as the CEO's short-run incentives.

Equation (7) is obtained by simply defining $p_t \equiv q_t^M/\theta$ and plugging it in the SDE of q_t^M. As previously mentioned, the contracting variable p_t captures the present value of the contract's future "slopes." This variable represents the incentive strength of deferred compensation, being related to the contract's long-term incentives.

The final step is to consider the CEO's saving choice. Using, the first order condition for consumption, $u_c = q^S$ and the SDE for the adjoint variables q_t^S, we get that the evolution of the marginal utility of consumption is

$$du_{c_t} = \nu_t^S \sigma dB_t. \tag{13}$$

Equation (13) is the continuous time version of the traditional Euler equation for consumption, which states that the expected marginal utility of consumption must be constant when the CEO's discount rate is equal to the market's interest rate, otherwise the contract would induce the CEO to save privately.

We complete the analysis by solving equation (13) using a guess and verify approach. Following He et al. (2014), we conjecture that the CEO's continuation utility is equal to ru_t. Using the CARA utility specification we find that $u_c = -\gamma u = -\gamma r W$. Plugging this relation into (13) and setting $\nu_t^S = r\gamma\beta_t W_t \sigma$ we get

$$du_{ct} = -r\gamma dW_t = r\gamma\beta_t W_t \sigma dB_t. \tag{14}$$

Dividing by $-r\gamma$ we verify that (14) coincides with (12) so $u_{c_t} = -r\gamma W_t$ solves the adjoint equation for the CEO savings. This result, due to He et al. (2014), combines two observations. First, the CEO can smooth consumption intertemporally hence his marginal utility follows a martingale. As mentioned above, predictable changes in the CEO's continuation value induce the CEO to save privately. Second, due to our CARA specification, the CEO's continuation value is linear in flow utility.

Finally, combining the first order conditions and the private saving condition $u = rW$ we

tional to the sensitivity of consumption to cash-flows that corresponds to the traditional definition of PPS in the empirical literature adapted to our setting.

get the necessary incentive compatibility constraints:

$$r\gamma h'(a_t) = \beta_t \tag{15}$$

$$r\gamma g'(m_t) = \beta_t + \theta \frac{p_t}{W_t}, \tag{16}$$

along with the respective complementary slackness condition. Equations (7) is derived by plugging $u_t = rW_t$ into equation (12) and defining $\zeta_t W_t \sigma \equiv \nu_t^M$ in equation (7).

At this point, it remains to show that the first order approach is valid so the necessary conditions are also sufficient. Being more technical, this step is relegated to the appendix.

4 Principal's Problem: the Optimal Contract

Having determined the incentive compatibility constraints, we can write the principal's problem recursively. Because we have a persistent state variable M_t it is no longer possible to use the continuation value as the sole state variable; we also have to keep track of long term incentives. Hence, in principle we need to solve a two dimensional stochastic control problem. However, the absence of wealth effects in the manager's CARA preferences, together with the presence of private savings, allow us to eliminate the continuation value from the principal's objective function. Moreover, the scale invariance of exponential utility allows us to write the problem as a function of only one state variable, namely $z_t \equiv -p_t/W_t$.

The principal's original optimization problem is

$$V(W_0, p_0) = \sup_{c,a,m,\beta,\zeta} V(c, a, m)$$

subject to

$$\begin{aligned} dW_t &= -\beta_t W_t \sigma dB_t \\ dp_t &= \left[(r+\kappa)p_t + \beta_t W_t\right] dt + \zeta_t W_t \sigma dB_t \\ rW_t &= u(c_t, a_t, m_t) \\ &\quad \text{IC constraints (3)-(5).} \end{aligned}$$

The private savings condition, $rW_t = u(c_t, a_t, m_t)$ determines the consumption process as a function of W_t, a_t, and m_t

$$c_t = h(a_t) + g(m_t) - \frac{\log(-r\gamma W_t)}{\gamma}. \tag{17}$$

Equation (17) allows us to eliminate the consumption process in the objective function and write the expected profits as a function of a_t, m_t, and W_t alone. If we replace (17) in the objective function we find that the principal's payoff is[5]

$$E\left[\int_0^T e^{-rt}\left(a_t - \lambda m_t - h(a_t) - g(m_t)\right)dt + \int_0^{T+\tau} e^{-rt}\left(\frac{\log(-W_t)}{\gamma} + \frac{\log(r\gamma)}{\gamma}\right)dt + \frac{\log(-r\gamma W_{T+\tau})}{r\gamma}\right]. \tag{18}$$

We can compute the expected value of $\log(-W_t)$ using Itô's Lemma and write the principal's payoff as

$$\text{constant}\times\log(-W_0)+E\left[\int_0^T e^{-rt}\left(a_t - \lambda m_t - h(a_t) - g(m_t)\right)dt - \int_0^{T+\tau} e^{-rt}\frac{\sigma^2\beta_t^2}{2r\gamma}dt\right]. \tag{19}$$

The first integral captures the cash flows realized throughout the CEO tenure. The second integral captures the cost of risk borne by the manager since the beginning till the the end of the clawback period, at time $T+\tau$.

Having eliminated the dependence of the principal's payoff upon the manager's continuation value, we proceed to eliminate such dependence from the constraints. We reduce the dimensionality of the problem by defining the state variable $z_t \equiv -p_t/W_t$, which captures the present value of the contract future *PPS* conveniently normalized by the CEO's continuation utility. Hereafter, we refer to z as the contract's long-run incentives. Using the law of motion of W_t and p_t in (6) and (7), along with Ito's lemma, we find that z follows the

[5]The expected value of $\log(-W_t)$ is

$$E[\log(-W_t)] = \log(-W_0) - \frac{1}{2}E\left[\int_0^t \sigma^2\beta_s^2 ds\right].$$

If we change the order of integration we obtain that

$$E\left[\int_0^{T+\tau} e^{-rt}\int_0^t \sigma^2\beta_s^2 ds dt\right] = E\left[\int_0^{T+\tau}\left(e^{-rt} - e^{-r(T+\tau)}\right)\frac{\sigma^2\beta_t^2}{r}dt\right].$$

Replacing this equation in in (18) we arrive at a compact characterization of the principal's payoff in (19).

following stochastic differential equation

$$dz_t = [(r+\kappa)z_t + \beta_t(\sigma v_t - 1)]dt + v_t dB_t, \tag{20}$$

where $v_t \equiv \sigma(\beta_t z_t - \zeta_t)$ and the incentive compatibility constraint is defined by $g'(m_t) = h'(a_t) - \phi z_t = (a_t - \phi z_t)/g$. We have reduced the principal's problem to a finite horizon one dimensional stochastic control problem:

$$F(z) = \sup_{a,m,\beta,v} E\left[\int_0^T e^{-rt}\left(a_t - \lambda m_t - h(a_t) - g(m_t)\right)dt - \int_0^{T+\tau} e^{-rt}\frac{\sigma^2\beta_t^2}{2r\gamma}dt\right]$$

subject to the law of motion of z_t in (21) and the manager's participation and incentive compatibility constraints. The principal chooses short-term incentives β_t and long-term incentives z_t to maximize firm value. He also chooses the volatility of long-term incentives v_t. The latter choice allows the principal to modify the vesting speed of long-term incentives based on the evolution of performance.

4.1 Infinite Tenure and Irrelevance of Manipulation

In this section we consider a variation of the moral-hazard-without-manipulation problem studied by Holmstrom and Milgrom (1987) but allow for both intermediate consumption and private savings. This problem serves as a benchmark for evaluating the effects of short-termism on firm value.

In the absence of manipulation (namely when the manager cannot manipulate the performance measure) deferring compensation beyond time T plays no role, hence the principal's problem boils down to

$$\max_{a_t} E\left[\int_0^T e^{-rt}\left(a_t - h(a_t) - \frac{\sigma^2 r\gamma h'(a_t)^2}{2}\right)dt\right].$$

We can maximize effort point-wise to obtain the optimal effort as given by

$$a^{HM} \equiv \frac{1}{1+r\gamma\sigma^2}.$$

This is the optimal effort in absence of performance manipulation concerns. We have thus recovered the result that with CARA preferences and Brownian shocks the optimal contract is linear and implements constant effort. This is also the level of effort arising in the private savings CARA model in He, Wei, and Yu (2014).

In practice, the benefits of non-linear contracts are controversial. In fact, the compensation literature often argues that firms should get rid of non-linearities in compensation schemes to prevent performance manipulation. For example, Jensen (2001, 2003) argues that non-linearities induce managers to manipulate compensation over time and distract them from focusing on the long-term. Consistent with this view, we next show that even when manipulation is possible, the above linear contract remains optimal as long as the CEO works forever. More precisely, we show that when $T = \infty$ the optimal contract is linear and involves zero manipulation. In fact, a perpetual linear contract aligns the incentives of the principal and the CEO eliminating the manager's incentive to manipulate performance over time. Later we show that a no manipulation contract is no longer optimal when $T < \infty$ even though it might still be feasible. The no manipulation constraint can be written as:

$$\frac{\beta_t}{r\gamma} - \phi E_t \left[\int_t^\infty e^{-(r+\kappa)(s-t)} \frac{\beta_s W_s}{W_t} ds \right] \leq 0.$$

If the pay-performance β_t is constant, then the no manipulation constraint can be reduced to

$$\beta \left(\frac{1}{r\gamma} - \phi \int_t^\infty e^{-(r+\kappa)(s-t)} ds \right) \leq 0,$$

where we use the result that W_t is a martingale. This condition is always satisfied under the conditions that manipulation destroys value (i.e., $\theta \geq r + \kappa$). Hence, we have verified that the optimal contract in the relaxed problem that ignores the no-manipulation constraint continues to be feasible when we incorporate the no-manipulation constraint. When the manager tenure is infinite, the possibility of manipulation is thus irrelevant and the optimal contract implements no manipulation, being identical to that in Holmstrom and Milgrom (1987). We summarize the previous discussion in the following proposition.

Proposition 2. *When $\theta \geq r + \kappa$ and $T = \infty$ the optimal contract entails zero manipulation. The optimal contract is stationary implementing $a_t = a^{HM}$.*

We next tackle the main problem of the paper, which arises when the CEO tenure is finite. We do it in two steps. In Section 4, we consider the case of deterministic incentives and then, in Section 5, we allow for stochastic incentives.

5 Deterministic Incentives

In general, the effort and manipulation processes implemented by the optimal contract are stochastic but, following Edmans et al. (2012) and He et al. (2014), we begin by looking at the subclass of contracts that implement deterministic sequences of effort and manipulation. By definition, long-term incentives z_t are given by

$$z_t = -\frac{p_t}{W_t} = E_t\left[\int_t^{T+\tau} e^{-(r+\kappa)(s-t)}\frac{\beta_s W_s}{W_t}ds\right].$$

When incentives are deterministic, we can substitute $E_t(\beta_s W_s) = \beta_s W_t$ to eliminate the expectation and get

$$z_t = \int_t^{T+\tau} e^{-(r+\kappa)(s-t)}\beta_s ds.$$

As mentioned above, long-term incentives are computed as the present value of the contract future "slopes" using an adjusted discount rate $r+\kappa$ to take into account the fact that the stock of manipulation depreciates at rate κ. Using the incentive compatibility constraint (3) we arrive at

$$z_t = r\gamma\int_t^{T} e^{-(r+\kappa)(s-t)}h'(a_s)ds + \int_T^{T+\tau} e^{-(r+\kappa)(s-t)}\beta_s ds.$$

Before retirement, effort is proportional to the contract *PPS*. After retirement, there is no effort but performance sensitivity remains positive to deter previous manipulations. The principal's problem boils down to the following deterministic optimal control problem

$$F(z_0) = \sup_{a,m,\beta}\int_0^T e^{-rt}\left(a_t - \lambda m_t - h(a_t) - g(m_t)\right)dt - \int_0^{T+\tau} e^{-rt}\frac{\sigma^2\beta_t^2}{2r\gamma}dt$$

subject to the equation describing the evolution of long-term incentives

$$\dot{z}_t = (r+\kappa)z_t - \beta_t. \tag{21}$$

and the incentive compatibility constraints. We solve the problem by backward induction, optimizing over two consecutive periods –pre and post-retirement– separated by the manager's retirement date T. In the next section, we study the optimal compensation during the post-retirement period, which specifies the optimal way to defer compensation after retirement.

5.1 Post Retirement Compensation

A potentially important aspect of a CEO contract is to define the compensation the manager will receive after retirement and how such compensation is tied to the firm's performance. Intuitively, by linking the evolution of the manager's wealth to the firm's post-retirement performance, in the spirit of clawbacks, the firm can mitigate the manager's short-termism in the final years in office. This is however costly: linking the manager's compensation to post-retirement performance imposes risk on the manager during a time the manager will not exert effort. In this section, we study how such trade-off determines the optimal structure of post-retirement compensation given the contract's promised post-retirement incentives, z_T.

Given a promise z_T the principal must distribute the payments to the manager over the clawback period $[T, T+\tau]$ to minimize the overall cost of providing such incentives. This problem can be formulated as the following cost minimization problem

$$\min_{\beta} \int_T^{T+\tau} e^{-r(t-T)} \frac{\sigma^2 \beta_t^2}{2r\gamma} dt \text{ s.t } z_T = \int_T^{T+\tau} e^{-(r+\kappa)(t-T)} \beta_t dt \tag{22}$$

This formulation shows that providing post-retirement compensation is particularly costly to the firm when cash flows are noisy or the manager is risk-averse. Risk aversion also explains why the flow cost to the firm is convex (quadratic) in β_t, which means the principal will prefer to smooth out the stream of payments to the manager over the entire claw-back period rather than cluster them around a shorter period of time within $[T, T+\tau]$.

The Lagrangian of this minimization problem is

$$\mathcal{L} = \int_T^{T+\tau} e^{-r(t-T)} \frac{\sigma^2 \beta_t^2}{2r\gamma} dt + \ell \left(z_T - \int_T^{T+\tau} e^{-(r+\kappa)(t-T)} \beta_t dt \right),$$

where ℓ is the multiplier for the z_T-constraint. We can maximize point-wise to get

$$\beta_t = \ell \frac{2r\gamma}{\sigma^2} e^{-\kappa(t-T)}$$

and find the value of ℓ by plugging β_t into the constraint. The evolution of PPS during the clawback is thus proportional to $e^{-\kappa(t-T)}$ and z_T. Consequently, the cost of providing the manager incentives z_T is

$$\Psi(z_T) = \frac{1}{2}\mathcal{C}z_T^2,$$

where

$$\mathcal{C} \equiv \frac{\sigma^2(r+2\kappa)}{r\gamma\left(1-e^{-(r+2\kappa)\tau}\right)}. \tag{23}$$

Hence, the cost of post-retirement incentives increases in the size of those incentives z_T in a convex fashion, which among other things means the principal dislikes uncertainty about the level of post-retirement incentives. On the surface, this might suggest that stochastic incentives are suboptimal, but as we will discuss later, the possibility of stochastic incentives provides other important contractual benefits to the principal.

We conclude by studying the determinants of the marginal cost of post retirement compensation:

Corollary 1. *The marginal cost of post-retirement incentives is increasing in the volatility of the performance measure σ, the CEO's risk aversion parameter γ^{-1}, the depreciation rate of manipulation stock κ. By contrast, the marginal cost of post-retirement incentives is decreasing in the interest rate r, and the length of the clawback period τ.*

As $\tau \to 0$, $\mathcal{C} \to \infty$ which means that the cost of post-retirement incentives becomes prohibitive if the clawback period shrinks ($\tau \to 0$), for any given $z_T > 0$. The reason is again risk-aversion: a significant portion of the manager's continuation value would be subject to risk in a short time period.

Notice that when $\tau = 0$ we have $z_T = 0$. When $\tau \to \infty$, we have $\mathcal{C} = \frac{\sigma^2(r+2\kappa)}{r\gamma} > 0$. So providing post-retirement incentives is costly even when compensation can be deferred forever, particularly if the manager is risk-averse or performance is too volatile. There is thus a significant difference between extending the CEO horizon T versus extending the clawback period τ. As we will see, the former can fully eliminate manipulation, the latter can only partially mitigate manipulation, but its effect is limited when managers are risk averse.

In general, providing post-retirement incentives is more costly when the performance measure is more volatile, the manager is more risk averse, and the stock of manipulation depreciates faster. In the next section, and before solving for the optimal pre-retirement compensation scheme, we study the best contract that implements zero manipulation.

5.2 Best No-Manipulation Contract

Prior research focuses on contracts that implement zero manipulation (Edmans, Gabaix, Sadzik, and Sannikov, 2012; Varas, 2013; Zhu, 2013). Here, we study the contract that induces zero manipulation $m_t = 0$ throughout the CEO tenure. Such a contract can be represented by the following optimization problem:

$$\begin{aligned}
&\max_{z_0,a} \int_0^T e^{-rt}\left(a_t - \frac{(1+r\gamma\sigma^2)a_t^2}{2}\right)dt - e^{-rT}\Psi(z_T)\\
s.t.&\\
&\dot{z}_t = (r+\kappa)z_t - r\gamma a_t\\
&a_t \leq \phi z_t.
\end{aligned}$$

The principal maximizes the firm's cash flows, net of the manager's pre and post-retirement compensation. The first constraint describes how long-term incentives must evolve to satisfy the incentive compatibility of the manager's actions. The second constraint is required to ensure that the manager always prefers zero manipulation. Intuitively, this constraint says that long-term incentives z should be large enough relative to short-term incentives to prevent manipulation.

We solve the problem using Pontryagin's maximum principle for problems with mixed-state constraints. The Hamiltonian of the optimal control problem is

$$\mathcal{H}(z_t, a_t, m_t, \psi_t) = \pi(a_t, m_t) + \psi_t((r+\kappa)z_t - r\gamma a_t).$$

Given the constraints of the optimization problem, we must consider the augmented Hamiltonian

$$\mathcal{L}(z_t, a_t, m_t, \psi_t, \mu_t) = \mathcal{H}(z_t, a_t, m_t, \psi_t) + \mu_t(\phi z_t - a_t)$$

where μ_t is a Lagrange multiplier and the adjoint variable ψ_t solves the initial value problem

$$\dot{\psi}_t = -\kappa\psi_t - \phi\mu_t,\ \psi_0 = 0.$$

with the transversality condition $\psi_T = -\mathcal{C}z_T$. The optimal effort solves the first order condition

$$a_t = \frac{1 - r\gamma\psi_t - \mu_t}{1 + r\gamma\sigma^2}.$$

If the no manipulation constraint is not binding before time t, then we have that $\psi_t = \mu_t = 0$ and so $a_t = a^{nm}$ and the effort implemented is the same as in the absence of manipulation concerns. However, when the no manipulation constraint is binding, the level of effort implemented needs to account for present and future constraints. When the no manipulation constraint is binding, increasing effort requires to increase long term incentives today. The multiplier μ_t represents the shadow cost of such increment at t while ψ_t represents the effect of the associated increment of long term incentives in future periods. Integrating the equation for ψ_t and using the transversality condition we get

$$\psi_t = -\mathcal{C}z_T + \phi \int_t^T e^{\kappa(s-t)}\mu_s ds. \tag{24}$$

The co-state variables has the interpretation of the derivative of the value function with respect to the state, that is $\psi_t = F_z(z_t, t)$. Equation (24) thus shows that the marginal effect of increasing long term incentives today has two effects. It relaxes the no manipulation constraint in all future periods with an associated shadow benefit of μ_t. However this increment in long term incentives also increases the cost of providing post-retirement compensation after time T. The following proposition presents the optimal effort path implemented by the zero manipulation contract.

Proposition 3. *In the best contract implementing zero manipulation there is $t^* \leq T$ such that $a_t = a^{nm}$, all $t \in [0, t^*)$ and $a_t = \phi z_t$ for all $t \in [t^*, T]$.*

In particular,

$$\phi^2 \frac{2\kappa + r - 2\theta + (\theta - \kappa)\, e^{-(\theta - r - \kappa)T} + e^{(\theta-\kappa)T}\,(\theta - r - \kappa)}{\mathcal{C}e^{-(\theta-r-\kappa)T}\,(2\theta - 2\kappa - r)\,(\theta - \kappa)} \leq a^{HM}$$

Then the optimal effort is $a_t = \phi z_t$, where $z_t = e^{-(\theta - r - \kappa)t} z_0$ and

$$z_0 = \frac{\phi\left(1 - e^{(\theta-\kappa)T}\right)/(\theta - \kappa)}{\mathcal{C}e^{(r+\kappa-\theta)T} - \phi^2\left(1 + r\gamma\sigma^2\right)\frac{e^{(r+\kappa-\theta)T} - e^{(\theta-\kappa)T}}{2\theta - 2\kappa - r}}$$

Otherwise, let $t^* > 0$ *be the unique solution to*

$$\frac{\phi}{\theta-\kappa} + \frac{\phi\left[e^{-(\theta-r-\kappa)(T-t^*)} + e^{(\theta-\kappa)(T-t^*)}\frac{\theta-r-\kappa}{\theta-\kappa}\right]}{2\kappa-2\theta+r} = \frac{-\mathcal{C}e^{-(\theta-r-\kappa)(T-t^*)}}{\phi}a^{HM}$$

The optimal effort is $a_t = a^{HM}$ *if* $t < t^*$ *and* $a_t = \phi z_t$ *if* $t \in [t^*, T]$*, where*

$$z_t = e^{-(\theta-r-\kappa)(t-t^*)}\frac{a^{HM}}{\phi}, \;\; t \in [t^*, T].$$

The optimal level of effort implemented by the contract takes the following form. The manager's tenure can be divided in two phases. In the early years of office, effort is relatively high, coinciding with the optimal effort implemented in absence of manipulation concerns, a^{HM}. In the second phase, as the manager approaches T the manipulation constraint binds, forcing the principal to lower the short-term incentives, which in turn leads to a decreasing effort profile.

Figure 1 shows the dynamics of long term incentives and effort. At the beginning of the manager's tenure, when the remaining tenure is long, the firm can implement the same level of effort that would prevail without the possibility of manipulation. However, this is optimal only for a limited period of time. After time t^*, the possibility of manipulation induces the firm to distort downward the contract *PPS*. In that sense, misreporting only has a significant effect to the extent that the manager's horizon is or has become short. As in Holmström (1999), moral hazard exacerbates over time but not because career concerns become weaker, but because the contract's incentives make the manager increasingly myopic. The principal implements positive manipulation by the end of the CEO tenure; the ability to provide long-term incentives and the relatively fast reversal of manipulation makes this optimal: the manager can be threaten with a sharp decrease in compensation if the firm performance suddenly declines, thereby making the manager internalize some of the consequences of manipulation.

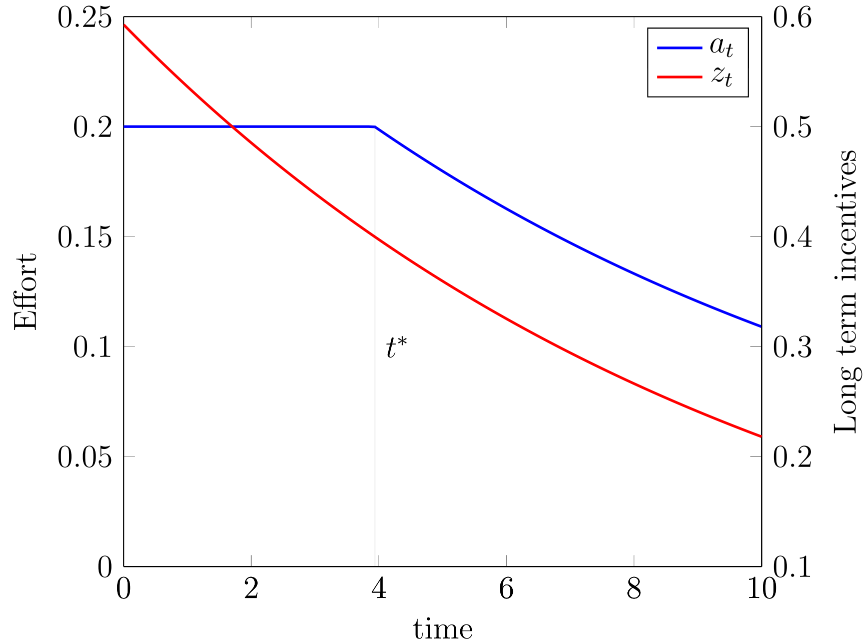

Figure 1: Parameters: $r = 0.1$, $\gamma = 1$, $g = 1$, $\theta = 0.5$, $\kappa = 0.3$, $\sigma = 2$, $T = 10$, $\tau = 1$

5.3 Optimal Deterministic Contract

The contract that induces zero manipulation is suboptimal in general. Tolerating some manipulation is desirable for two reasons: first, it allows the firm to implement higher levels of effort than a zero manipulation contract and, second, it exposes the manager to lower levels of risk by reducing the use of deferred compensation at the end of tenure.

In this section, we characterize the best contract among the subclass of contracts that implement deterministic sequences of effort and manipulation. Hence optimal contract in this section means the best contract among those implementing deterministic actions. As in the previous section, the evolution of long-term incentives specializes to

$$\dot{z}_t = (r + \kappa)z_t - r\gamma a_t. \tag{25}$$

Equation (25) reveals a fundamental limitation of a deterministic contract: effort and long-term incentives are intertwined: to reduce long term incentives, the contract must increase the level of effort and vice versa. Later in Section 6, we show that a fully history dependent contract (with stochastic effort) relaxes this link. Let us shorten notation by defining the *flow* profit function

$$\pi(a,m) \equiv a - \lambda m - \frac{gm^2}{2} - \frac{\left(1 + r\gamma\sigma^2\right)a^2}{2}.$$

We consider the following optimal control problem for the principal

$$\max_{z_0,a,m\geq 0} \int_0^T e^{-rt}\pi(a_t,m_t)dt - e^{-rT}\Psi(z_T)$$

$$\begin{aligned}
&s.t. \\
&\dot{z}_t = (r+\kappa)z_t - r\gamma a_t \\
&m_t \geq \frac{a_t - \phi z_t}{g} \\
&m_t \geq 0.
\end{aligned}$$

The formulation of the incentive compatibility constraint here is slightly different from that in Proposition 1. The benefit of this formulation is that the feasible set becomes convex. Though seemingly more restrictive, this formulation is equivalent to that in Proposition 1 because the principal would never implement positive manipulation if the manipulation incentive compatibility constraint were slack. The previous problem is an optimal control problem with mixed state constraints.[6] The main challenge in the characterization of the solution is the determination of the time intervals over which each of these constraints are binding. We can solve for the optimal path in closed form (up to the solution of a nonlinear equation); however, the solution is lengthy and tedious so we relegate it to the Appendix. The following proposition characterizes the optimal path of manipulation.

Proposition 4. *In the optimal contract, the effort, manipulation and long-term incentives satisfy the following:*

1. *Long term incentives, z_t, are non-increasing.*

2. *Manipulation, m_t, is non-decreasing.*

3. *In general, there are three regions characterized by thresholds $t^* \leq t^{**}$.*

    - *In the first region, $[0, t^*)$ there is no manipulation and the level of effort is the same as in the problem without manipulation (that is, $a_t = a^{HM}$).*

[6]Hartl et al. (1995) presents a survey of the maximum principle for this kind of problems.

- *In the second region, $(t^*, t^{**}]$, there is no manipulation but the level of effort is limited by long term incentives ($a_t = \phi z_t$).*
- *In the third region, $(t^{**}, T]$ manipulation is positive and increases over time.*
- *Depending on parameters the regions $(t^*, t^{**}]$ and $(t^{**}, T]$ can be empty.*

The case in which the region $(t^{**}, T]$ is empty corresponds to the case in which the no manipulation contract identified in the previous section is optimal. Figure 2 shows a numerical example in which the three regions identified above are present. Surprisingly, effort is non-monotone, it decreases at the beginning and increases toward the end. This is not true in general, and depending on the parameters, effort can be either increasing or decreasing in the final region $(t^{**}, T]$. Why is effort sometimes increasing? What happens here is that vesting of long term compensation accelerates toward the end of the CEOs tenure boosting short term incentives. This acceleration of vesting has a different nature than that in Edmans et al. (2012). In their model, the CEO is finitely lived so vesting accelerates at the end because there are fewer period to compensate the CEO before the final date: payments have to be spread within a shorter time span. In our setting, the CEO is infinitely lived so in principle we do not have to accelerate vesting to satisfy the promise keeping constraint. In our setting, vesting accelerates because deferring compensation after retirement is more costly than deferring compensation while the CEO is working at the company. In fact, having a significant level of deferred compensation when the CEO retires is so costly that the amount of vesting accelerates towards the end of the CEO tenure to lower the level of post-retirement incentives. Hence, PPS increases significantly thereby increasing both the effort and manipulation levels.

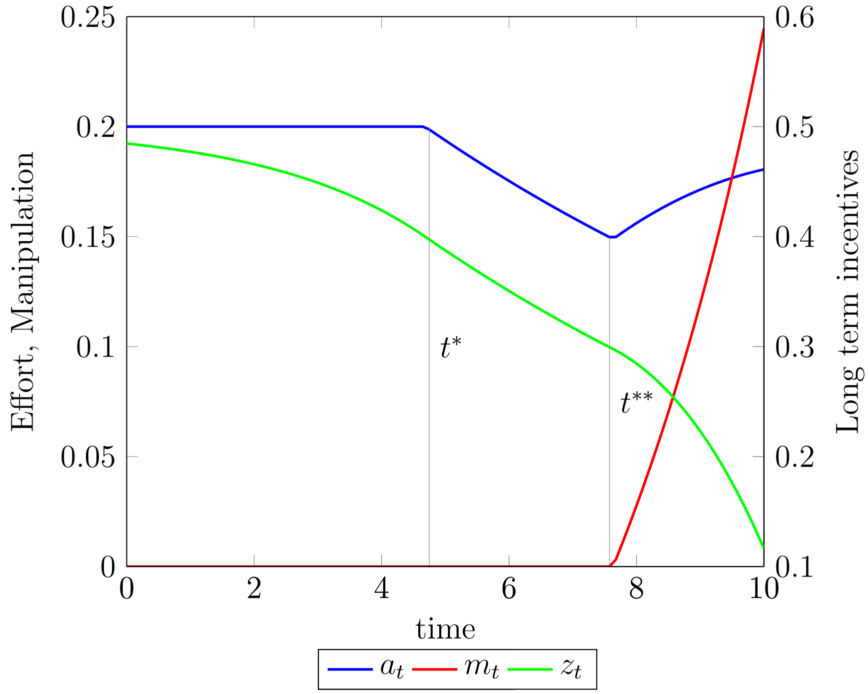

Figure 2: Parameters: $r = 0.1$, $\gamma = 1$, $g = 1$, $\theta = 0.5$, $\kappa = 0.3$, $\sigma = 2$, $T = 10$, $\tau = 1$

The optimal contract induces positive manipulation but not necessarily in every instant of the manager's tenure. In fact the CEO tenure consists of three distinct phases ranked by the intensity of manipulation. During the first phase, manipulation incentives are weak, because the CEO horizon is long which means the principal has enough time to "detect" and penalize the manager's manipulation. As a consequence, short run incentives are high and the manager exerts high effort and zero manipulation. During the second phase, the manager's manipulation incentives bind, but the contract still implements zero manipulation. However, to prevent manipulation, the principal is forced to distort the contract *PPS* downward which leads to a pattern of decreasing effort. During the third phase, manipulation incentives become even stronger, so preventing manipulation becomes more difficult and is no longer optimal. Long-term incentives have to mature over time as the manager approaches retirement, and this process strengthens short-term incentives. In turn, this triggers manipulation but may also boost effort in the final years. We can think of these two effects as mirror images: providing high post-retirement compensation is costly. To reduce it, some of the contract's long-term incentives must mature which in turn increases short run incentives.

The relative length of the three phases in the manager's tenure depends on the severity of the manipulation problem. Thus, for instance when the reversal is slow (low θ), enforcement is weak (low τ), or manipulation is easy (low g), the relative importance of the third phase grows at the expense of the other two phases, especially the first one. We discuss this and other comparative statics in Section 7 where we also look at the empirical implications of the model.

In the remainder of this section we analyze the principal's optimization problem. We solve the problem using Pontryagin's maximum principle. The current value Hamiltonian for this optimal control problem is

$$\mathcal{H}(z_t, a_t, m_t, \psi_t) = \pi(a_t, m_t) + \psi_t((r+\kappa)z_t - r\gamma a_t).$$

We incorporate the different constraints by considering the augmented Hamiltonian

$$\mathcal{L}(z_t, a_t, m_t, \psi_t, \eta_t, \nu_t) = \mathcal{H}(z_t, a_t, m_t, \psi_t) + \eta_t\left(m_t - \frac{a_t - \phi z_t}{g}\right) + \nu_t m_t$$

where η_t and ν_t are Lagrange multipliers. The co-state variable ψ_t solves the initial value problem

$$\dot{\psi}_t = -\kappa\psi_t - \frac{\phi}{g}\eta_t, \;\; \psi_0 = 0.$$

together with the transversality condition $\psi_T = -\mathcal{C}z_T$. Maximizing the augmented Hamiltonian, we find the optimal effort and manipulation solving the first order conditions

$$\begin{aligned} 1 - \left(1 + r\gamma\sigma^2\right) a_t - r\gamma\psi_t - \frac{\eta_t}{g} &= 0 \\ -\lambda - g m_t + \eta_t + \nu_t &= 0. \end{aligned}$$

In addition, we also consider the complementary slackness conditions

$$\begin{aligned} &\eta_t = 0 \text{ if } m_t > \frac{a_t - \phi z_t}{g} \\ &\nu_t = 0 \text{ if } m_t > 0. \end{aligned}$$

We have that $\nu_t = 0$ If $m_t > 0$, in which case we get the optimal effort

$$a_t = \frac{g - \lambda + \phi z_t - r\gamma g \psi_t}{1 + g\left(1 + r\gamma\sigma^2\right)}. \tag{26}$$

This equation shows the determinants of effort. The relation between effort and ψ_t is intuitive: recall that $\psi_t < 0$ represents the shadow marginal cost of providing long-term incentives. The level of effort is higher when maintaining long term incentives is more costly because the firm has stronger incentives to accelerate vesting of long-term incentives. Similarly, providing more long term incentives (higher z_t) leads to higher effort as we can increase the amount of effort while keeping manipulation at moderate levels. Manipulation is interior only if $m_t = (a_t - \phi z_t)/g > 0$ which requires that $a_t > \phi z_t$.

We can replace a_t to find the condition

$$\frac{g - \lambda - r\gamma g \psi_t}{g\left(1 + r\gamma\sigma^2\right)} \geq \phi z_t \tag{27}$$

Whenever this condition is not satisfied, the optimal solution must have zero manipulation. With zero manipulation we must consider the two cases that we already considered in Section 5.2: i) $a_t = \phi z_t$ and ii) $a_t > \phi z_t$. When the solution is given by case i), we compute the multiplier η_t using the first order condition for a_t. This yields

$$\eta_t = g\big(1 - \left(1 + r\gamma\sigma^2\right)\phi z_t - r\gamma\psi_t\big). \tag{28}$$

This solution satisfies the optimality condition only if the multiplier η_t is positive. If neither inequality (27) nor $\eta_t > 0$ in equation (28) are satisfied, then it must be the case that $a_t < \phi z_t$. In this final case, $\eta_t = 0$ and so

$$a_t = \frac{1 - r\gamma\psi_t}{1 + r\gamma\sigma^2}.$$

Figure 3 summarizes the previous analysis and illustrates the optimal effort and manipulation for different combinations of (z_t, ψ_t) as we move through the phase diagram.

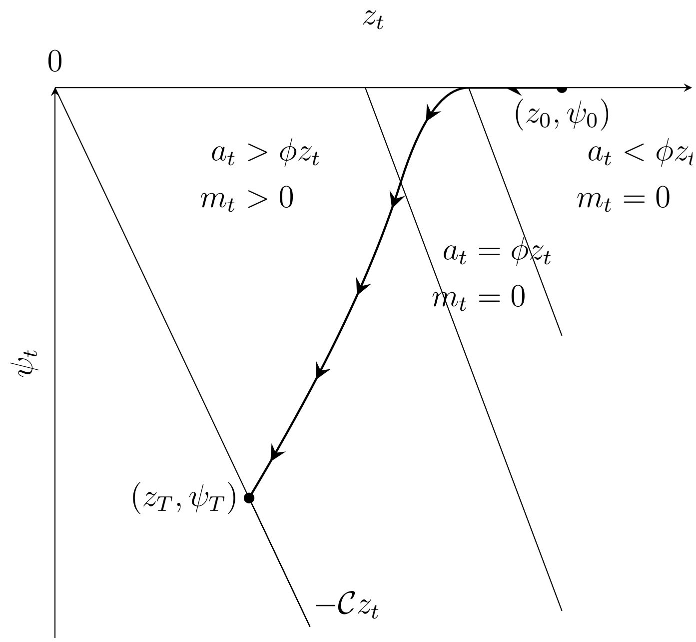

Figure 3: This figure summarizes the optimal effort and manipulation as we move through the phase diagram. There are three regions: in the first region we have positive manipulation, in the second region we have zero manipulation and effort is determined by the binding no-manipulation constraint, in the final region we have zero manipulation and the no-manipulation constraint is slack.

If we replace the solution for (a_t, m_t) in the differential equation for (z_t, ψ_t), we find a linear system of ordinary differential equations that can be solved in closed form for each time interval identified in the proposition. We identify the thresholds t^* and t^{**} using the transversality condition and pasting the solution for (z_t, ψ_t) in the different intervals. The proof of Proposition 4 follows from an analysis of the behavior of this dynamic system as we move through the three regions identified in Figure 3. In particular, in Figure 3, we show that the system moves through the phase diagram from the northeast towards the southwest. The detailed formal analysis is provided in the appendix.

6 The Benefit of Stochastic Incentives

In general, the optimal contract's effort and manipulation paths are history dependent: long-term incentives and CEO behavior is adjusted based on the history of performance.

In this section, we look at the optimal contract in general, allowing for stochastic incentives that evolve over time based on the history of performance. The objective of this section is twofold. First, to show as a robustness check that the main qualitative aspects of the contracts implementing deterministic actions remain the same when one considers general stochastic contracts. Second, we show how the mix of long versus short term incentives respond to performance shocks throughout the CEO tenure. Anticipating the results, we find that early on in the CEO tenure, the duration of incentives increases in response to positive shocks, shifting the mix of incentives toward the long-term. On the contrary, by the end of the CEO tenure, positive performance shocks reduce the duration of incentives shifting incentives toward the short term. In other words, vesting accelerates after positive performance shocks.

The principal now solves the following stochastic control problem

$$F(z,0) = \sup_{a,m,v} E\left[\int_0^T e^{-rt}\pi(a_t,m_t)dt - e^{-rT}\Psi(z_T)\right]$$

subject to

$$\begin{aligned}
dz_t &= [(r+\kappa)z_t + r\gamma a_t(\sigma v_t - 1)]dt + v_t dB_t \\
m_t &\geq \frac{a_t - \phi z_t}{g} \\
m_t &\geq 0.
\end{aligned}$$

For analytical considerations, it is convenient to bound the volatility of incentives so the set of controls is closed and bounded. In what follows, we restrict attention to bounded sensitivity such that $|v_t| \leq L_v$ for some constant L_v. In our numerical computations, we pick L_v large enough so that the constraint is not binding.

The Hamilton-Jacobi-Bellman equation for the principal optimization problem is

$$rF(z,t) = \max_{a,m,v}\ \pi\big(a,m\big) + F_t(z,t) + [(r+\kappa)z + r\gamma a(\sigma v - 1)]F_z(z,t) + \frac{1}{2}v^2 F_{zz}(z,t) \quad (29)$$

where the maximization is subject to the incentive compatibility constraints. The value function must also satisfy the terminal condition

$$F(z,T) = -\Psi(z).$$

One difficulty that arises when dealing with Equation (29) is that the diffusion coefficient v_t is not bounded away from zero. Hence, the HJB equation is a PDE of the degenerate parabolic type, which means that a classical solution may fail to exist. For this reason, we must rely on the theory of viscosity solutions for the analysis of the HJB equation. The fact that the value function is a viscosity solution of the HJB equations follows from the principle of dynamic programming. The uniqueness of the solution follows from the comparison principle in Fleming and Soner (2006). Equation (29) is a highly non-linear PDE so there is no hope of finding an analytical solution; hence, we solve the HJB equation numerically. In the Appendix, we provide a finite difference scheme that is guaranteed to converge to the unique viscosity solution of Equation (29). Once we have solved for the value function, we initialize the contract at $z_0 = \arg\max_z F(z, 0)$.

As mentioned above, exposing the CEO to risk after retirement is costly. It is inefficient from the perspective of risk sharing and serves no incentive purpose at this point. Accordingly, the principal wishes to reduce long term incentives just before time T. The only way to achieve this when one uses deterministic incentives is to distort the evolution of effort. By contrast, in the case of stochastic incentives, we can reduce long term incentives without distorting effort by adjusting the sensitivity of long term incentives to cash-flows shocks. The optimal contract adjusts the duration of incentives based on the realization of performance. This adjustment is not costless. Using a high sensitivity has a cost if the value function is concave. Looking at the first order condition for v_t we get

$$v_t = -r\gamma\sigma \frac{F_z(z_t, t)}{F_{zz}(z_t, t)} a_t. \tag{30}$$

If the value function is concave then the sign of v_t will be negative when z_t is high. Figure 4 shows that the value function $F(z, t)$ is concave and that for high values of z the sensitivity of long term incentives with respect to shocks $v(z, t)$ becomes negative. Consistent with the previous intuition, Figure 4 also shows that the drift of z_t is positive whenever z is low and negative when z is high. Hence, the evolution of z_t resembles a mean reverting process that follows a time-varying target that converges to zero as T becomes closer. In fact, the drift of z becomes negative over a wider range of z when $T - t$ becomes low. This shows that incentives shift toward the short term as the CEO gets closer to T. In fact, at time T we have that $F_z(z_T, T) = -\mathcal{C} z_T$. Accordingly, the sensitivity parameter v is likely to become negative close to time T.

Figure 5 compares the expected path in the contract with stochastic incentives versus that in the contract implementing deterministic actions studied in Section 6. The main features characterizing the contract with deterministic incentives are also present in the characterization of the optimal stochastic contract. Long term incentives and effort are front loaded and decrease over time while manipulation is back-loaded and tends to increase as the CEO approaches T. The main new feature is the presence of stochastic long term incentives whose volatility is characterized by the sensitivity parameter v_t. In principle, long term incentives could be positively or negatively correlated with performance. However, our numerical computations suggest that the sensitivity v_t is decreasing over time and tends to be negative around T. This means that by the end of tenure the maturity of incentives is shortened in the presence of positive shocks. The magnitude of v_t tends to be small in all the numerical examples we have analyzed, which means that the optimal deterministic contract appears to be close to optimal. From a quantitative perspective, the benefit of using fully history dependent incentives are arguably small relative to the additional complexity costs of writing such contracts.

7 Empirical Implications

In this section we perform comparative statics, discuss the empirical implications of the model, and relate them to the existing evidence. Edmans et al. (2013) study how a CEO behaves in years in which he has a significant amount of shares and options vesting. CEOs typically sell their equity upon vesting to diversify their portfolios. Equity vesting makes CEOs particularly concerned about short-term stock prices. The authors find that, in years in which the CEOs experience significant equity vesting, they cut investments in *R&D*, advertising, and capital expenditures. Moreover, in these years, CEOs are more likely to meet or just beat analyst earnings forecasts. For example, if the forecast is \$1.27 per share, they report earnings of \$1.27 or \$1.28. The authors find that the magnitude of investment cuts is just enough to allow CEOs meet their targets. Seemingly, vesting induces CEOs to act myopically in order to meet short-term targets.

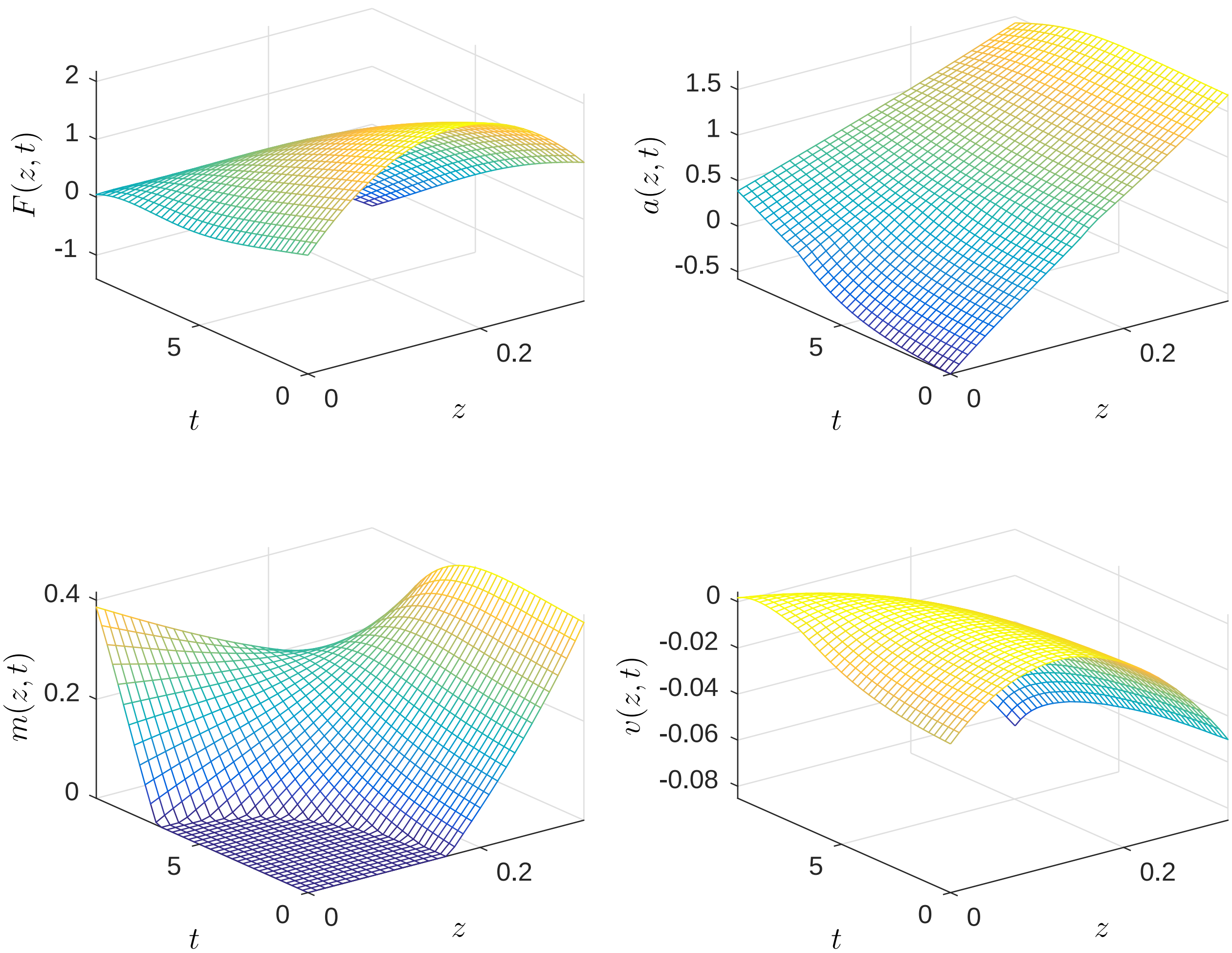

Figure 4: Solution principal's optimization problem. Parameters: $r = 0.1$, $\gamma = 1$, $g = 1$, $\theta = 0.4$, $\kappa = 0.3$, $\sigma = 2$, $T = 10$, $\tau = 5$

Horizon, Short-Termism and Pay Duration. The executive compensation literature hypothesizes the existence of a "CEO Horizon problem" whereby CEO short-termism would be particularly severe in the final years of CEO office, in so far as the manager is unable to internalize the consequences of his actions. Gibbons and Murphy (1992) indeed hypothesize that existing compensation policies induce executives to reduce investments during their last years of office but do not find conclusive evidence of greater manipulation. Gonzalez-Uribe and Groen-Xu (2015) find that "CEOs with more years remaining in their contract pursue more influential, broad and varied innovations." Dechow and Sloan (1991) document that managers tend to reduce *R&D* expenditures as they approach retirement, and the reductions

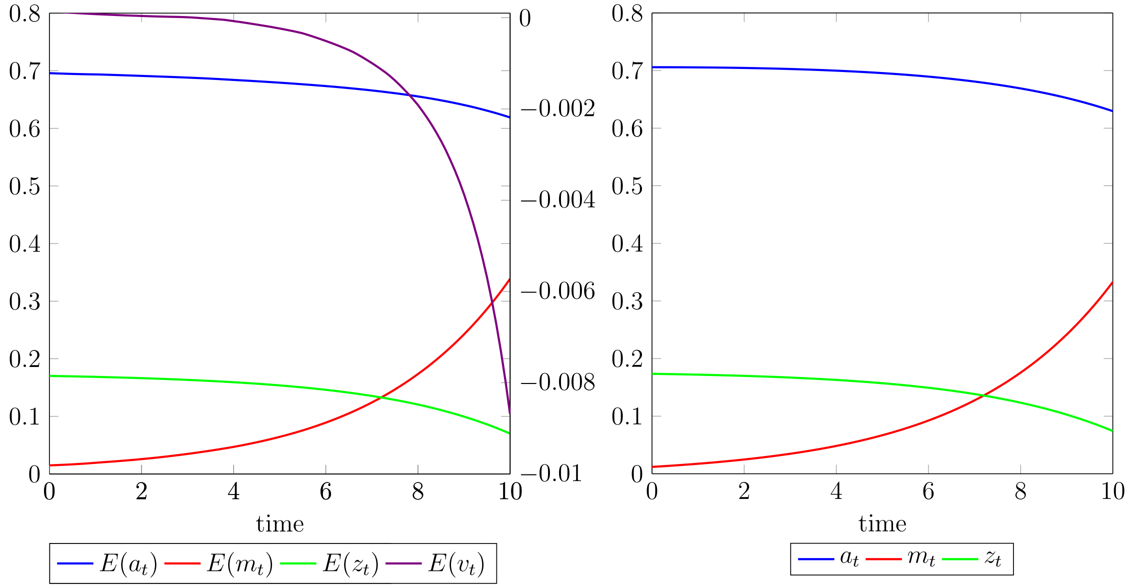

Figure 5: Expected path optimal contract vs. the optimal contract with deterministic incentives. The expected path in the stochastic case was computed using Monte Carlo simulation. Parameters: $r = 0.1$, $\gamma = 1$, $g = 1$, $\theta = 0.4$, $\kappa = 0.3$, $\sigma = 2$, $T = 10$, $\tau = 5$

in *R&D* are mitigated by CEO stock ownership.

Although intuitive, the CEO horizon hypothesis seems to ignore that manager incentives are endogenous. If shareholders anticipate the CEO horizon problem, arguably they will adjust compensation contracts accordingly. This can explain why the empirical evidence regarding the relation between manipulation and tenure is ambivalent (Gibbons and Murphy (1992)). Cheng (2004), for instant finds evidence that compensation contracts become particularly insensitive to accounting performance measures that are easily manipulable by the end of the manager's tenure, suggesting that compensation committees are able to anticipate the manager's incentives. In this paper we show that the an horizon problem exists even in the presence of endogenous incentives. The finite nature of CEO tenure and the fact that deferring compensation after retirement is particularly costly, explain why optimal contracts implement manipulation in our setting. In Section 4.1 we show that when the manager horizon grows large ($T \to \infty$) the possibility of manipulation is irrelevant. Linear contracts such as the one analyzed by Holmstrom and Milgrom (1987) suffice to eliminate manipulation. This result is consistent with Jensen (2001, 2003) who recommends linear contracts to prevent managers from gaming compensation systems. Our analysis shows that when the CEO has a limited horizon, linear contracts don't prevent short-termism but may even induce too much.[7]

From a contracting perspective, two instruments are effective at dealing with the possibility of manipulations: *i*) Pay Duration and *ii*) Clawbacks. Both of these tools are used in practice. Some empirical evidence suggests that after SOX the average duration of CEO compensation increased, and firms have started to rely more on restricted equity to compensate managers. Gopalan et al. (2014) for example provide evidence that the duration of stock-based compensation is about three to five years. They document a negative association between the duration of incentives and measures of manipulation such as discretionary accruals. In particular, they find that duration is shorter for older executives and executives with longer tenure. The second instrument is clawbacks. A clawback is a contractual clause included in employment contracts whereby the manager is obliged to return previously awarded compensation due to special circumstances, described in a contract, for example a fraud or restatement. The growing popularity of clawback provisions is due, at least in part, to the Sarbanes-Oxley Act of 2002, which requires the U.S. Securities and Exchange Commission (SEC) to pursue the repayment of incentive compensation from senior executives that are involved in a fraud or a restatement.[8] Although we do not incorporate clawbacks – as a discrete event triggered by say a restatement – in our model, the fact the manager's income depends on post-retirement performance captures some the essence of clawbacks as an incentive mechanism. In the model, the parameter τ incorporates the fact that clawbacks might be limited by some unmodeled institutional reasons. In the model, a longer τ means the manager's wealth can be tied to the firm performance for a longer period after retirement. We have allowed for $\tau = \infty$ to consider the case in which there is no enforcement problem. It is natural to conjecture that unrestricted clawbacks can eliminate manipulation altogether: Allowing the principal to impose severe penalties if a sudden performance reversal is observed after retirement could allow the firm to prevent manipulations. However, imposing risk on the manager after retirement is so costly that firms might prefer to give up on harsh clawback provisions. Dehaan et al. (2013) indeed provide evidence that managers who are subject to clawback provisions demand an increase in base salary.

[7]Kothari and Sloan (1992) provides evidence that accounting earnings commonly take up to three years to reflect changes in firm value

[8]The prevalence of clawback provisions among Fortune 100 companies increased from less than 3% prior to 2005 to 82% in 2010.

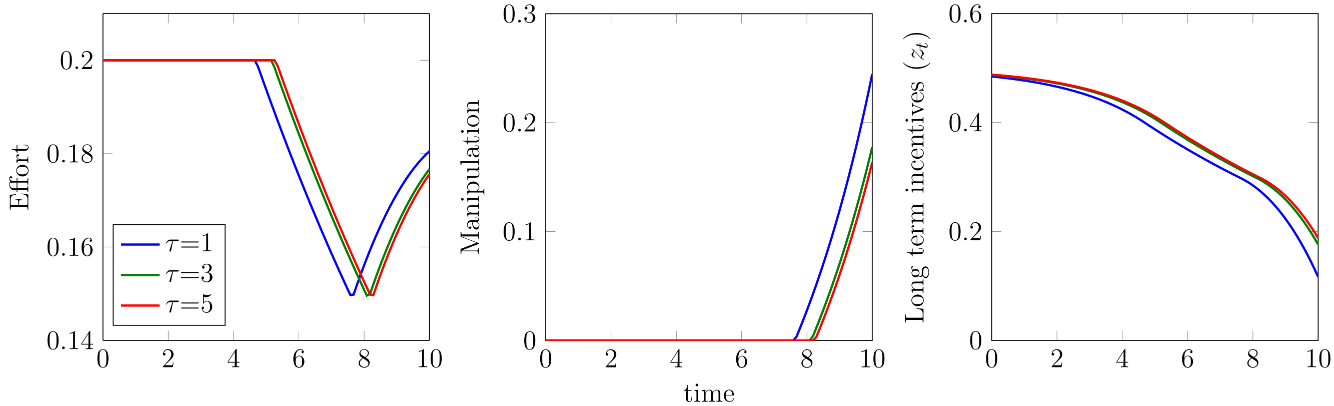

Figure 6: Parameters: $r = 0.1$, $\gamma = 1$, $\theta = 0.5$, $\kappa = 0.3$, $\sigma = 10$, $T = 10$.

Figure 6 shows the comparative statics of effort, manipulation, and long term incentives when we change τ. As expected, a shorter τ reduces long-term incentives and increases manipulation. Surprisingly, the effect on effort is non-monotonic. This happens because the fast vesting taking place before T effectively boosts short term incentives.

To test the implications of our model, it is not enough to look at the duration of incentives in isolation. If we compare the evolution of long term incentives under the optimal contract in Figure 2, versus the evolution of long term incentives under the best no manipulation contract in Figure 1, we observe that the evolution of long-term incentives is similar. In that sense both contracts are observationally equivalent. In order to test the predictions of our model versus previous models in the literature, one would have to look jointly at payment duration, equity vesting, and short-termism in relation to the tenure of the CEO. Some of these predictions have already been tested. As mentioned above, Gopalan et al. (2014) show that pay duration is shorter for older executives and executives with longer tenure. This is consistent with the prediction that long term incentives should fall as the CEO approaches the end of the contract or retirement. Gonzalez-Uribe and Groen-Xu (2015) shows that CEOs with more years remaining in office pursue more innovation, which is consistent with the idea of CEOs investing in long term projects at the beginning of their tenure and focusing more in the short term towards the end. Finally, Edmans et al. (2013, 2014) show that equity vesting is negatively correlated with long term investment while Ladika and Sautner (2016) show that executives reduce long-term investing when their option vesting is accelerated.

Pay-for-Performance The executive compensation literature has documented at least two puzzles regarding pay-performance sensitivity. First, pay for performance evolves with CEO tenure (Brickley et al. (1999)). Unlike in Holmstrom and Milgrom (1987) a constant *PPS* is not optimal in our setting. Indeed, a constant *PPS* would lead to excessive manipulation, especially around the retirement date. Our model predicts a profile of increasing manipulation along with a relatively low but potentially increasing *PPS*. Some evidence suggests that the *PPS* of CEO compensation increases over time, as manager's stock ownership grows (Gibbons and Murphy (1992)). At first blush this fact seems to contradict the predictions of our model. In our setting *PPS* may increase over time; however, it is never higher than at the start and it is non monotonic in time; it only increases at the end. A time profile of increasing *PPS* is consistent with an extended version of the model, in which the performance measure is a distorted version of the firm's cash flows (for example the firm earnings). Following Baker (1992), we can consider the case in which the firm only observes a distorted performance measure. For example, consider the case when the firm does not observe output X_t but only observes the distorted performance measure Y_t as given by

$$dY_t = (a_t + m_t - \theta_Y M_t)dt + \sigma dB_t.$$

The main difference between X_t and Y_t is that the latter does not capture the whole extent of short-termism. Of course, the performance measure could differ in other characteristics such as κ and the effect would be similar. We can readily solve the model in this case by adjusting the parameters, the effect of manipulation of cash flows, λ would be the same as before; however, θ_Y would take the role of θ in the rest of the analysis. If $\theta_Y < r + \kappa$, then short-termism will be problematic even if $T = \infty$ and the PPS may increase over time (see Figure 7).

A second empirical puzzle that intrigued the compensation literature in the 1990 is the low *PPS* in CEO contracts (see e.g., Jensen (2001)). Our model predicts that such low *PPS* can be the result of the possibility of manipulation, as already suggested by Goldman and Slezak (2006).

Corporate Governance and Short-Termism The CEO horizon problem is ultimately a corporate governance weakness reflecting the inability of the firm to monitor CEO actions. If

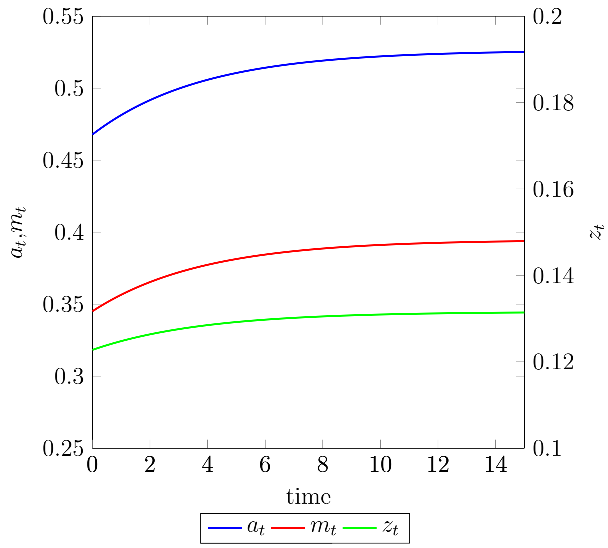

Figure 7: Evolution of effort, manipulation and long term incentives with imperfect performance measures. Parameters: $r = 0.1$, $\gamma = 1$, $\theta = 0.4$, $\kappa = 0.3$, $\sigma = 2$, $T = \infty$, $\theta_Y = 0.1$.

we understand corporate governance as a set of mechanisms (some of which are exogenous to the firm) that make it more costly for the manager to manipulate performance (for example by increasing the cost of manipulation g) then our model predicts that better corporate governance would result in relatively more short-term compensation (lower duration) and greater firm value. Maybe paradoxically, it does not predict that the levels of manipulation will be lower, as can be seen in Figure 8. If better corporate governance makes short-run incentives relatively more effective at stimulating effort, vis-a-vis manipulation, then the firm may find it optimal to offer stronger short-term incentives, even at the expense of tolerating greater manipulations. This effect is present in previous static models of costly state falsification. For example, Lacker and Weinberg (1989) show that no manipulation is optimal when the cost of manipulation is not too convex. In our setting, with a quadratic falsification cost, this condition translates into a low value of g. From the IC constraint we find that the sensitivity of manipulation to changes in effort (for a fixed z) is $1/r\gamma g$. This means that when the marginal cost of manipulation g is low, the trade-off between higher effort and higher manipulation is too high. A small increment in effort generates so much manipulation that makes the no-manipulation optimal. In fact, we have that when $g = 0$ the optimal contract implements no manipulation. Hence, in this case the optimal contract is given by the best no-manipulation contract derived in Section 5.2. Of course one needs

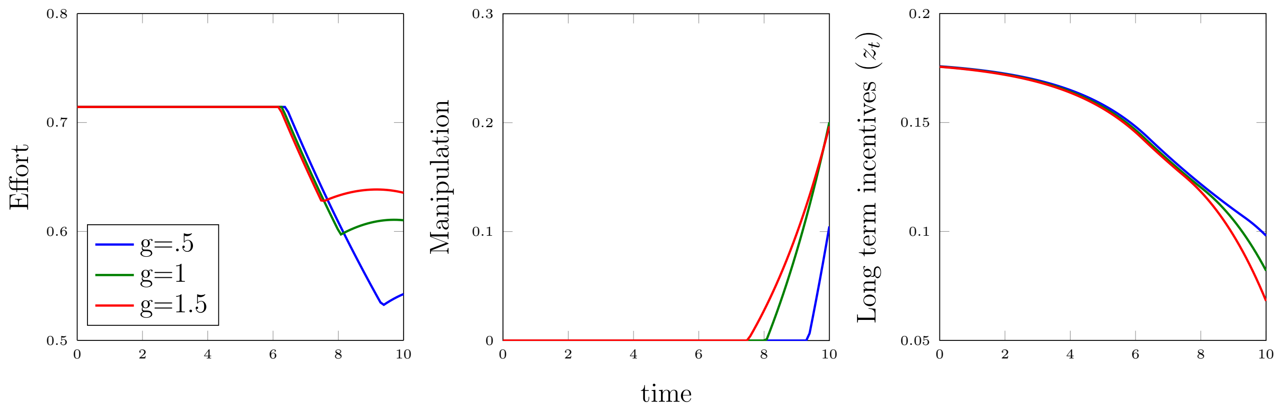

Figure 8: Parameters: $r = 0.1$, $\gamma = 1$, $\theta = 0.5$, $\kappa = 0.3$, $\sigma = 2$, $T = 10$, $\tau = 2$

to be careful when interpreting this observation as evidence that short-run incentives cause manipulation (Bergstresser and Philippon (2006)). In the limit as the cost of manipulation g grows large the manager's manipulation incentives are vanishingly low. The contract then becomes stationary –with constant *PPS*– because short-term incentives suffice to induce effort.

One of the goals of the Sarbannes Oxley act (SOX) enacted in 2002, was to curb fraud and manipulation by making it more costly to the manager. Cohen et al. (2008) provide evidence that SOX did mitigate accrual-based manipulation but, as an unintended consequence, exacerbated real earnings management. We cannot fully speak to this evidence because we only consider one type of manipulation with two possible effects: timing and cash flow effects. However, one could think of SOX as a regulatory shift from an environment characterized by a low g and $\lambda = 0$ to an environment characterized by a high g and $\lambda > 0$. While an increase in g leads in the model to a greater focus on short-term compensation, a higher λ can have an ambiguous effect on the duration of incentives: though manipulation becomes more costly to the firm it, at the same time, becomes less attractive to the manager.

8 Conclusion

This paper studies optimal CEO contracts when managers can increase short term performance at the expense of firm value. The model is flexible, encompassing both the case in which the CEO can increase short-term performance by distorting cash-flows and the case in which observed performance can be manipulated by means of accrual earnings management. We consider a setting in which the manager horizon is finite. We find that long term incentives decrease over time, managerial short-termism increases, and effort may be non-monotonic in time, increasing at the end of the CEO career. The optimal compensation scheme includes deferred compensation. Vesting of the manager incentives accelerates at the end of tenure, thus shifting the balance of incentives towards short-term compensation. This, process gives rise to a CEO horizon problem -as an inherent feature of optimal contracts- whereby managers intensify performance manipulation in the final years in office. We characterize the optimal mix of short and long-term incentives and the optimal duration and vesting of incentives along the manager's tenure.

We explore the optimality of deferred compensation as a contracting tool for alleviating the effects of CEO manipulation. Though potentially effective, clawbacks and deferred compensation impose significant risk on the CEO during a time when incentives are not needed to stimulate effort. This issue makes these tools costly from the firm perspective, limiting their effectiveness: firms may prefer to set relatively short clawback periods particularly when the firm's fundamentals are too volatile.

Unlike in Holmstrom and Milgrom (1987) the optimal contract is non-linear in performance. Moreover, it implements stochastic effort and manipulation, effectively making the firm's performance more noisy. Under a deterministic contract, the firm can modify the long-term incentives only by distorting the CEO's effort (for the contract to preserve incentive compatibility). Stochastic incentives help because they allow the firm control the evolution of long-term incentives without having to distort the CEO's effort. The optimal contract is such that the sensitivity of long-term incentives to firm performance at the beginning of the CEO's tenure and at the end are qualitatively different. At the beginning, positive performance shocks increase the use of long-term incentives, in other words, the duration of incentives increases when the firm is performing well. On the contrary, at the end of the CEO's tenure long term incentives are negatively correlated to firm's performance. Positive performance shocks lead to an acceleration of incentive vesting.

Managerial short-termism has been a concern among policy makers, practitioners and academics but it has taken a particularly prominent role in recent years after the Enron scandal and the financial crisis of 2008. These events prompted the design of new regulation for executive compensation practices. For example, as part of the Dodd-Frank act, financial regulators have proposed rules that regulate equity vesting and the use of clawbacks.[9] In this paper, we look to understand the factors determining the use of long-term incentives by firms' compensation policies and the unintended consequences this practice triggers. We show that given the cost of deferring compensation, companies are likely to offer contracts that induce some managerial short-termism and that this problem is particularly acute if the horizon of the manager is short. In our model, the CEO horizon is exogenous and fixed. However, the main intuitions should hold in the case with endogenous turnover: contracts should become more short-sighted when the manager is close to separation. We expect that managerial short-termism will be particularly problematic in industries with high turn-over rates in which CEO horizons are expected to be short.

In our model, allowing for manipulation is optimal from the firm perspective. Hence, to rationalize the social benefits of regulation, a model should consider the presence of externalities. For example, if some of the losses caused by manipulations are borne by the government (e.g., due to bailouts) or by other firms in the industry (e.g., due to credibility contagion). In these cases, firms will tend to offer contracts that are too short sighted and generate excessive short-termism. This possibility could justify regulations, such as those currently debated in the U.S and Europe, aimed at increasing the use of deferred compensation and at lowering the incentive power of CEO contracts.

We conclude by noting that the design of monetary incentives alone is not enough to eliminate managerial short-termism. In practice, other corporate governance tools may complement the disciplining role of compensation. For example, we can think that CEOs' discretion to make short term investments or cut long term ones evolves over time, being a function of the manager's horizon. There are different ways in which this could be addressed. For example, the level of discretion a CEO receives affects the freedom he has to manipulate performance, but it also makes him less productive. In other words, the CEO might not be able to manipulate performance as freely as before, but the associated lack of flexibility can also reduce his productivity. Specifically, assume that under low discretion the manager's effort only produces a fraction of what it produces otherwise (that is, the marginal productivity of effort is αa_t for some $\alpha < 1$). Our analysis suggests that CEOs should be given more discretion at the beginning of their tenure with an increment in board oversight taking place as he gets close to retirement. Of course, this policy recommendation must be taken

[9]See "New Rules Curbing Wall Street Pay Proposed," *Wall Street Journal*, Apr. 22, 2016.

with a grain of salt. There are additional factors that we have ignored in the model. One of these factors is learning, a young and inexperienced CEO's talent may be uncertain and the board may want to monitor his actions more closely.

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Appendix

Proof Lemma 1

Proof. The discounted value of cash-flows is

$$E\left[\int_0^\infty e^{-rt}dX_t\right] = E\left[\int_0^\infty e^{-rt}a_t dt + \int_0^\infty e^{-rt}(m_t - \theta M_t)dt\right]$$

Using integration by parts we get that

$$\int_0^\infty e^{-rt}M_t dt = \frac{1}{r+\kappa}\int_0^\infty e^{-rt}m_t dt = \frac{1}{r+\kappa}\int_0^T e^{-rt}m_t dt,$$

as $m_t = 0$ for $t > T$. Replacing above,

$$E\left[\int_0^\infty e^{-rt}dX_t\right] = E\left[\int_0^T e^{-rt}a_t dt + \int_0^T e^{-rt}\left(1 - \frac{\theta}{r+\kappa}\right)m_t dt\right].$$

The result follows from taking derivative with respect to m_t in the expression above.

□

Proof Proposition 1: Sufficient Condition

Proof. To prove sufficiency, we follow Sannikov (2014) and He et al. (2014) in the construction of an upper bound for the payoff after a deviation. In particular, following He et al. (2014), we construct an upper bound of the form

$$\hat{W}_t = \xi_t W_t e^{-r\gamma(S_t+Y_t)},$$

where

$$dY_t = (rY_t + Q_t)dt,\ Y_0 = 0.$$

for some process Q_t to be determined later. Using Ito's lemma, the evolution of $\hat{W}_t$ is

$$d\hat{W}_t = -r\gamma\hat{W}_t(dS_t + dY_t) + \Gamma_t\hat{W}_t dB_t,$$

where

$$\Gamma_t = -\frac{\Delta a_t + \Delta m_t - \theta \Delta M_t}{\sigma} - \sigma\beta_t = -\frac{\Delta a_t + \Delta m_t - \theta \Delta M_t}{\sigma} - r\gamma\sigma a_t.$$

Next, we define the process

$$G_t \equiv \int_0^t e^{-rs}\xi_s u_s^{\Delta} ds + e^{-rt}\hat{W}_t.$$

Differentiating G_t we get

$$e^{rt}dG_t = \xi_t u_t^{\Delta} - r\hat{W}_t dt - r\gamma\hat{W}_t(dS_t + dY_t) + \Gamma_t \hat{W}_t dB_t.$$

Using the fact that $rW_t = u_t \equiv u(c_t, a_t, m_t)$, we can write the previous expression as

$$e^{rt}dG_t = \xi_t u_t \left[\frac{u_t^{\Delta}}{u_t} e^{r\gamma(S_t+Y_t)} dt - dt - \gamma(dS_t + dY_t)\right] + \Gamma_t \hat{W}_t dB_t.$$

Because $e^x \geq 1 + x$, we have that

$$\begin{aligned}
\frac{u_t^{\Delta}}{u_t} e^{r\gamma(S_t+Y_t)} - 1 - \gamma(rS_t - \Delta c_t + rY_t + Q_t) &\geq 1 - \gamma\Delta c_t + \frac{\gamma(a_t + \Delta a_t)^2}{2} - \frac{\gamma a_t^2}{2} \\
&\quad + \frac{\gamma g(m_t + \Delta m_t)^2}{2} - \frac{\gamma g m_t^2}{2} + r\gamma S_t + r\gamma Y_t \\
&\quad - 1 - r\gamma S_t + \gamma \Delta c_t - r\gamma Y_t - \gamma Q_t \\
&= \frac{\gamma(a_t + \Delta a_t)^2}{2} - \frac{\gamma a_t^2}{2} + \frac{\gamma g(m_t + \Delta m_t)^2}{2} - \frac{\gamma g m_t^2}{2} - \gamma Q_t \\
&= \gamma \left[a_t \Delta a_t + \frac{\Delta a_t^2}{2} + g m_t \Delta m_t + g\frac{\Delta m_t^2}{2} - Q_t\right]
\end{aligned}$$

Hence, if we choose

$$Q_t = a_t \Delta a_t + \frac{\Delta a_t^2}{2} + g m_t \Delta m_t + g\frac{\Delta m_t^2}{2},$$

and given that $\xi_t u_t < 0$, we get that G_t has a negative drift so

$$G_t \leq G_0 + \int_0^t e^{-rs}\Gamma_s \hat{W}_s dB_s.$$

Taking expectation, we find that

$$E[G_\infty] = E\left[\int_0^T e^{-rt}\xi_t u_t^\Delta dt + \int_T^\infty e^{-rt}\xi_t u^R(c_t + \Delta c_t)dt\right] \leq W_0.$$

Hence, the necessary incentive compatibility constraint is also sufficient.

□

Proof Proposition 3

Proof. The proof proceeds in several steps. First, we show that the optimal path of effort takes the form $a_t = a^{HM}\mathbf{1}_{\{t<t^*\}} + \phi z_t \mathbf{1}_{\{t\geq t^*\}}$ and then we determine the value of t^* and the law of motion of z_t.

Lemma 3. *Suppose that $\theta \geq r + \kappa$. If $\phi z_t \leq a^{HM}$ then $a_s = \phi z_s$ for all $s \in [t, T]$.*

Proof. Because $\psi_t \leq 0$ it follows that if $\phi z_t \leq a^{HM}$ then it must be the case that $a_t = \phi z_t$. To see why this is the case suppose first that $\psi_t < 0$. If $\mu_t = 0$ then we have that

$$a_t = \frac{1 - r\gamma\psi_t}{1 + r\gamma\sigma^2} > a^{HM} \geq \phi z_t,$$

which would violate the constraint. Then it must be the case that $\mu_t > 0$ which means that $a_t = \phi z_t$. Suppose next that $\psi_t = 0$ so that

$$a_t = \frac{1 - \mu_t}{1 + r\gamma\sigma^2}.$$

If $\mu_t = 0$, we have $a_t = a^{HM}$ which can only satisfy the constraint if $\phi z_t = a^{HM}$, in which case the equality $a_t = \phi z_t$ is trivially satisfied. On the other hand, if $\phi z_t < a^{HM}$ then it must be the case that $\mu_t > 0$ which implies that $a_t = \phi z_t$. Hence, we can conclude that $a_t = \phi z_t$.

The next step is to show that $a_s = \phi z_s$ for $s \in (s, T]$. Replacing $a_t = \phi z_t$ in the law of motion for z_t we get that $\dot{z}_t = (r + \kappa - r\gamma\phi)z_t = (r + \kappa - \theta)z_t \leq 0$. Hence, we have that $\phi z_s \leq a^{HM}$ for some h and all $s \in (t, t+h)$. Repeating the same argument we get that $a_{t+h} = \phi z_{t+h}$ and $\dot{z}_{t+h} \leq 0$ so we can conclude that $\phi z_s \leq a_{nm}$ and so $a_s = \phi z_s$ for all $s \in [t, T]$. □

Lemma 4. *Let $t^* \equiv \inf\{t \in [0,T] : \phi z_t \le a^{HM}\}$. The solution to the optimal control problem is $a_t = a^{HM}$ if $t \in [0,t^*)$ and $a_t = \phi z_t$ if $t \in [t^*,T]$.*

Proof. We have already proven in Lemma 3 that $a_t = \phi z_t$ for $t \in [t^*,T]$. Hence, the only step left is to show that $a_t = a^{HM}$ for $t < t^*$. If $\phi z_0 \le a^{HM}$ then by Lemma 3 we have $t^* = 0$ and there is nothing to prove. Suppose next that $\phi z_0 > a^{HM}$. Using the initial condition $\psi_0 = 0$ we get that the optimal effort at time zero satisfies the first order condition

$$a_0 = \frac{1 - r\gamma\psi_0 - \mu_0}{1 + r\gamma\sigma^2} = \frac{1 - \mu_0}{1 + r\gamma\sigma^2} \le a^{HM}.$$

If $\mu_t > 0$ then we have $\phi z_0 = a_0 < a^{HM}$ which contradicts the hypothesis that $\phi z_0 > a^{HM}$. Thus, we can conclude that $\mu_0 = 0$ and $a_0 = a^{HM}$. Replacing in the law of motion of ψ_t we get that $\dot{\psi}_0 = 0$ and $\psi_0 = 0$. This means that we can extend the previous argument to the interval $[0,t^*)$ and get $\psi_t = 0$, $\mu_t = 0$, and $a_t = a^{HM}$ for all $t \in [0,t^*)$. □

Using the previous two lemmas, we can solve for the optimal path of effort, for $t < t_*$ we have that $\psi_t = 0$, $a_t = a^{HM}$, and

$$\dot{z}_t = (r+\kappa)z_t - r\gamma a^{HM}.$$

For $t \ge t^*$ we have that $a_t = \phi z_t$

$$\dot{z}_t = (r+\kappa-\theta)z_t,$$

and

$$\dot{\psi}_t = -\kappa\psi_t - \phi\mu_t,$$

where $\mu_t = 1 - r\gamma\psi_t - (1 + r\gamma\sigma^2)\phi z_t$. In addition, the system of equations satisfies the terminal condition $\psi_T = -\mathcal{C} z_T$. For $t \ge t_*$, the solution is

$$z_t = e^{-(\theta-r-\kappa)(t-t^*)} z_{t^*}.$$

Solving for ψ_t and using the initial condition $\psi_{t^*} = 0$ we get

$$\psi(t)=\frac{\phi}{\theta-\kappa}+\frac{\phi\left[e^{-(-r-\kappa+\theta)(t-t^{*})}+e^{(\theta-\kappa)(t-t^{*})}\frac{\theta-r-\kappa}{\theta-\kappa}\right]}{-2\theta+2\kappa+r} \tag{31}$$

Using the terminal condition we get the equation $\psi_T = -Cz_T$ $t^* = 0$ if and only if $\phi z_0 \leq a^{HM}$ which means that $t^* = 0$ if and only if

$$\phi^2\frac{-2\theta+2\kappa+r+(\theta-\kappa)\,e^{-(-r-\kappa+\theta)T}+e^{(\theta-\kappa)T}\,(\theta-r-\kappa)}{-Ce^{-(\theta-r-\kappa)T}\,(-2\theta+2\kappa+r)\,(\theta-\kappa)}\leq a^{HM}$$

If this condition is not satisfied, then $t^* > 0$ and $z_{t^*} = a^{HM}/\phi$. Using this relation, we get that t^* is the unique solution to

$$\frac{\phi^2}{\theta-\kappa}+\frac{\phi^2\left[e^{-(-r-\kappa+\theta)(T-t^{*})}+e^{(\theta-\kappa)(T-t^{*})}\frac{\theta-r-\kappa}{\theta-\kappa}\right]}{-2\theta+2\kappa+r}=-Ce^{-(\theta-r-\kappa)(T-t^{*})}a^{HM}$$

Given t^*, we can find z_0 solving the differential equation

$$\dot{z}_t=(r+\kappa)z_t-r\gamma a^{HM},$$

with the terminal condition $z_{t^*} = a^{HM}/\phi$. This yields,

$$z_t=\left[\frac{r\gamma}{\kappa+r}-\frac{(\theta-r-\kappa)e^{-(r+\kappa)(t^{*}-t)}}{(r+\kappa)\phi}\right]a^{HM}$$

□

Proof Proposition 4

We start with 3 claims that we will use extensively.

Claim 1 If $\phi z_t > a^{HM}$ and $\psi_t = 0$ then $a_t = a^{HM}$, $m_t = 0$ and $\dot{\psi}_t = 0$.

Proof. Suppose that $m_t > 0$. If this is the case, then we have that

$$\begin{aligned} a_t - \phi z_t &= \frac{g - \lambda + \phi z_t}{1 + g(1 + r\gamma\sigma^2)} - \phi z_t \\ &= \frac{g - \lambda - \phi z_t g(1 + r\gamma\sigma^2)}{1 + g(1 + r\gamma\sigma^2)} \\ &\leq \frac{g - \lambda - a^{HM} g(1 + r\gamma\sigma^2)}{1 + g(1 + r\gamma\sigma^2)} \\ &= \frac{-\lambda}{1 + g(1 + r\gamma\sigma^2)} < 0, \end{aligned}$$

which implies that $m_t < 0$, contradiction. Accordingly, we have

$$a_t = \frac{1 - r\gamma\psi_t}{1 + r\gamma\sigma^2} = a^{HM}.$$

Hence, we have that $m_t = 0 > a_t - \phi z_t$ which yields $\eta = 0$. Replacing in the law of motion for ψ_t we find that $\dot{\psi}_t = 0$. □

Claim 2 If $\phi z_t \in \left[\frac{1-\lambda/g - r\gamma\psi_t/g}{1+r\gamma\sigma^2}, a^{HM}\right]$ then $a_t = \phi z_t$, $m_t = 0$ and $\dot{z}_t < 0$.

Proof. Suppose that $m_t > 0$, then $a_t - \phi z_t < 0$ which means that $m_t < 0$ a contradiction. Hence, it must be the case that $m_t = 0$. Using the first order condition a we get

$$a_t = \frac{1 + \eta/g}{1 + r\gamma\sigma^2}.$$

If $\eta = 0$ we get $a_t = a^{HM}$ which violates the constraint $a_t \leq \phi z_t$ for $m_t = 0$. Hence, it must be the case that $\eta > 0$, which means that $a_t = \phi z_t$. Replacing in the ODE for z_t we get $\dot{z}_t < 0$. □

Claim 3 $\phi z_t < \frac{1-\lambda/g - r\gamma\psi_t/g}{1+r\gamma\sigma^2} \Rightarrow m_t > 0$, and $\dot{z}_t < 0$.

Proof. Suppose that $m_t = 0$, then we have that $\eta = 0$ and

$$a_t = \frac{1 - r\gamma\psi_t}{1 + r\gamma\sigma^2} \geq a^{HM} > \phi z_t$$

which means that the constraint $a_t \leq \phi z_t$ is violated. Thus, we have that $m_t > 0$, $\eta_t > 0$ and $a_t > \phi z_t$. Using the law of motion of z_t we get that

$$\dot{z}_t = (r+\kappa)z_t - r\gamma a_t < (r+\kappa)z_t - r\gamma\phi z_t = (r+\kappa-\theta)z_t < 0.$$

$\square$

Proof Proposition 4.

1. Suppose not. Then we have that $\phi z_0 > \frac{\theta}{r+\kappa}a^{HM} > a^{HM}$. By Claim 1 we have that $a_0 = a^{HM}$, $\dot{\psi}_0 = 0$ and $\dot{z}_0 > 0$. Repeating the same argument at any time t we find that $\phi z_t > a^{HM}$, $a_t = a^{HM}$ and $\psi_t = 0$, all $t \in [0, T]$. However, this violates the transversality condition $\psi_T = -\Psi'(z_T)$.

2. Combining Claim 1 and 1 above, we find that $a_0 = a^{HM}$, $\dot{z}_0 < 0$ and $\dot{\psi}_0 = 0$. We also can conclude from Claim 1 that $a_t = a^{HM}$, $\dot{z}_t < 0$ and $\dot{\psi}_t = 0$ for all $t \in [0, t^*]$ where $t^* = \inf\{t > 0 : \phi z_t = a^{HM}\}$. Moreover, it must be the case that $t^* < T$, as otherwise we wold have $\psi_T = 0$, which violates the transversality condition.

   Claim 2, implies that that $a_{t^*} = \phi z_{t^*}$, $m_{t^*} = 0$ and $\dot{z}_{t^*} < 0$. Let's define $t^{**} = \inf\{t > 0 : \phi z_t = \frac{1-\lambda/g - r\gamma\psi_t/g}{1+r\gamma\sigma^2}\}$. If $t^{**} > T$ then there is nothing else to prove. Hence, suppose that $t^{**} < T$. Claim 2 implies that $a_t = \phi z_t$, $m_t = 0$ and $\dot{z}_t < 0$ for all $t \in [t^*, t^{**}]$. Next, we show that it is also the case that $\dot{\psi}_t < 0$, all $t \in [t^*, t^{**}]$. Using the law of motion of ψ_t we get that $\dot{\psi}_{t^*} < 0$. Differentiating $\dot{\psi}_t$ we get that

   $$\ddot{\psi}_t = -\kappa\dot{\psi}_t - \frac{\phi}{g}\dot{\eta}_t,$$

   where

   $$\eta_t = g - g\left(1 + r\gamma\sigma^2\right)a_t - \psi_t r\gamma g = g - g\left(1 + r\gamma\sigma^2\right)\phi z_t - \psi_t r\gamma g$$

   so

   $$\dot{\eta}_t = -g\left(1 + r\gamma\sigma^2\right)\phi\dot{z}_t - \dot{\psi} r\gamma g.$$

   Suppose there is $\tilde{t} \in [t^*, t^{**}]$ such that $\dot{\psi}_{\tilde{t}} > 0$, then there is $\hat{t} < \tilde{t}$ such that $\dot{\psi}_{\hat{t}} = 0$. Replacing in the law of motion of η and $\dot{\psi}_t$ we find that $\dot{\eta}_{\hat{t}} > 0$ and $\ddot{\psi}_t < 0$. Hence, $\dot{\psi}_t$ can never cross zero which means that $\dot{\psi}_t < 0$ for all $t \in [t^*, t^{**}]$.

The final step is to analyze the behavior of (z_t, ψ_t) for $t \in [t^{**}, T]$. We know that $\dot{z}_{t^{**}} < 0$ and $\dot{\psi}_{t^{**}} \leq 0$, which means that $\frac{d}{dt}\left(\phi z_t - \frac{1-\lambda/g - r\gamma\psi_t/g}{1+r\gamma\sigma^2}\}\right)\Big|_{t=t^{**}} < 0$. Accordingly, $\phi z_t < \frac{1-\lambda/g-r\gamma\psi_t/g}{1+r\gamma\sigma^2}$ for $t \in (t^{**}, t^{**}+\epsilon)$. Claim 3 implies that $m_t > 0$ and $\dot{z}_t < 0$ for all $t \in (t^{**}, t^{**}+\epsilon)$. Differentiating $\dot{\psi}_t$, we get that

$$\ddot{\psi}_t = -\kappa\dot{\psi}_t - \frac{\phi}{g}\dot{\eta}_t = -\kappa\dot{\psi}_t - \phi\dot{m}_t.$$

Differentiating the solution for m_t we get that $\dot{\psi}_t = 0$ implies $\dot{m}_t < 0$. Thus, $\dot{\psi}_t = 0$ implies $\ddot{\psi}_t < 0$ which implies that $\dot{\psi}_t < 0$ for all $t \in (t^{**}, t^{**}+\epsilon)$. We can repeat the argument beyond $t^{**}+\epsilon$ to conclude that $m_t > 0, \dot{z}_t < 0, \dot{\psi}_t < 0$ all $t \in (t^{**}, T]$. Manipulation is given by

$$\begin{aligned}
m_t &= \frac{g - \lambda + \phi z_t - r\gamma g\psi_t}{1 + g\left(1 + r\gamma\sigma^2\right)} - \phi z_t \\
&= \frac{g - \lambda - \phi g\left(1 + r\gamma\sigma^2\right) z_t - r\gamma g\psi_t}{1 + g\left(1 + r\gamma\sigma^2\right)}
\end{aligned}$$

Hence, we get that

$$\dot{m}_t = -\frac{\phi g\left(1 + r\gamma\sigma^2\right)\dot{z}_t + r\gamma g\dot{\psi}_t}{1 + g\left(1 + r\gamma\sigma^2\right)} > 0.$$

3. We repeat the proof in 2 using the initial condition $\psi_0 = 0$ and setting $t^* = 0$.

4. We repeat the proof in 2 using the initial condition $\psi_0 = 0$ and setting $t^{**} = 0$.

$\square$