

**Detecting Financial Statement Irregularities:  
Evidence from the Distributional Properties of Financial Statement Numbers**

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**Abstract**

Anecdotal evidence suggests that a significant portion of financial statement irregularities, whether created in error or to mislead, are ignored by reporting firms, their auditors, and the SEC. Motivated by a method used by forensic investigators and auditors to detect irregularities in a variety of settings, such as elections, tax return data, and individual financial accounts, we create a composite financial statement measure to estimate the degree of financial reporting irregularities for a given firm-year. The measure assesses the extent to which features of the distribution of a firm's financial statement numbers diverge from a theoretical distribution posited by Benford's Law, or the law of first digits. Whether in aggregate, by year, or by industry, we find that the empirical distribution of the numbers in firms' financial reports generally conform to the theoretical distribution specified by Benford's Law. In a battery of construct validity tests, we show that i) the divergence measure is positively correlated with commonly used earnings management proxies, ii) the restated financial reports of misstating firms exhibit greater conformity, and iii) divergence decreases in the years following restatements. Turning to the informational implications of Benford's Law, we provide evidence that as divergence increases, information asymmetry increases and earnings persistence decreases in the year following the disclosure of the financial report. These results suggest that the degree of divergence from Benford's Law can be used as a tool to detect possible financial irregularities.

## **1. Introduction**

Irregularities in financial statements, whether created in error or to mislead, are difficult to detect and lead to inefficiencies in capital allocation. When a firm's financial statements mislead investors and regulators, the impact is pervasive and can consequently trickle down from the firm's institutional investors to its employees, who risk losing their jobs as well as a substantial portion of their savings, as was seen in the collapse of Enron. In the last decade, the Securities and Exchange Commission has dramatically reduced its sparse resources for detecting accounting fraud to instead focus on insider trading and issues surrounding the financial crisis. During that time, the number of financial frauds and financial restatements dropped dramatically (Whalen et al., 2013). While the former SEC enforcement director, Robert Khuzami, claims that the decline was due to fewer accounting and disclosure irregularities, other SEC officials hint that this is not the case (McKenna, 2013). For example, Mary Jo White, the current SEC chairperson, raised concerns regarding the drop in the number of accounting fraud cases the agency has brought in recent years, a sentiment echoed by Andrew Ceresney, the new codirector of the SEC's Division of Enforcement, who has acknowledged that fraud is going on undetected. In response to criticism regarding lax enforcement, the SEC has only recently announced a plan to create a unit focusing on accounting fraud.

In this study, we attempt to examine an empirically interesting and challenging question that arises from this debate: Do financial statement irregularities go undetected by auditors and regulators? Prior literature has struggled to provide insights into this question for at least two related reasons. First, researchers can only observe firms that were caught by the SEC or those that restated their financial statements. If indeed, as suggested by the anecdotal evidence, there are many irregularities that remain undetected, then a plausible explanation for the differences between detected irregularities and undetected irregularities is the ability of the filer to better

engage in activities that allow it to avoid detection. Second, most existing measures of accounting irregularities are generally based on abnormal accruals models that are inherently correlated with firms' business models and growth opportunities. As a result, if such models classify firms with complicated or high-growth business structures as having low accounting quality (Owens et al., 2013), they may generate false positives by erroneously flagging firms with more complicated business environments. The limitations of these accruals-based measures are especially concerning since the SEC has begun to construct an accruals-based model to detect financial statement irregularities, a model that may lead to increased and unnecessary regulatory scrutiny for complex registrants.

We attempt to overcome these concerns by constructing and validating a parsimonious measure that may serve as a red flag for irregularities solely based on the distribution of features of the numbers in financial reports. The primary measure we use is based on the mean absolute deviation (MAD) statistic as applied to the composite distribution of features of the numbers in annual financial statement data, which we term 10-K MAD for short. This measure is motivated by the forensic accounting literature which has used various techniques to assess single digit, as opposed to distributional, conformity to detect anomalous individual income tax reporting data and corporate financial data at the individual account level. The measure allows us to compare the empirical distribution of features of the numbers in a firm's annual financial report to that of a theoretical or expected distribution. Deviations from the theoretical distribution may prove useful to regulators in their renewed attempt to detect irregularities as 10-K MAD is a highly scalable and machine-readable tool that can potentially detect financial irregularities before such reporting becomes detrimental to the firm and its stakeholders (Eaglesham, 2013; Hoberg and

Lewis, 2013). Moreover, such a tool may also prove useful to researchers who attempt to study the causes and consequences of financial reporting irregularities (Dechow et al., 2010).

Forensic investigators, auditors, and prior literature suggest that a commonly used tool in practice to detect irregularities in numerical data is an examination of the distribution of the first digits in the numbers appearing in underlying data (Hill, 1998). More specifically, the distribution of the first digits in a set of numbers generated by natural interactions (i.e., unaltered) of varying amounts can be described by a very peculiar, non-uniform distribution: Benford's distribution or Law.<sup>1</sup> This distribution states that the first digits of all numbers in a data set containing numbers of varying magnitude will appear with decreasing frequency (that is, 1 will appear 30.1% of the time, 2 will appear 17.6% of the time, etc. – please refer to Appendix A for the exact distribution). Divergence from Benford's distribution has been used in practice to detect irregularities in published scientific studies, fraudulent election data in Iran, suspicious macroeconomic data from Greece prior to the country joining the EU, accounts receivables fraud, tax returns misreporting, and so forth. However, we are unaware of an attempt to apply Benford's Law to the entire set of numbers contained in an annual financial report in order to investigate whether divergence from Benford's Law can be used as a firm-year measure of the degree of financial reporting irregularities.

Since the reported values of the line items in financial statements are determined by many transactions in the period and over multiple periods, the leading digits of those numbers can be considered to be randomly generated and of varying magnitude. Because of this feature, it is possible that firms' financial statements follow Benford's Law. If firms' financial numbers generally follow Benford's Law, then deviations from it could provide investors, auditors, and

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<sup>1</sup> By "natural", we mean non-truncated or uncensored. For example, a petty cash account with a reimbursement limit of \$25 would not be expected to follow Benford's Law.

regulators with a “first-pass” filter or red flag in summary fashion. The flag itself could be built into models of accounting or information environment quality or simply be used to classify firms as “high risk” and therefore in need of further examination.<sup>2</sup>

A necessary step in examining the usefulness of Benford’s Law to assess irregularities in firms’ accounting data is to establish whether firms’ financial statements, on average, follow the law. This may not be the case if the accounting process does not conform to the natural or unaltered interactions that give rise to the law. We first show that our aggregate sample of all annual financial statement variables for the period 2001-2011 follows Benford’s Law. We further analyze our data and show that every year and industry in our sample closely follows the law. Lastly, when examining each firm-specific annual report independently, we find that roughly 85% of firm-years conform to the law.

Once conformity in general is established, we start exploring whether divergence from Benford’s Law can be used as a measure of accounting irregularities. As mentioned above, we measure the level of divergence using the mean absolute deviation (MAD) statistic as applied to all numbers contained in an annual financial statement (10-K MAD), which takes the sum of the absolute value of the difference between the empirical distribution of first digits and the theoretical Benford distribution, and divides that by the number of non-zero digits.<sup>3</sup> A higher (lower) 10-K MAD statistic implies that the composite distribution of the leading digits of the numbers contained in annual financial statement data exhibit greater divergence from (conformity to) Benford’s distribution.

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<sup>2</sup> We do not take a stance on the type of financial reporting irregularity—i.e., intentional errors (fraud) or unintentional errors—that is captured by divergence from Benford’s Law.

<sup>3</sup> See Appendix A for an example of the calculation.

We first conduct a battery of validation tests to establish the construct validity of the 10-K MAD statistic as a measure of financial statement irregularities. We start by showing that the 10-K MAD statistic is significantly positively correlated with other commonly used accounting irregularities measures, such as the Dechow-Dichev and modified Jones model measures. This is consistent with the fact that the 10-K MAD statistic captures some of the underlying irregularities measured by those commonly used measures. We continue our examination of the usefulness of the 10-K MAD statistic as a measure of irregularities by designing a powerful test to directly examine our conjecture. We identify a sample of firms that restated their financial statements and compare the 10-K MAD statistic for the restated and unrestated numbers. This test provides a unique setting to examine the usefulness of the MAD statistic since we compare the same firm-year to itself, thus keeping all else equal except for the reported numbers. We show that the restated numbers have significantly lower divergence (lower 10-K MAD statistic) from Benford's Law as compared to the same firm-year's unrestated numbers. This result provides strong evidence that divergence from Benford's Law is a useful tool for detecting irregularities. Moreover, we then go on to show that in the years following the restatement, financial statements more closely conform to Benford's distribution.

In our next set of tests, we examine the informational implications of divergence from Benford's Law. Prior literature has found that a decrease in the quality of financial disclosures leads to information asymmetries (Healy et al., 1999). The negative relation occurs because some investors have access to private information or are better at processing information while others do not have such access (Brown and Hillegeist, 2007). If the 10-K MAD statistic is indeed a measure of financial reporting irregularities, it should capture any reduction in the informativeness of financial statements, which would then imply greater information asymmetry

(Diamond and Verrecchia, 1991). We use three measures to quantify information asymmetry: one-quarter-ahead bid-ask spread, one-year-ahead bid-ask spread, and one-quarter-ahead probability of informed trading (Venter and de Jongh, 2006). With all three variables, we find a positive relation with the 10-K MAD statistic, which implies that information asymmetry increases as divergence from Benford's distribution increases, a finding that is consistent with the 10-K MAD statistic capturing financial statement irregularities.

In our last test of the informational implications of Benford's Law, we examine the relation between the level of conformity to the law and earnings persistence. If a higher 10-K MAD statistic captures a higher degree of financial statements irregularities, it is likely that current earnings are less likely to explain future earnings for such firms (Richardson et al., 2005). Li (2008) provides qualitative support for this argument by showing a negative relation between low financial report readability and earnings persistence. Similar to Li (2008), we expect there to be a negative relation between the 10-K MAD statistic and earnings persistence. We find that, for firms with the greatest amount of divergence, the 10-K MAD statistic is negatively related to earnings persistence.

Taken together, the findings from our tests suggest that divergence from Benford's Law is a useful tool for measuring financial statement irregularities. Our validation tests allow us to next examine the question of whether or not financial statements irregularities go undetected, given the current regulatory and enforcement environment at the SEC. We show that the MAD statistic negatively predicts SEC Accounting and Auditing Enforcement Releases actions and financial restatements. The negative relation is plausible given the current enforcement environment at the SEC and anecdotal evidence suggestive of lax enforcement. The result provides strong evidence that firms engage in activities that allow their financial statement

irregularities to remain undetected by auditors and enforcement bodies, yet such activities leave a trace of the irregularities in features within the distributional properties of accounting numbers.

In addition to informing the accounting quality and fraud detection literatures, our study contributes to the debate over recent calls by the investment community that the SEC should ramp up its resources and efforts to detect accounting fraud. Our paper provides a parsimonious and efficient “first-pass” approach for assessing financial statements for the possibility of financial irregularities. Moreover, as discussed in detail in Section 3, our measure of financial statement irregularities has significant advantages over previously used measures. For example, it is purely statistically driven, does not require a time series or cross-sectional data, and is available to essentially every firm with accounting information. Importantly, our paper contributes to the public debate by providing evidence consistent with the claim that financial irregularities are escaping detection. As such, our results collectively show promise for the 10-K MAD statistic having the potential for investors, auditors, researchers, and regulators to easily assess financial irregularities.

## **2. Institutional Setting: Motivation and the Need for Enhanced Detection Tools**

Other than a spike in the 2005-2006 period, the number of financial restatements has been historically low during the last decade (Whalen et al., 2013). For fiscal 2011, the SEC brought 735 enforcement actions against companies and individuals—a record for the agency—but only 89 of those actions focused on accounting irregularities at public companies (McKenna, 2012). In addition, the magnitude of the effect misstated numbers have on net income, a measure of the severity of the restatement, has also declined. Part of the reason for this decline is ostensibly due to the dismantling of the SEC’s accounting fraud task force. After missing Bernie Madoff’s \$65 billion Ponzi scheme, the SEC enforcement division, under Robert Khuzami,

dismantled the task force to divert resources to units that focused on crimes like Madoff's, as well as bribery and market manipulation (McKenna, 2012). Khuzami claimed that fewer revisions meant there were fewer irregularities, but during Khuzami's time in the enforcement division, revision restatements, or "restatements lite" proliferated (McKenna, 2012).<sup>4</sup> Such restatements do not require an 8-K Item 4.02 non-reliance filing. Instead, they are merely included in other periodic reports. Revision restatements comprised 65% of all restatements in 2012, the highest number since 2005 (Whalen et al., 2013).

Despite the low number of full restatements, the SEC undoubtedly believes that accounting fraud still exists. According to Scott Friestad, a senior SEC enforcement official: "We have to be more proactive in looking for it...[t]here's a feeling internally that the issue hasn't gone away"—a sentiment supported by those in industry (McKenna, 2013). A 2009 survey of 204 executives found that 65% of respondents considered financial reporting fraud and misconduct to be "a significant risk" for their industries (KPMG, 2009). And a recent survey of 169 CFOs of publicly traded companies estimated that about 20% of companies manipulate their earnings every year (Dichev et al., 2013). This evident contradiction between the opinions raised by the anecdotal evidence leads us to question whether firms' financial statements contain irregularities that go undetected by the SEC, the auditor, or the firm's internal control mechanisms.

Moreover, SEC Chair Mary Jo White has also made statements publicly about her concern regarding the drop in accounting irregularities cases (Gallu, 2013) and announced in July 2013 that the agency is creating a financial reporting and audit task force with a principle

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<sup>4</sup> Khuzami's logic is reminiscent of the fabled tale attributed to the Commissioner of the U.S. Patent & Trademark Office in the early 1900's in that the patent office (accounting fraud task force) should be closed (disbanded) since "everything's been invented" (there are fewer restatements).

goal of “fraud detection and increased prosecution of violations involving false or misleading financial statements and disclosures . . . including on-going review of financial statement restatements and revisions, analysis of performance trends by industry, and use of technology-based tools such as the [newly developed] Accounting Quality Model” (Ellsworth and Newkirk, 2013), which has been dubbed “Robocop” because of its reliance on algorithms and automated tasks to detect financial reporting irregularities.

Aligned with the SEC’s plans to rely more heavily on technology, such as its nascent Accounting Quality Model, to search for possible accounting irregularities (Eaglesham, 2013), we show that, while financial statement numbers generally conform to Benford’s distribution, a significant shift in a firm’s conformity to the distribution, as exhibited by the 10-K MAD statistic, can serve as a potential red flag for financial misreporting.<sup>5</sup> Such a tool is timely given the current institutional dynamics in play. With the rise of electronic disclosure and enhancements in computing technology, investors, auditors, researchers, and the SEC can efficiently and parsimoniously use 10-K MAD to augment existing modeling techniques. The 10-K MAD statistic also plays off one of Robocop’s weaknesses in that it does not rely on financial comparisons within industry peer groups to flag a company for suspicious behavior. In addition, Robocop relies, in part, on XBRL data to flag firms for potential irregularities. Like all cat-and-mouse games, the recent efforts by the SEC are already being anticipated and countered: Firms like RDG Filings are including in their marketing materials claims about how they can help firms reduce the likelihood of getting flagged by Robocop.<sup>6</sup> In contrast to Robocop’s

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<sup>5</sup> We use “financial misreporting” and “financial statement irregularities” interchangeably throughout the draft.

<sup>6</sup> “RDG Filings has the knowledge, expertise, and experience to ensure that the AQM-Robocop tool being deployed daily by the SEC are far less likely to flag your XBRL filings,” the company claimed in an article in August 2013.

strategy of using industry benchmarks, where fraud detection can be averted by following the industry pack, it is less likely that firms are able to systematically alter the conformity to Benford's Law of the entirety of the numbers they report on an annual basis.

### **3. Benford's Law**

#### *3.1 Mathematical Foundations*

Benford's Law is a mathematical property discovered in 1881 by astronomer Simon Newcomb, who noticed that the earlier pages of books of logarithms were more worn than the latter pages, which contain larger first digits. He inferred from this observation that scientists looked up smaller digits more often than larger digits and determined that the probability that a number has a first digit,  $d$ , is:

$$P(d) = \frac{1}{d \ln(10)}, \text{ where } d = 1, 2, \dots, 9.$$

This equation gives us the theoretical distribution of what is now commonly referred to as Benford's Law, or the expected frequency of the first digits 1 through 9 in a randomly generated data set (see Appendix A for the theoretical distribution).

In 1938, physicist Frank Benford tested Newcomb's discovery on a variety of data sets, including the surface areas of rivers, molecular weights, death rates, and the numbers contained in an issue of *Reader's Digest*, and found that the law held in each dataset (Benford, 1938). Some years later, Hill (1995) provided a formal derivation of Benford's Law. Intuitively, as explained by Durtschi et al. (2004), an asset with a value of \$1,000,000 will have to double in size before the first digit becomes 2, whereas it only needs to grow by 50% to get to 3 and by 33% to get to 4. While Boyle (1994) demonstrated that datasets containing numbers that have been multiplied, divided, or raised to a power often follow Benford's distribution, Hill proved that datasets that conform to Benford's distribution consist of convex combinations of other

distributions. Since the numbers contained in a financial reporting system, on the basis of double-entry accounting, are often endogenous combinations of other journal entries, accounting numbers are expected to frequently conform to Benford’s distribution.

### 3.3 Natural and “Unnatural” Sequences

Many natural number sequences, such as the Fibonacci Sequence, follow Benford’s distribution. The Fibonacci Sequence consists of a series of numbers where the next number equals the sum of the previous two:

$$F_n = F_{n-1} + F_{n-2}.$$

$F_0 = 0$  and  $F_1 = 1$ , so the sequence begins, 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144...

The distribution of the first digits of the first 200 numbers in the sequence is:

<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>	<u>9</u>
0.300	0.180	0.125	0.090	0.085	0.060	0.055	0.060	0.045

The distribution is very similar to Benford’s distribution, with a MAD statistic of 0.0041. Quite distinct from naturally occurring sequences, we can compare Benford’s distribution with the empirical distribution of the first digits from the monthly returns of the *Fairfield Sentry Fund*, a fund-of-funds that invested solely with Bernie Madoff, during the 215 months in which it reported returns (Blodget, 2008):

<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>	<u>9</u>
0.396	0.142	0.104	0.071	0.075	0.066	0.061	0.066	0.019

One would expect unaltered returns to conform to Benford's distribution, but this distribution differs significantly from the theoretical distribution with a MAD statistic of 0.0252, six times greater than that of the Fibonacci Sequence. While the empirical distribution of the Sentry Fund isn't direct evidence of fraud, such a large deviation should be a red flag that demands closer scrutiny from investors, regulators, auditors, and researchers.

### *3.3 Measuring Conformity*

Measuring whether a data set conforms to Benford's distribution has been the subject of some debate in the field of mathematics (Pike, 2008; Morrow, 2010). Test statistics can be strongly influenced by sample size, with some statistics requiring near perfect adherence to the distribution as the sample becomes large (Nigrini, 2012). We use two statistics when measuring conformity to Benford's distribution, the Kolmogorov-Smirnoff (KS) statistic and the Mean Absolute Deviation (MAD) statistic. The KS statistic uses the maximum deviation from Benford's distribution, determined by the cumulative difference between the empirical distribution of the digits from 1 to 9 and the theoretical distribution (see Appendix A for the distribution). This statistic is useful for firm-level examinations of conformity to Benford's distribution since there exists a critical value to test against:

$$\text{Critical value at the 5\% level} = 1.36/\sqrt{N},$$

where N is the total number of digits used.

The KS statistic becomes less useful as the sample size increases. In order establish (fail to reject) the null hypothesis of distributional conformity at the 5% level, the statistic requires near perfect conformity of the underlying empirical distribution to Benford's distribution for large data sets (Nigrini, 2012). As a result, the KS statistic tends towards over-rejection as sample size increases. The MAD statistic, on the other hand, does not take sample size into

account. The MAD statistic is calculated as the sum of the absolute difference between the empirical frequency of each digit from 1 to 9 and the theoretical frequency in Benford's distribution, divided by the number of digits (9) (see Appendix A for the distribution). The scale invariance aspect of the MAD statistic makes it useful for examining large data sets, such as the population of financial results over a decade, as well as for assessing conformity over time, especially since the number of line items in an annual report can vary across industries and through time. Consequently, we use the KS statistic only in our descriptive tests where we examine the number of individual firm-years that conform to Benford's distribution. For our empirical tests, where we do not require a firm-year critical value, we rely exclusively on the MAD statistic to assess the shift in the empirical distribution.

### *3.4 Use of Benford's Law in Accounting*

Carslaw (1988) used a variant of Benford's distribution to argue that New Zealand firms whose earnings didn't conform to the law were rounding up their earnings numbers. While Thomas (1989) showed similar results for U.S. firms, he further found that the relation inverts for loss firms by demonstrating a greater (lower) than expected frequency of 9's (0's) for such firms. Nigrini (1996) used Benford's Law to examine items such as the interest received and interest paid on individual tax returns and found a higher (lower) than expected frequency of 1's (9's) on interest received (paid). Note that in all three of the preceding papers, conformity is assessed on the basis of single digit frequency deviations from expectation, rather than on a composite distributional deviation. More recently, Durtschi et al. (2004) provided a practitioner's guide for auditors on potential uses of Benford's Law to uncover fraud in individual accounts. The limited amount of prior research has alluded to the large-scale application of Benford's Law to detect possible earnings management or financial irregularities, but has thus far only restricted itself to tax settings and individual financial accounts.

With the advent of machine-readable financial reports and computing technology that can quickly assess thousands of reports at a time, the financial reporting applications of Benford's Law have only recently become tractable on a large scale. Distinct from prior studies, we employ a measure of financial statement conformity to Benford's Law on a firm-year basis for the composite distribution of the leading digits from all numbers contained in a firm's annual financial report, the 10-K MAD statistic. We further differ from prior literature in that we then examine how this composite measure relates to financial reporting irregularities, informational properties of the firm, and predictive ability.

### *3.5 Advantages of 10-K MAD*

In addition to the implications for its use by practitioners, the 10-K MAD statistic can benefit researchers since it has significant advantages over existing measures of financial reporting irregularities or accounting quality that are commonly used in the literature. First, it does not require a time series or cross-sectional data to estimate on a firm-year basis as do essentially all other accounting quality measures (e.g., Dechow and Dichev measure, Jones model, smoothness, conservatism, earnings persistence, relevance, etc.). Second, the measure is purely statistically driven, and therefore, we believe there should be no ex-ante relation to underlying firm characteristics, such as complexity. This aspect of the measure is a significant advantage in that, theoretically, it should not be related to the firm's business model or the accrual generating process, which is a major limitation of the accruals-based models (Owens et al., 2013). Third, it does not require forward-looking information as required by some of the abnormal accruals models (e.g., Dechow and Dichev measure). Fourth, it does not require returns or price information. Fifth, it is completely scale independent and thus fits to every currency or size. Sixth, it is available to essentially every firm with accounting information. Finally, unlike

some strategies used by regulators, this measure does not rely on industry peer comparisons to flag firms for potential irregularities.

While the advantages are substantial, a significant deficiency of this measure is that it does not give the user any insights as to the origination of the irregularity (i.e., upward accrual management, downward accrual management, smoothing, mistakes, etc.). In that sense, it can be used as an indicator that can quickly flag potentially high-risk financial reports for further review. A second major concern is that it requires the assumption that if the irregularity is purposefully done by the firm, the firm either is not aware of, not concerned with, or unable to change the distribution of the first digits.

#### **4. Prediction Development**

##### *4.1 Validation Tests: Establishing Unconditional Conformity*

A data generating process comprised of combinations from other distributions frequently follows Benford's distribution (Boyle, 1994). Financial information reported in companies' annual reports (10-K's) appears to follow such a process. For example, reported revenue consists of the sums of many individual transactions, and each of those transactions is calculated as selling price multiplied by quantity. This insight into Benford's distribution leads us to our first prediction, that the empirical distribution of an aggregate sample of reported financial results will conform to Benford's distribution.

An aggregate sample over a ten-year period will provide more than 1 million first digits to use to calculate how closely the empirical distribution follows Benford's distribution. As stated above, the numbers in this sample are generated in a manner similar to those in many other samples that conform to the law. However, the empirical distribution might fail to conform to the law because, unlike naturally occurring empirical distributions, the numbers in the financial

statements are generated by economic agents who may have intent. To test conformity, we need to use the 10-K MAD statistic (see Appendix A for the calculation) and assess how close it is to zero. The closer the 10-K MAD statistic is to zero, the closer the empirical distribution is to Benford's distribution. If the aggregate empirical distribution conforms, we would expect that samples sorted by year and by industry would also conform. If this empirical distribution conforms to Benford's distribution, it is likely that we will see conformity at the firm-year level as well, which leads us to our second prediction that, on average, each firm-year's financial results will conform to Benford's distribution.

Unlike at the aggregate, year, and industry levels, we use a sample of less than 250 digits to calculate the empirical distribution for a given firm-year. The advantage of the smaller pool of first digits from which to generate the empirical distribution on a firm-year basis is that the KS statistic, which has a defined critical value, can be calculated. The disadvantage to examining conformity at the firm-year level is that the smaller sample size introduces the potential for erratic distributions.

#### *4.2 The Change in Conformity Around Restatements*

Next, based on insights from prior literature, we argue that firms that misstate their financial results, either unintentionally or to disguise actual results, may report numbers with first digits that are not, in expectation, randomly generated. Given the nature of double-entry bookkeeping, this lack of randomness will trickle through the financial statements. For example, a firm that is trying to increase earnings for the current period may underreport depreciation. This artificial manipulation will affect net property, plant and equipment, accumulated depreciation, depreciation expense, operating income, taxable income, and net income. On the other hand, after a restatement, financial reports should more closely represent the true nature of

the distribution of leading digits found within the financial statements. This expected impact of misreporting leads us to our next prediction: The empirical distribution of restated financial statements will more closely follow Benford's distribution than the empirical distribution of misstated financial reports. The difference in the level of conformity between the restated and misstated results can be measured through the change in the 10-K MAD statistic. For firms that restate their financial results, a lower 10-K MAD statistic for the restated results compared to the misstated results signifies that changes in the 10-K MAD statistic may be used to gauge the extent of financial misreporting.

Prior literature has found that firms that restate attempt to "right the ship" after detection of events which question financial statement credibility (Farber, 2005; Bartholdy et al., 2013). This leads us to the following prediction: A firm's empirical distribution in the years following a restatement will more closely follow Benford's distribution than the empirical distribution before the restatement. We therefore would expect the 10-K MAD statistic to decrease in the years following a restatement since firms' less altered financial results should tend to gravitate toward reporting naturally generated first digits. As a result, the empirical distribution of these more accurately reported numbers will more closely conform to Benford's distribution.

#### *4.3 Informational Implications of Divergence*

If financial statement irregularities, whether created in error or intentionally, reduce the conformity of the empirical distribution of financial reports to Benford's distribution, an increase in the 10-K MAD statistic should signal a decrease in disclosure quality. We therefore argue that the 10-K MAD statistic can be used to measure the quality of reported financial results. As the quality of disclosure decreases, information asymmetry increases as traders are concerned that they are trading against an informed trader (Diamond and Verrecchia, 1991; Healy et al., 1999).

This argument leads us to our next prediction: Information asymmetry will increase as a firm's divergence from Benford's distribution increases.

Building off the argument that deviation from Benford's distribution signals a decrease in the quality of reported financial results, we expect that, following Li (2008), greater divergence from Benford's distribution should signal lower earnings persistence. The idea is based on the notion that it is less likely that current earnings will be as informative about future earnings in firms with lower accounting quality (Richardson et al., 2005). Li (2008) provides qualitative support for this argument by showing a negative relation between low financial report readability and earnings persistence. If the 10-K MAD statistic does capture irregularities in financial statements, current earnings are expected to exhibit less persistence for firms with greater divergence, giving us our next prediction: Earnings persistence will decrease as a firm's conformity to Benford's distribution decreases.

#### *4.4 The Predictive Ability of 10-K MAD*

Finally, in directly responding to the recent debate and nascent efforts by the SEC surrounding accounting fraud and detection, we examine whether financial irregularities are escaping detection by the SEC, the auditor, or the firm's internal control mechanisms. Based on the criticism leveled at the SEC during Khuzami's tenure after disbanding the accounting fraud group and the severe underfunding of the SEC to pursue detection and enforcement, during the period of our study, we expect that 10-K MAD will negatively predict financial restatements and AAER's if there are indeed undetected financial statements irregularities. If, however, the SEC's efforts during the period sufficiently identified and prosecuted fraud, and deterred "restatements lite", then we would expect a positive relation between 10-K MAD, financial restatements, and AAER's.

## **5. Sample Selection, Variable Measurement, and Descriptive Statistics**

### *5.1 Sample Selection and Variable Measurement*

Our sample consists of all annual data from Compustat for the period 2001-2011. For simplicity and objectivity, we use all Compustat variables that appear in the Balance Sheet, Income Statement, and Statement of Cash Flow to calculate the 10-K MAD and 10-K KS statistics. For variables reported with a value of less than 1, we take the first non-zero digit. We set missing variables to 0, as they do not affect our calculations of the 10-K MAD and 10-K KS statistics, which require only digits 1 through 9.

As previously discussed, the primary measure we use throughout the paper to assess an empirical distribution's conformity to Benford's theoretical distribution is the 10-K MAD statistic, as it is insensitive to sample size and is therefore useful when examining large samples. While the 10-K KS statistic also tests conformity to the law and, unlike the 10-K MAD statistic, has established critical values against which to test, it becomes unreliable as the sample size increases. We therefore only rely on the 10-K KS statistic when gauging the conformity of individual firm-years. In terms of other primary variables of interest, RESTATED is an indicator variable equal to 1 if a firm restated its financial statements in that year, according to the Audit Analytics database, and is zero otherwise. FRAUD is an indicator variable equal to 1 if a firm was included in the annual AAER database (Dechow et al., 2011) for allegedly misstating their financial information in that year. RESTATED\_NUMS is an indicator variable assigned to all firms that have both restated and originally reported numbers in a year available through Compustat. It is equal to 1 if the reported numbers are restated and zero if the numbers are what was originally reported. CAUGHT equals 1 if a firm ever restated its financial statements, according to Audit Analytics database, and POST equals 1 for the years after a firm has restated its financial information.

We use three measures to examine information asymmetry, one-quarter-ahead average bid-ask spread (QTRBASPREAD), one-year-ahead average bid-ask spread (YRBASPREAD), and one-quarter-ahead probability of informed trading (PIN). QTRBASPREAD is calculated from data available in the monthly CRSP stock return files for 2001-2011. We take the difference between the absolute values of the bid and ask prices and divide that by the absolute value of the closing price for each month for all firms. We then take the average of that number for the three months following the end of the fiscal year. YRBASPREAD is calculated in a similar manner but uses the 12 months following the end of the fiscal year. Given the known interpretation issues in using bid-ask spreads, we follow Daske et al. (2013) and control for share turnover, the volatility of returns, size, and financial leverage to capture other determinants of information asymmetry such as liquidity, inventory holding costs, and inventory risk (Amiram et al., 2012).

In contrast to bid-ask spreads, PIN is a firm-specific estimate of the probability that a trade originates from a privately informed investor. Hence, PIN directly captures the extent of information asymmetry *among investors* in the secondary market. As Brown and Hillegeist (2007) argue, an advantage of PIN over spread-based measures is that PIN can be disaggregated “into its component parameters, each of which represents a different aspect of the firm’s trading and information environment.” On the other hand, similar to bid-ask spreads, PIN may also measure trading liquidity rather than only information asymmetry (Duarte and Young, 2009). Therefore, similar to the bid-ask spread regressions, we include the Daske et al. (2013) controls in this specification as well. Our PIN data is downloaded from Stephen Brown’s website and is calculated based on Brown and Hillegeist (2007), which implements the Venter and de Jongh (2006) extension in order to improve the Easley et al. (1997) estimation technique.

We remove from the sample any firm-years where the total number of first digits used to calculate the MAD and KS statistics for a given firm-year is less than 100 in order to increase the power of the test statistics (including those with less than 100 first digits does not alter our results). We also remove firms with negative total assets. All non-indicator variables in the total sample of 41,863 firm-years are then winsorized at the 1% and 99% levels to eliminate the influence of outliers. See Appendix B for further details, as well as for the definitions of the control variables.

### *5.2 Descriptive Statistics*

Table 1 provides descriptive statistics for the full sample of firms from 2001-2011. The 10-K MAD statistic's mean is 0.029 with a standard deviation of 0.009. 16.1% of firm-years in our sample restate and 0.6% of firm-years are subject to an AAER. QTRBASPREAD, the one-quarter-ahead average bid-ask spread, has a mean of 0.178 and a standard deviation of 0.114. The mean of the one-year-ahead average bid-ask spread, YRBASPREAD, is 0.184, and its standard deviation is 0.099. PIN, the one-quarter-ahead probability of informed trading, has a mean of 0.231 and a standard deviation of 0.127.

Table 2 presents Spearman correlations above the diagonal and Pearson correlations below the diagonal. The Pearson correlation between the 10-K MAD statistic and RESTATED is -0.0095 and not significant at the 5% level, but the relation is negative and significant at the 5% level (-0.0305) with the Spearman calculation. We find similar results in the correlation between 10-K MAD and FRAUD, with both Pearson and Spearman correlations negative and significant (-0.0180 and -0.0390, respectively). Importantly, Table 2 reveals that the 10-K MAD statistic is positively correlated with our two measures of accounting quality, the absolute value of the modified Jones model residual, ABS\_JONES\_RESID, and the standard deviation of the

Dechow-Dichev (2002) residual, STD\_DD\_RESID. For ABS\_JONES\_RESID, the Spearman correlation with the 10-K MAD statistic is 0.0579, which is significant at the 5% level or better. The Pearson correlation is also significant at 0.0740. The Spearman correlation between the 10-K MAD statistic and STD\_DD\_RESID is 0.0028, which is not significant, but the Pearson correlation is significant at 0.0097. These relationships may imply that these measures of accounting quality and our measure of deviation from Benford's distribution are capturing low accounting quality. In addition, a positive and significant correlation exists between 10-K MAD and all the variables used to measure information asymmetry, which implies that conformity to Benford's distribution deteriorates as information asymmetry increases. Finally, the correlations between 10-K MAD and net income for periods  $t$ ,  $t+1$ , and  $t+2$  are all negative and significant, implying that conformity to Benford's distribution decreases as performance and persistence decrease.

## **6. Methodology and Results**

### *6.1 Investigating the Unconditional Distribution of First Digits in Financial Reports*

Table 3 shows how the *aggregate* empirical distribution conforms to Benford's Law. That is, the 10-K MAD statistic is calculated by measuring the frequencies of the first digits from *all* firm-years in the sample. In the aggregate, the 10-K MAD statistic is 0.0015, well below 0.006, which can be considered close conformity to the law (Nigrini, 2012). This result can also be seen graphically in Figure 1. Panels B and Panel C of Table 3 show similar results when examining aggregate financial results by industry based on the Fama-French 17-industry classification and by fiscal year. This table supports our first prediction that the empirical distribution of the frequency of first digits in aggregate financial results conforms to Benford's distribution.

Table 4 examines individual firm-year conformity to Benford's Law based on the 10-K KS statistic. Of the 41,863 firm-years in our sample, 35,456, or 86%, conform to the law at the 5% level or better, as shown in Panel A. Figure 2 provides examples of the empirical distributions for two firm-years, one that conforms to Benford's distribution at the 5% level (AT&T, 2003) and one that does not conform (Sprint Nextel, 2001). While there are some kinks in AT&T's distribution, the overall divergence from Benford's distribution is visually apparent for Sprint Nextel. Panel B of Table 4 shows similar results when firms are sorted by industry, with a minimum conformity of 83% of all firms in a given industry and a maximum conformity of 91%. Panel C shows similar results when firms are sorted by fiscal year, with all years exhibiting between 84% and 87% conformity. This table supports our second prediction that a significant majority of firm-year empirical distributions conforms to Benford's Law.

### *6.2 Misstated versus Restated Financial Statements*

We have established that the empirical distributions of most firms' financial results conform to Benford's Law. Hypothetically speaking, unlike the results reported in an unaltered 10-K, results that contain irregular numbers may not be generated through an endogenous process arising from double-entry bookkeeping. Therefore, when firms restate their financial results, the empirical distribution of the restated results should more closely conform to Benford's Law than the misstated results. To test this prediction, we investigate a sample of firms that have restated their financial results and compare the 10-K MAD statistics of the misstated financial results with those of the restated results. Consequently, we expect that the 10-K MAD statistic will decrease, or more closely conform to Benford's Law, for the restated results.

To conduct our test, we examine all firm-years in Compustat from 2001-2011 where both misstated and restated financial results are available (in Compustat, datafmt=STD for original and datafmt=SUMM\_STD for restated). We then create an indicator variable, RESTATED\_NUMS, which is equal to 1 for results that have been restated and 0 for the originally reported results. We regress this indicator variable on the 10-K MAD statistic and include two variables to control for accounting quality, the five-year moving standard deviation of the Dechow-Dichev residual (Dechow and Dichev, 2002), as suggested by Kothari et al. (2005), and the absolute value of the accruals quality residual from the modified Jones model (Jones, 1991), as suggested by Francis et al. (2005). Please refer to Appendix B for calculations of these variables. Since the regression compares the firm to itself, we do not include additional firm control variables.

Table 5 presents the results of our test of our third prediction that the relation between the empirical distribution of firms' financial results and Benford's Law changes for misstated versus restated results in the same firm-year. Consistent with this prediction, the coefficient on RESTATED\_NUMS in Column (1) is -0.0004, which is statistically significant at the 5% level. To ensure that our measure of conformity to Benford's Law isn't merely a proxy for existing measures of accounting quality, in Column (2) we control for two standard accounting quality measures, modified Jones (ABS\_JONES\_RESID) and Dechow-Dichev residuals (STD\_DD\_RESID), as discussed above. When adding these two additional measures, we find similar results, with the coefficient on RESTATED\_NUMS equal to -0.0004 and significant at the 5% level. Consequently, for the sample of firms that have both original and restated financial results available through Compustat from 2001-2011, the 10-K MAD statistic is lower for the restated results, which implies that the empirical distribution of restated financials more closely

conforms to Benford's Law. As our result is incremental to standard accounting quality proxies, this result also implies that our measure, while somewhat correlated with these proxies, is distinct from it. Consequently, our measure of conformity may be useful in augmenting existing accounting quality models.

### *6.2 Benford's Law in the Years Following a Restatement*

Firms that restate their financial results may be more cautious about reporting in years following a restatement (Farber, 2005; Bartholdy et al., 2013). As a result, if firms are more cautious with their financial reporting after a restatement, one may expect greater conformity to Benford's Law in the years following a restatement. This is our fourth prediction. To test this prediction, we first create two indicator variables. CAUGHT equals 1 for all firm-years if a firm is included in the Audit Analytics database as having restated its financial results at some point between 2001 and 2011. POST equals 1 for the years after a firm has restated its financial results. For firms included in Compustat from 2001-2011 that have not restated their financial results, we assign a random year to each firm to calculate POST. Next, we interact CAUGHT and POST with the expectation that the interaction will be negative, signifying that the 10-K MAD statistic decreases in the years following a restatement. In addition to the accounting quality controls in Table 5, we follow Sun (2012) and control for managerial discretion by including working capital accruals (WCACC) and soft assets (SOFTAT) in our regression. We also include changes in ROA (CHROA) and sales (CHCSALE) to control for firm performance, assets (AT) to control for size, and book-to-market (BTM) to control for growth.

Table 6 presents the results of our test on whether firms that restate their financial results more closely follow Benford's distribution in the years following a restatement. Consistent with our fourth prediction, in Column (1), the coefficient on POST x CAUGHT, -0.0005, is

statistically significant at the 5% level. Adding accounting quality controls, as well as controls for firm characteristics, in Column (2) we find similar results, with the coefficient on POST x CAUGHT equal to -0.0005 and significant at the 5% level. This implies that conformity to Benford's Law increases in the years following a restatement.

### *6.3 Information Asymmetry*

If 10-K MAD speaks to the possibility of detecting irregularities in financial reports, divergence from Benford's Law may be associated with lower quality financial reports, which may manifest itself in higher information asymmetry among investors, our fifth prediction. We rely on three measures of information asymmetry to test this hypothesis, QTRBASPREAD, YRBASPREAD, and PIN. To test this hypothesis, we regress the 10-K MAD statistic on the measures of information asymmetry. In addition to the accounting quality controls from previous tests, we follow Daske et al. (2013) and control for size by including market value (MKT\_VAL) and total assets (AT), trading volatility by including share turnover (SHR\_TURN), the volatility of returns by including the annual standard deviation of monthly returns (RET\_VOL), and financial leverage by including the ratio of total liabilities to total assets (FIN\_LEV). These controls are intended to capture other determinants of information asymmetry such as liquidity, inventory holding costs, and inventory risk (Amiram et al., 2012).

Table 7 presents the results of our test of our fifth prediction on the informational consequences of firm-year level of conformity to Benford's distribution. Consistent with this hypothesis, information asymmetry, as measured by QTRBASPREAD, YRBASPREAD, and PIN, increases as the 10-K MAD statistic increases (that is, diverges from Benford's Law). This relation implies that the information environment becomes more opaque as firm-year conformity to Benford's distribution diverges. Specifically, in Columns (1), (3), and (5), the coefficients for

QTRBASPREAD, YRBASPREAD, and PIN are 1.179, 1.350, and 1.263, respectively. All coefficients are significant at the 1% level. When we add accounting quality, firm characteristics and market controls in Columns (2), (4), and (6), the coefficients for QTRBASPREAD, YRBASPREAD, and PIN are 0.252, 0.503, and 0.973, respectively. Again, all coefficients are significant at the 1% level. This result corroborates our findings above regarding restatements in that there appears to be a relation between the 10-K MAD statistic and the informational quality of financial disclosures. While we have remained largely agnostic about the form of financial irregularities Benford's Law captures (e.g., intentional or unintentional errors), this finding with respect to PIN is especially noteworthy as it implies that divergence from Benford's Law may be evidence of intentional manipulation of a firm's financial report.

#### *6.4 Benford's Distribution and Earnings Persistence*

Our sixth prediction states that earnings persistence is likely to be lower when firms significantly deviate from Benford's distribution. To test this hypothesis, we regress the interaction between net income and the 10-K MAD statistic for the most divergent firms in year  $t$  on net income in years  $t+1$  and  $t+2$ . Following Li (2008), we control for managerial discretion by including accruals (ACC), whether the firm pays a dividend (DIV), the market value of equity (SIZE), growth (MTB), special items (SI), survivorship (AGE), return volatility (RET\_VOL), and the volatility of net earnings (NI\_VOL).

Our results, presented in Table 8, support our prediction. Both columns show that, when looking at firms with a 10-K MAD statistic that substantially diverges from Benford's Law, there is a negative and significant relation between earnings persistence and the 10-K MAD statistic. Specifically, for year  $t+1$ , the coefficient on the interaction between the MAD statistic and net income in year  $t$ , -66.136, is significant at the 5% level. For year  $t+2$ , the coefficient on the

interaction variable, -80.091 is significant at the 10% level. This evidence, combined with that provided in our information asymmetry tests, corroborates the view that divergence from Benford's Law, as exhibited by annual financial reports and captured by 10-K MAD, may reflect the informational quality of financial disclosures.

### *6.5 Benford's Distribution and the Predictability of AAER and Financial Restatements*

Having validated our 10-K MAD measure of financial reporting irregularities with respect to restatements and demonstrated its informational implications, in our final test, we directly examine whether financial statement irregularities appear to go undetected by the SEC, the auditor, or the firm's internal control mechanisms. To do so, we employ logit regressions to test whether the 10-K MAD statistic is predictive of SEC AAER's and financial restatements. As before, we control for accounting quality and firm characteristics as in Table 6.

On the presumption that our measure is a valid approximation of financial reporting irregularities, our results presented in Table 9 support our hypothesis that irregularities appear to escape detection. Column (1) tests the 10-K MAD statistic's relation to RESTATED, an indicator variable equal to 1 if a firm appears in the Audit Analytics database as having restated its financial results in that year. The coefficient on the 10-K MAD statistic, -2.979, is significant at the 10% level. When we include controls in Column (2), the coefficient, -5.559, is significant at the 1% level, signaling that financial statements containing irregularities escape detection by the SEC and/or firms' auditors. Columns (3) and (4) show similar results for FRAUD, an indicator variable equal to 1 if a firm was the subject of an AAER in that year. In both columns, the coefficient on the 10-K MAD statistic (-27.933 and -27.147, respectively) is negative and significant at the 1% level, again corroborating the view that financial statements containing irregularities escape detection. As such, consistent with critics' views that the SEC should ramp

up its accounting fraud detection efforts, these results provide evidence that firms engage in activities that enable them to avoid detection of financial statements irregularities, yet such activities still leave traces in the distributional properties of firms' financial statements in the form of deviations from Benford's Law.

## **7. Summary and Conclusion**

We provide a first attempt to answer the question of whether financial statement irregularities go undetected by the SEC, the auditor, or the firm's internal control mechanisms. Building on forensic research and practice, this paper provides a much-needed tool to assess financial reporting irregularities based on the level of divergence from Benford's Law. In particular, we propose that interested parties may find a firm's level of divergence from Benford's Law to be a useful tool to augment existing techniques to uncover financial reporting irregularities. This law states that the first digits of all numbers in a data set containing numbers of varying magnitude will follow a particular theoretical and mathematically derived distribution where the leading digits 1 through 9 appear with decreasing frequency. In the context of financial reporting, numbers that exogenously arise in financial statements due to financial misreporting, as opposed to a data generating process that endogenously arises in the accounting system on the basis of double-entry bookkeeping, should not follow Benford's Law.

We construct a composite, firm-year measure of financial statement irregularities based on the divergence between the observed first digits distribution in annual financial statements and the theoretical Benford distribution. This measure has significant advantages over other measures of accounting quality that are currently used in the literature. That is, it does not require a time series or cross-sectional data, is purely statistically driven, does not require forward looking information, does not require returns or price data, is completely scale independent, is

available for essentially every firm with accounting information, and does not rely on peer group benchmarking. In our initial, unconditional validation tests, we find that at the aggregate level, financial statement numbers conform to Benford's Law in all industries and years. When assessing the conformity of individual firm-years, we find that roughly 85% of firm-years conform to the law as well.

We then further show that when restatements occur, the restated numbers are significantly closer to Benford's Law relative to the misstated numbers. In addition, conformity increases in the years following the restatement. These results, which are incremental to standard accounting quality proxies, suggest that the measure we use to determine the level of conformity to Benford's distribution, the 10-K MAD statistic, can serve as a distinct tool to detect financial reporting irregularities. Consequently, in today's environment of increasingly electronic, machine-readable disclosures, the investment community (investors, regulators, auditors, and researchers) can easily and parsimoniously deploy such a tool on a large scale as a "first-pass" filter to flag companies that potentially file suspect financial disclosures.

The suggestion that the 10-K MAD statistic can be used as a tool to detect financial misreporting is bolstered by its relation to a firm's information environment and earnings persistence. We find that as firms' financial statements diverge from Benford's Law, their information environments deteriorate and earnings persistence decreases. These negative relations with the 10-K MAD statistic further support our claim that there exists a relation between the level of divergence from Benford's distribution and the informational quality of reported financial results.

Finally, returning to the primary question that motivated our study, we provide evidence that the 10-K MAD statistic negatively predicts SEC AAERs and restatements. In light of the

current regulatory and enforcement environment at the SEC, and in the wake of Robert Khuzami's tenure, this result supports claims by critics that a significant number of financial statement irregularities escape detection by the SEC, the auditor, or the firm's internal control mechanisms. As such, our study is timely in that it answers recent calls by critics for the SEC to ramp up its resources and efforts to detect financial reporting fraud by suggesting a measure that the SEC, as well as auditors, researchers and, investors, can use for such purposes.

To our knowledge, this paper is the first to document how firms' composite annual financial statement conformity to Benford's Law changes after financial restatements and is also the first to demonstrate the informational implications of firms' divergence from the law. In the age of information overload and burgeoning financial reports, our paper provides a parsimonious, efficient approach for assessing financial statements for the possibility of financial misreporting. Future research in this vein could explore the relation between conformity to Benford's Law and the likelihood of accounting fraud in periods of high enforcement. In particular, as data becomes available, future studies should follow the SEC's progress in implementing its Accounting Quality Model to determine if its efforts will bear fruit with respect to fraud detection and enforcement. Additionally, exploring how investors punish or reward firms based on conformity could provide further insights into the ability of Benford's Law to assess the informational quality of financial reports.

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**APPENDIX A:** How to calculate conformity to Benford's Law, an example

<b>Assets</b>		<b>Liabilities</b>	
Cash	<b>1,364</b>	Accounts payable	<b>1,005</b>
Accounts receivable	<b>931</b>	Short-term loans	<b>780</b>
Inventory	<b>2,054</b>	Income taxes payable	<b>31</b>
Prepaid expenses	<b>1,200</b>	Accrued salaries and wages	<b>37</b>
Short-term investments	<b>38</b>	Unearned revenue	<b>405</b>
Total short-term assets	<b>5,587</b>	Current portion of long-term debt	<b>297</b>
Long-term investments	<b>1,674</b>	Total short-term liabilities	<b>2,555</b>
Property, plant, and equipment (Less accumulated depreciation)	<b>4,355</b> <b>2,215</b>	Long-term debt	<b>6,507</b>
Intangible assets	<b>608</b>	Deferred income tax	<b>189</b>
Other	<b>84</b>	Other	<b>587</b>
Total assets	<b>14,523</b>	Total liabilities	<b>9,838</b>
		<b>Equity</b>	
		Owner's investment	<b>1,118</b>
		Retained earnings	<b>2,732</b>
		Other	<b>835</b>
		Total equity	<b>4,685</b>
		Total liabilities and equity	<b>14,523</b>

Above is a sample balance sheet. To test its conformity to Benford's Law, take the first digit of each number (in bold), and calculate the distribution of the occurrence of each digit. In this case, there are 28 total numbers and eight appearances of the number 1, so 1's distribution is  $8/28 = .2857$ .

Compare the digit distributions to those of Benford's theoretical distribution:

Digit	1	2	3	4	5	6	7	8	9
Total occurrences	8	5	3	3	2	2	1	2	2
Empirical Distribution	0.2857	0.1786	0.1071	0.1071	0.0714	0.0714	0.0357	0.0714	0.0714
Theoretical Distribution	0.3010	0.1761	0.1249	0.0969	0.0792	0.0669	0.0580	0.0512	0.0458

The **Mean Absolute Deviation (MAD) statistic** and the **Kolmogorov-Smirnoff (KS) statistic** are computed from these distributions to test the conformity of the sample to Benford's Law.

1.) The **KS** statistic is calculated as follows:

$$KS = \text{Max}(|AD_1 - ED_1|, |(AD_1 + AD_2) - (ED_1 + ED_2)|, \dots, |(AD_1 + AD_2 + \dots + AD_9) - (ED_1 + ED_2 + \dots + ED_9)|)$$

where AD (actual digit) is the empirical frequency of the number and ED (expected digit) is the theoretical frequency expected by Benford's distribution.

In this example,

$$\begin{aligned} & \text{Max}(|0.2857 - 0.3010|, |(0.2857 + 0.1786) - (0.3010 + 0.1761)|, \dots, \\ & (|(0.2857 + 0.1786 + 0.1071 + 0.1071 + 0.0714 + 0.0714 + 0.0357 + 0.0714 + 0.0714) - \\ & (0.3010 + 0.1761 + 0.1249 + 0.0969 + 0.0792 + 0.0669 + 0.0580 + 0.0512 + 0.0458)|) = 0.0459 \end{aligned}$$

To test conformity to Benford's distribution at the 5% level based on the KS statistic, the test value is calculated as  $1.36/\sqrt{N}$ , where N is the total number of occurrences. The test value for the sample balance sheet is  $1.36/\sqrt{28} = 0.2570$ . Since the calculated KS statistic of 0.0459 is less than the test value, we cannot reject the hypothesis that the sample distribution follows Benford's theoretical distribution.

2.) The **MAD** statistic is calculated as follows:

$$MAD = (\sum_{i=1}^K |AD - ED|) / K, \text{ where } K \text{ is the number of leading digits being analyzed.}$$

In this example,

$$\begin{aligned} & (|0.2857 - 0.3010| + |0.1786 - 0.1761| + |0.1071 - 0.1249| + |0.1071 - 0.0969| + |0.0714 - 0.0792| + |0.0714 - \\ & 0.0669| + |0.0357 - 0.0580| + |0.0714 - 0.0580| + |0.0714 - 0.0458|) / 9 = 0.0140. \end{aligned}$$

Since the denominator in MAD is K, this test is insensitive to scale (sample size, or N). This test becomes more useful as the total sample size increases, while the KS test become more sensitive as N increases.

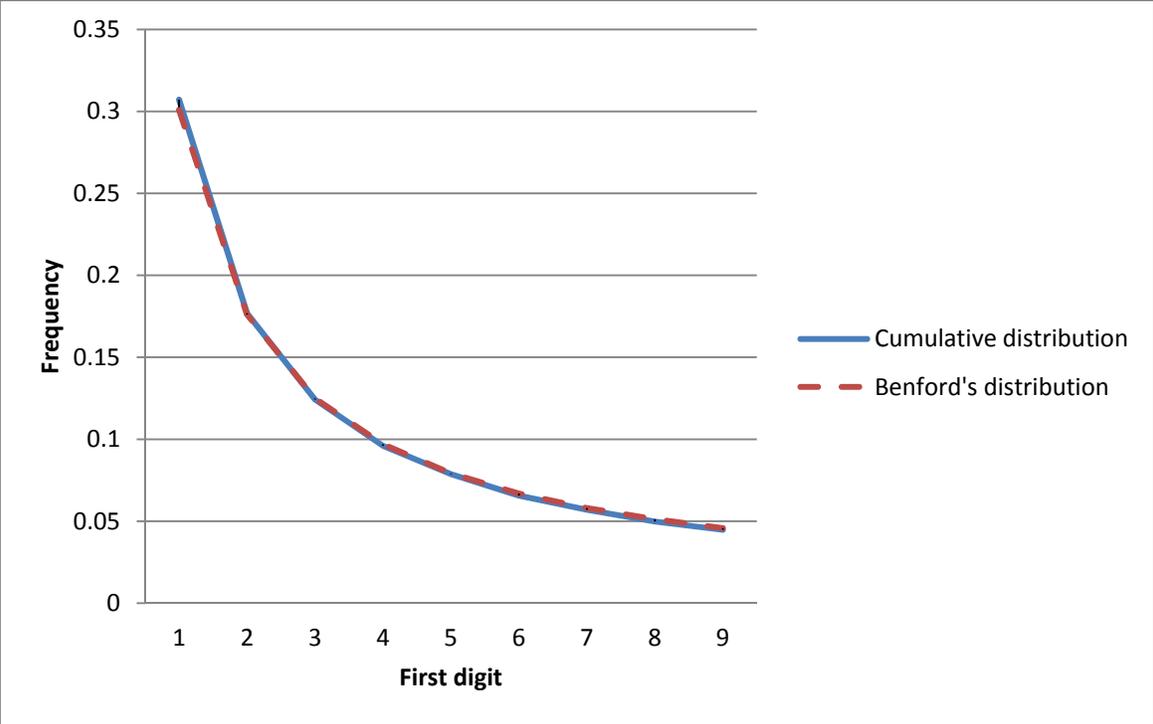
There are no determined critical values to test the distribution using MAD.

## Appendix B: Variable definitions

VARIABLE	DESCRIPTION	DEFINITION
10-K MAD	Mean absolute deviation test statistic for annual financial statement data	The sum of the absolute difference between the empirical distribution of leading digits in annual financial statements and their theoretical Benford distribution, divided by the number of leading digits. See Appendix A for a sample calculation of MAD.
10-K KS	Kolmogorov-Smirnoff test statistic for annual financial statement data	The maximum deviation of the cumulative differences between the empirical distribution of leading digits in annual financial statements and their theoretical Benford distribution. See Appendix A for a sample calculation of KS.
RESTATED	Indicator that equals 1 if a firm restated its financial information in that year	Firms that publicly disclosed financial restatement and non-reliance filings from 2001-2011, based on the Audit Analytics database.
FRAUD	Indicator that equals 1 if a firm was the subject of an SEC Accounting and Auditing Enforcement action in that year	Firms that were included in the annual Accounting and Auditing Enforcement Releases (AAER) database (Dechow et al., 2011) for allegedly misstating their financial information.
QTRBASPREAD	Average bid-ask spread in the quarter following the end of the fiscal year	Monthly bid-ask spreads, using the CRSP monthly database, are calculated as $(\text{abs}(\text{ask or hi price}) - \text{abs}(\text{bid or lo price}))/\text{abs}(\text{price})$ . The mean of the three months following the end of the fiscal year is then calculated.
YRBASPREAD	Average bid-ask spread in the year following the end of the fiscal year	Monthly bid-ask spreads, using the CRSP monthly database, are calculated as $(\text{abs}(\text{ask or hi price}) - \text{abs}(\text{bid or lo price}))/\text{abs}(\text{price})$ . The mean of the 12 months following the end of the fiscal year is then calculated.
PIN	Probability of informed trading in the quarter following the end of the fiscal year	Calculated using the method in Brown and Hillegeist (2007) and obtained from Professor Stephen Brown's website.
NI	Net income	Reported net income.
ABS_JONES_RESID	Absolute value of the residual from the modified Jones model, following Kothari et al. (2005)	The following regression is estimated for each industry year: $tca = \Delta \text{sales} + \text{net PPE} + \text{ROA}$ , where $tca = (\Delta \text{current assets} - \Delta \text{cash} - \Delta \text{current liabilities} + \Delta \text{debt in current liabilities} - \text{depreciation and amortization})$ , ROA is defined as below, and all variables are scaled by beginning-of-period total assets.
STD_DD_RESID	Five-year moving standard deviation of the Dechow-Dichev residual, following Francis et al. (2005)	The following regression is estimated for each industry year: $tca = cfo_{t-1} + cfo + cfo_{t+1}$ , where $tca$ is defined as above, and $cfo = (\text{interest before extraordinary items} - (\text{wacc} - \text{depreciation and amortization}))$ . All variables are scaled by average total assets. The five-year rolling standard deviations of the residuals are then calculated.

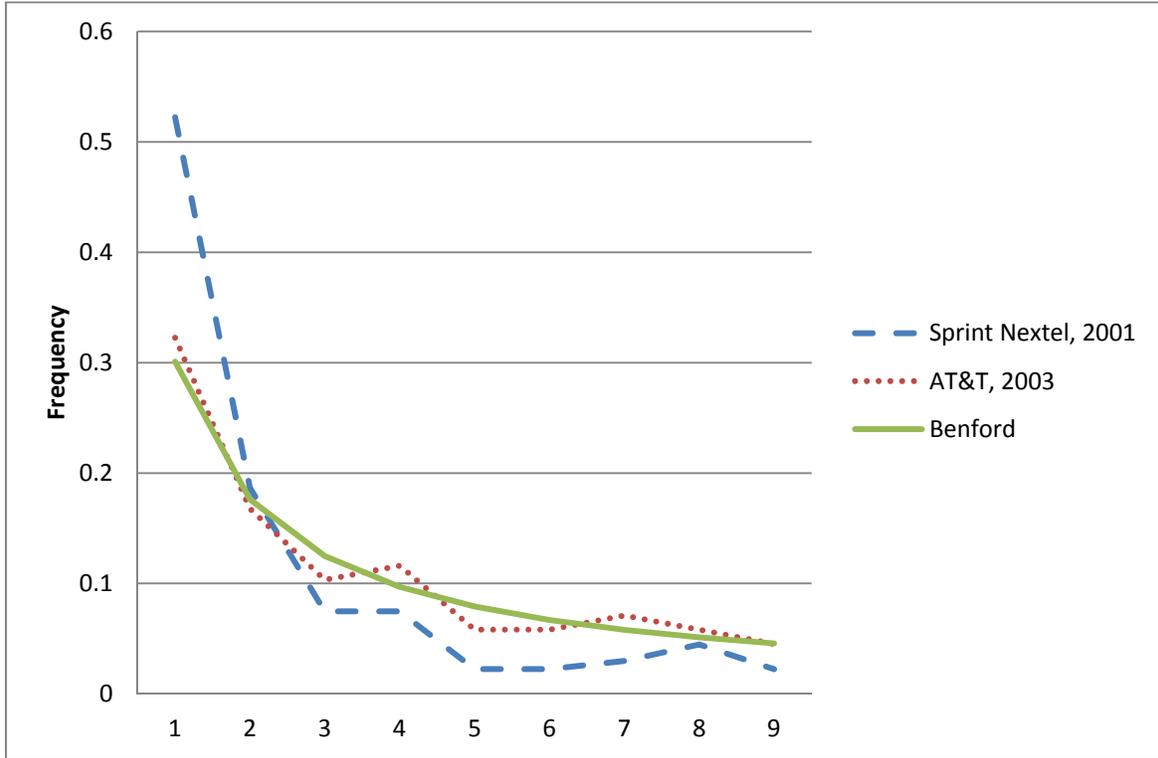
WCACC	Working capital accruals	Calculated as ( $\Delta$ current assets - $\Delta$ cash - $\Delta$ current liabilities + $\Delta$ debt in current liabilities) scaled by average total assets.
CHCSALE	Change in cash sales	Cash sales <sub>t</sub> - cash sales <sub>t-1</sub> /cash sales <sub>t-1</sub> , where cash sales = total revenue - $\Delta$ total receivables.
CHROA	Change in ROA	ROA <sub>t</sub> - ROA <sub>t-1</sub> , where ROA = income before extraordinary items <sub>t</sub> /total assets <sub>t-1</sub> .
SOFTAT	Soft assets	(Total assets - net PPE - cash)/total assets <sub>t-1</sub> .
ISSUANCE	Indicator variable that equals 1 if the company issued debt or equity in that year	When long-term debt issuance (Compustat DLTIS) > 1 or sale of common or preferred stock (SSTK) > 1, then issuance = 1.
BTM	Book-to-market	Total stock holders' equity (Compustat SEQ)/(closing price at the end of the fiscal year (Compustat PRCC_F) * common shares outstanding (Compustat CSHO).
AT	Total assets	Compustat AT.
CAUGHT		Indicator variable that equals 1 if a firm ever restated its financial statements, according to the Audit Analytics database.
POST		Indicator variable that equals 1 for all years after a firm restates, according to the Audit Analytics database.
INDUSTRY	Industry classification	Groups companies into 17 industry portfolios based on the Fama-French.
MKT_VAL	Market value	Stock price x shares outstanding.
SHR_TURN	Share turnover	Annual trading volume/MKT_VAL.
RET_VOL	Return volatility	Standard deviation of monthly stock returns in the last year.
FIN_LEV	Financial leverage	Total liabilities/total assets.
ACC	Accruals	(Operating income after depreciation – operating activities net cash flow)/total assets.
DIV	Indicates if the firm paid a dividend	Equals 1 if the firm paid a dividend in that year.
SIZE	Log of market value of equity	Log(common shares outstanding * price at the end of the fiscal year).
MTB	Market-to-book ratio	(SIZE + total liabilities)/total assets.
AGE	Age of the firm	Number of years the firm appears in the CRSP monthly stock return file.
SI	Special items	Total special items/total assets.
NI_VOL	Earnings volatility	Standard deviation of net income for the last five years.
RESTATED_NUMS	Indicator variable that equals 1 if reported numbers are restated	For all firms from 2001-2011 where RESTATED=1 and both restated and original financial numbers are available in Compustat (datafmt=STD for original and datafmt=SUMM_STD for restated), we separate the original from the restated financial numbers and create an indicator equaling 1 for restated numbers.

**Figure 1:** Aggregate Distribution and Benford's Distribution



The above figure shows graphically the similarity between Benford's distribution and the aggregate distribution of all financial statement variables available on Compustat for the period 2001-2011. Not shown are distributions by industry and year, which similarly conform to Benford's Law.

**Figure 2:** Conformity to Benford's Distribution, Firm Examples



The above figure shows graphically the conformity to Benford's distribution for two firm years, Sprint Nextel, 2001, which does not conform to Benford's Law (10-K KS=0.224, 10-K MAD=0.052) and restated its financial results for that year, and AT&T, 2003, which does conform to Benford's Law (10-K KS=0.028, 10-K MAD=0.013).

**Table 1**  
*Descriptive Statistics*

VARIABLE	N	MEAN	SD	P25	P50	P75
10-K MAD	41,863	0.029	0.009	0.023	0.028	0.035
RESTATED	41,863	0.161	0.368	0.000	0.000	0.000
FRAUD	41,863	0.006	0.079	0.000	0.000	0.000
QTRBASPREAD	36,262	0.178	0.114	0.102	0.147	0.218
YRBASPREAD	36,348	0.184	0.099	0.114	0.160	0.227
PIN	10,881	0.231	0.127	0.136	0.202	0.296
NI	37,230	180.73	728.11	-5.74	9.40	77.78
NI(t+1)	37,230	200.39	785.24	-5.34	10.63	88.10
NI(t+2)	32,040	232.34	862.10	-4.00	12.50	103.95
ABS_JONES_RESID	41,863	0.167	0.288	0.028	0.070	0.170
STD_DD_RESID	41,863	0.010	0.174	-0.050	0.005	0.066
WCACC	41,863	0.004	0.139	-0.036	0.003	0.045
CHCSALE	41,863	0.168	0.681	-0.046	0.075	0.224
CHROA	41,863	0.017	0.382	-0.044	0.001	0.041
SOFTAT	41,863	0.671	0.405	0.429	0.642	0.829
ISSUANCE	41,863	0.914	0.281	1.000	1.000	1.000
BTM	41,863	0.562	0.977	0.259	0.479	0.803
AT	41,863	3,419	8,717	78	373	1,887
MKT_VAL	36,953	2,519,550	6,969,421	87,857	377,426	1,499,309
SHR_TURN	36,953	2.994	6.189	0.445	1.031	2.584
ACC	37,224	-0.029	0.092	-0.064	-0.019	0.019
DIV	37,230	0.396	0.489	0.000	0.000	1.000
SIZE	37,230	6.143	2.127	4.641	6.119	7.559
MTB	37,114	1.888	1.287	1.104	1.470	2.179
AGE	37,230	20.031	11.837	12.000	17.000	27.000
SI	36,859	-0.024	0.079	-0.014	-0.001	0.000
RET_VOL	36,744	0.147	0.094	0.083	0.123	0.181
NI_VOL	37,230	143.90	402.35	6.66	21.23	79.01

See Appendix A for the calculation of 10-K MAD. RESTATED is an indicator that equals 1 if a firm restated its financial information in that year. FRAUD is an indicator that equals 1 if a firm was the subject of an SEC Accounting and Auditing Enforcement action in that year. QTRBASPREAD is the average bid-ask spread in the quarter following the end of the fiscal year. YRBASPREAD is the average bid-ask spread in the year following the end of the fiscal year. PIN is the probability of informed trading in the quarter following the end of the fiscal year. NI is reported net income. NI(t+1) and NI(t+2) are net incomes in the year after and two years after, respectively. ABS\_JONES\_RESID is the absolute value of the residual from the modified Jones model. STD\_DD\_RESID is the five-year moving standard deviation of the Dechow-Dichev residual. WCACC is working capital accruals. CHCSALE is the change in cash sales. CHROA is the change in ROA. SOFTAT is total assets less net PPE and cash, scaled by beginning of period total assets. BTM is the book-to-market ratio. AT is total assets. MKT\_VAL is the market value. SHR\_TURN is share turnover. ACC is total accruals. DIV is an indicator variable equal to 1 if the firm paid a dividend. SIZE is the log of the market value of equity. MTB is the market-to-book ratio. AGE is the

number of years a firm appears in the CRSP monthly stock returns file. SI is special items. RET\_VOL is the volatility of returns. NI\_VOL is the volatility of net income. See Appendix B for further detail.

**Table 2**  
*Correlations*

	10-K MAD	RESTATED	FRAUD	QTRBASPREAD	YRBASPREAD	PIN	NI	NI(t+1)	NI(t+2)	ABS_JONES_RESID	STD_DD_RESID
10-K MAD		-0.0305*	-0.0390*	0.0942*	0.1250*	0.1002*	-0.1033*	-0.1112*	-0.1084*	0.0579*	0.0028
RESTATED	-0.0095		0.1434*	0.0363*	0.0284*	-0.0379*	-0.0496*	-0.0520*	-0.0468*	-0.0147	0.0112
FRAUD	-0.0180*	0.1354*		-0.0001	-0.0147	-0.0638*	0.0115	0.0095	-0.0022	-0.0045	0.0131
QTRBASPREAD	0.0895*	0.0332*	-0.0015		0.8118*	0.0824*	-0.4387*	-0.4213*	*0.3444*	0.1737*	-0.0312*
YRBASPREAD	0.1173*	0.0321*	-0.0090	0.7583*		0.1171*	-0.4946*	-0.5270*	-0.4421*	0.2044*	-0.0477*
PIN	0.0854*	-0.0409*	-0.0575*	0.0498*	0.0899*		-0.2249*	-0.2081*	-0.1816*	0.0431*	-0.0605*
NI	-0.0966*	-0.0709*	-0.0115*	-0.1866*	-0.2117*	-0.0963*		0.6577*	0.5292*	-0.1606*	0.1799*
NI(t+1)	-0.0999*	-0.0695*	-0.0092	-0.1866*	-0.2326*	-0.0910*	0.8853*		0.6624*	-0.1295*	0.0698*
NI(t+2)	-0.0961*	-0.0695*	-0.0105	-0.1600*	-0.2098*	-0.0867*	0.8380*	0.8874*		-0.1146*	0.480*
ABS_JONES_RESID	0.0740*	0.0019	0.0007	0.0853*	0.0959*	0.0314*	-0.0484*	-0.0392*	-0.0377*		-0.0588*
STD_DD_RESID	0.0097*	-0.0044	0.0091	-0.0230*	-0.0363*	-0.0641*	0.0000	-0.0157*	-0.0159*	-0.0373*	

Pearson (Spearman) correlations are below (above) the diagonal. \* indicates significance at the 5% level. All variables are defined in Appendix B.

**Table 3**  
*Aggregate Conformity to Benford's Distribution: All Firm Years*

Panel A: All financial statement numbers

NUMBER OF FIRM YEARS	10-K MAD
41,863	0.0015

Panel B: All financial statement numbers by industry

Panel C: All financial statement numbers by fiscal year

INDUSTRY	NUMBER OF FIRM YEARS	10-K MAD
1	1,340	0.0016
2	709	0.0021
3	1,937	0.0013
4	819	0.0021
5	1,075	0.0015
6	1,016	0.0019
7	1,914	0.0017
8	1,098	0.0016
9	697	0.0015
10	345	0.0013
11	6,887	0.0014
12	692	0.0011
13	1,923	0.0023
14	1,256	0.0020
15	2,678	0.0027
17	17,477	0.0016

FISCAL YEAR	NUMBER OF FIRMS	10-K MAD
2001	4,327	0.0015
2002	4,205	0.0019
2003	4,119	0.0019
2004	4,007	0.0018
2005	3,944	0.0019
2006	3,791	0.0014
2007	3,715	0.0012
2008	3,649	0.0013
2009	3,542	0.0014
2010	3,453	0.0015
2011	3,111	0.0013

Table 1 computes the *aggregate* 10-K MAD statistic from all financial statement variables available on Compustat for the period 2001-2011. See Appendix A for the calculation of 10-K MAD. Panel A shows the distribution for the entire sample. Panel B calculates the distributions by Fama-French industry portfolios. Panel C calculates the distribution by fiscal years. In all instances, 10-K MAD is well below 0.006, which can be considered close conformity to the law (Nigrini, 2012).

**Table 4**  
*Conformity to Benford's Distribution: By Individual Firm Year*

Panel A: Total number of firm-years that follow Benford's distribution

FREQUENCY	PERCENT
35,456	85.75

Panel B: Total number of firm-years by industry that follow Benford's distribution

INDUSTRY	FREQUENCY	PERCENT
1	1,155	86.65
2	576	82.64
3	1,671	87.62
r	710	87.01
5	906	84.75
6	878	87.54
r	1,526	81.56
8	966	88.95
9	607	87.97
10	303	88.08
11	5,865	86.09
12	602	87.12
13	1,627	85.63
14	1,120	91.06
15	2,231	84.00
17	14,713	85.34

Panel C: Total number of firm-years by year that follow Benford's distribution

FIRM YEAR	FREQUENCY	PERCENT
2001	3,668	85.94
2002	3,594	86.60
2003	3,479	85.84
2004	3,378	85.32
2005	3,370	86.68
2006	3,200	85.40
2007	3,168	86.37
2008	3,121	86.53
2009	2,959	84.35
2010	2,887	84.54
2011	2,632	85.34

Table 2 computes the 10-K KS statistic for each firm-year from 2001-2011 and shows the percentage of individual firm-years out of a total of 41,863 firm-years that conform to Benford's Law, where conformity is assessed as having a KS statistic that is not significantly different from zero at the 5% level. In Panel A, 86% of all firm-years are not different from zero at the 5% level. Panel B (Panel C) shows similar conformity to Benford's Law across industries (years). See Appendix A for the calculation of the 10-K KS statistic.

**Table 5**  
*Benford's Distribution: Misstated Versus Restated Financial Statements*

$$10\text{-K MAD}_{i,t} = \alpha + \beta_1 \text{RESTATED\_NUMS}_{i,t} + \beta_2 \text{ABS\_JONES\_RESID}_{i,t} + \beta_3 \text{STD\_DD\_RESID}_{i,t} + \varepsilon_{i,t}$$

VARIABLES	10-K MAD	
	(1)	(2)
RESTATED_NUMS	-0.0004** (-2.02)	-0.0004** (-2.06)
ABS_JONES_RESID		0.0061*** (17.99)
STD_DD_RESID		0.0009* (1.77)
Observations	8,734	8,734
R2	0.001	0.036

Table 5 examines the relation between restated data and Benford's Law. The OLS regressions use financial statement data from firms that restated their financial statements for the period 2001-2011. We require that firms have both restated and original financial data available in Compustat. RESTATED\_NUMS is an indicator that equals 1 for restated numbers and 0 for misstated numbers used in the calculation of the 10-K MAD statistic. 10-K MAD is the mean absolute deviation between the empirical distribution of leading digits contained in a firm's financial statements and Benford's Law. See Appendix A for the calculation of the 10-K MAD statistic. See Appendix B for definitions of the control variables. t-statistics are reported in parentheses in the table. \*, \*\*, and \*\*\* indicate significance at the 0.10, 0.05, and 0.01 levels, respectively.

**Table 6**  
*Benford's Distribution in the Years Following a Restatement*

$$10\text{-K MAD}_{i,t} = \alpha + \beta_1 \text{CAUGHT}_{i,t} + \beta_2 \text{POST}_{i,t} + \beta_3 (\text{POST X CAUGHT}_{i,t}) + \beta_4 \text{ABS\_JONES\_RESID}_{i,t} + \beta_5 \text{STD\_DD\_RESID}_{i,t} + \beta_6 \text{WCACC}_{i,t} + \beta_7 \text{CHCSALE}_{i,t} + \beta_8 \text{CHROA}_{i,t} + \beta_9 \text{SOFTAT}_{i,t} + \beta_{10} \text{ISSUANCE}_{i,t} + \beta_{11} \text{BTM}_{i,t} + \beta_{12} \text{AT}_{i,t} + \varepsilon_{i,t}$$

VARIABLES	10-K MAD	
	(1)	(2)
CAUGHT	0.0002 (0.85)	-0.0001 (-0.44)
POST	0.0002 (1.44)	0.0004** (2.34)
POST X CAUGHT	-0.0005** (-2.15)	-0.0005** (-2.34)
ABS_JONES_RESID		0.0019*** (11.45)
STD_DD_RESID		0.0011*** (3.20)
WCACC		-0.0002 (-0.48)
CHCSALE		0.0005*** (6.93)
CHROA		0.0002* (1.87)
SOFTAT		-0.0013*** (-9.69)
ISSUANCE		-0.0019*** (-10.06)
BTM		-0.0001*** (-2.77)
AT		-0.0000*** (-22.36)
Observations	41,863	41,863
R2	0.000	0.036

Table 6 examines if the conformity of firms' financial statements to Benford's Law improves in the years following a restatement. The OLS regressions use all financial statement data for the period 2001-2011. CAUGHT is an indicator variable equal to 1 if a firm restated its financial statements. POST is an indicator variable equal to 1 for all years after a firm has restated. For firms where CAUGHT=0, POST is determined using a randomly generated year. 10-K MAD is the mean absolute deviation between the empirical distribution of leading digits contained in a firm's financial statements and Benford's Law. See Appendix A for the calculation of the 10-K MAD statistic. See

Appendix B for definitions of the control variables. t-statistics are reported in parentheses in the table. \*, \*\*, and \*\*\* indicate significance at the 0.10, 0.05, and 0.01 levels, respectively. Standard errors are clustered by firm.

**Table 7**  
*Benford's Distribution and Information Asymmetry*

$$DV_{i,t} = \alpha + \beta_1 10\text{-K MAD}_{i,t} + \beta_2 \text{ABS\_JONES\_RESID}_{i,t} + \beta_3 \text{STD\_DD\_RESID}_{i,t} + \beta_4 \text{MKT\_VAL}_{i,t} + \beta_5 \text{SHR\_TURN}_{i,t} + \beta_6 \text{AT}_{i,t} + \beta_7 \text{FIN\_LEV}_{i,t} + \beta_8 \text{RET\_VOL}_{i,t} + \varepsilon_{i,t}$$

VARIABLES	QTRBASPREAD		YRBASPREAD		PIN	
	(1)	(2)	(3)	(4)	(5)	(6)
10-K MAD	1.179*** (14.72)	0.252*** (4.06)	1.350*** (18.07)	0.503*** (9.32)	1.263*** (8.25)	0.973*** (6.62)
ABS_JONES_RESID		0.003 (1.43)		0.005*** (3.16)		0.007 (1.53)
STD_DD_RESID		-0.005 (-1.21)		-0.013*** (-4.12)		-0.041*** (-5.88)
MKT_VAL		-0.000*** (-8.69)		-0.000*** (-7.01)		-0.000*** (-6.62)
SHR_TURN		0.004*** (25.47)		0.004*** (29.75)		0.0002 (1.38)
AT		-0.000*** (-8.76)		-0.000*** (-14.52)		-0.000*** (-3.94)
FIN_LEV		0.013*** (6.26)		0.016*** (8.60)		0.026*** (4.83)
RET_VOL		0.500*** (60.35)		0.435*** (62.22)		-0.005 (-0.36)
Observations	36,262	35,848	36,348	35,872	10,881	10,720
R2	0.008	0.316	0.014	0.337	0.007	0.081

Table 7 examines the relation between Benford's Law and several proxies for information asymmetry. The OLS regressions use all financial statement data for the period 2001-2011. QTRBASPREAD is the average bid-ask spread in the quarter following the end of the fiscal year. YRBASPREAD is the average bid-ask spread in the year following the end of the fiscal year. PIN is the probability of informed trading in the quarter following the end of the fiscal year. 10-K MAD is the mean absolute deviation between the empirical distribution of leading digits contained in a firm's financial statements and Benford's Law. See Appendix A for the calculation of the 10-K MAD statistic. See Appendix B for definitions of the control variables. t-statistics are reported in parentheses in the table. \*, \*\*, and \*\*\* indicate significance at the 0.10, 0.05, and 0.01 levels, respectively. Standard errors are clustered by firm.

**Table 8**  
*Benford's Distribution and Earnings Persistence*

$$DV_i = \alpha + \beta_1 NI_{i,t} + \beta_2 10\text{-K MAD}_{i,t} + \beta_3 NI_{i,t} * MAD_{i,t} + \beta_4 ACC_{i,t} + \beta_5 DIV_{i,t} + \beta_6 SIZE_{i,t} + \beta_7 MTB_{i,t} + \beta_8 SI_{i,t} + \beta_9 AGE_{i,t} + \beta_{10} RET\_VOL_{i,t} + \beta_{11} NI\_VOL_{i,t} + \varepsilon_{i,t}$$

VARIABLES	NI(t+1)	NI(t+2)
	(1)	(2)
NI	3.734*** (3.23)	4.237** (2.30)
10-K MAD	256.927 (0.60)	-2,350.133*** (-2.65)
NI*10-K MAD	-66.163** (-2.40)	-80.091* (-1.86)
ACC	-19.501 (-0.90)	72.405 (1.16)
DIV	15.496 (1.13)	27.030 (1.36)
SIZE	16.165* (1.89)	17.424 (1.55)
MTB	-0.943 (-0.30)	4.331 (0.68)
SI	-153.831** (-2.19)	-58.219 (-0.54)
AGE	0.400 (0.79)	-0.024 (-0.03)
RET_VOL	-2.661 (-0.08)	-42.445 (-1.01)
NI_VOL	0.125 (0.73)	0.669*** (2.78)
Observations	3,328	2,824
R-squared	0.696	0.618

Table 8 examines the relation between Benford's Law and earnings persistence in years t+1 and t+2. We break firms into deciles based on 10-K MAD and use the top decile of firms from 2001-2011, clustering standard errors by firm (gvkey). NI is reported net income. NI(t+1) and NI(t+2) are net incomes in the year after and two years after, respectively. 10-K MAD is the mean absolute deviation between the empirical distribution of leading digits contained in a firm's financial statements and Benford's Law. See Appendix A for the calculation of the 10-K MAD statistic. Control variables are based on those used in Li (2008). See Appendix B for definitions of the control variables. t-statistics are reported in parentheses in the table. \*, \*\*, and \*\*\* indicate significance at the 0.10, 0.05, and 0.01 levels, respectively.

**Table 9**  
*Prediction of Financial Statements Irregularities Using 10-K MAD*

$$DV_{i,t} = \alpha + \beta_1 10\text{-K MAD}_{i,t} + \beta_2 \text{ABS\_JONES\_RESID}_{i,t} + \beta_3 \text{STD\_DD\_RESID}_{i,t} + \beta_4 \text{WCACC}_{i,t} + \beta_5 \text{CHCSALE}_{i,t} + \beta_6 \text{CHROA}_{i,t} + \beta_7 \text{SOFTAT}_{i,t} + \beta_8 \text{ISSUANCE}_{i,t} + \beta_9 \text{BTM}_{i,t} + \beta_{10} \text{AT}_{i,t} + \varepsilon_{i,t}$$

VARIABLES	RESTATED		FRAUD	
	(1)	(2)	(3)	(4)
10-K MAD	-2.979*	-5.559***	-27.933***	-27.147***
	(-1.73)	(-3.19)	(-3.34)	(-3.22)
ABS_JONES_RESID		-0.091*		-0.111
		(-1.73)		(-0.49)
STD_DD_RESID		-0.071		0.776*
		(-0.71)		(1.94)
WCACC		-0.369***		-1.137***
		(-3.03)		(-2.89)
CHCSALE		0.048**		-0.111*
		(2.50)		(-1.86)
CHROA		-0.007		0.001
		(-0.20)		(0.02)
SOFTAT		0.249***		0.749***
		(5.94)		(7.20)
ISSUANCE		0.397***		1.146**
		(6.09)		(2.18)
BTM		-0.021		-0.041
		(-1.12)		(-0.89)
AT		-0.000***		-0.000
		(-8.01)		(-0.95)
Observations	41,863	41,863	41,863	41,863
Pseudo R-squared	0.0001	0.011	0.005	0.023

Table 9 examines the relation between Benford's Law, SEC AAER's, and firm restatements. The OLS regressions use all financial statement data for the period 2001-2011. RESTATED is an indicator that equals 1 if a firm restated its financial information in that year. FRAUD is an indicator that equals 1 if a firm was the subject of an SEC Accounting and Auditing Enforcement action in that year. 10-K MAD is the mean absolute deviation between the empirical distribution of leading digits contained in a firm's financial statements and Benford's Law. See Appendix A for the calculation of the 10-K MAD statistic. See Appendix B for definitions of the control variables. t-statistics are reported in parentheses in the table. \*, \*\*, and \*\*\* indicate significance at the 0.10, 0.05, and 0.01 levels, respectively. Standard errors are clustered by firm.