Voluntary Disclosure by Levered Firms

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Abstract

This paper contains a model of a public company’s manager’s voluntary disclosure decisions when the company has liabilities some of which are publicly traded and where the manager is compensated based on a weighted average of the market prices of the firm’s equity and debt.

We show: 1. in general, the relationship between the firm’s traded debt and its manager’s voluntary disclosures is not monotonic: more debt leads to less voluntary disclosure if the initial level of debt is low, but more debt leads to more voluntary disclosure if the initial debt level is high; 2. increasing the weight placed on the price of the firm’s equity in the manager’s contract always induces the manager to disclose less voluntary information; 3. the managerial contract that puts equal weight on the debt and equity prices maximizes the probability the manager will disclose the private information she receives, and this contract also has the “Modigliani-Miller property” that the manager’s disclosure policy is independent of the firm’s level of indebtedness.¹

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1 Introduction

The first models of voluntary disclosure in accounting (e.g., Dye [1985], Jung and Kwon [1988], Verrecchia [1983]) studied disclosures by what were regarded as all-equity firms: either there was no debt in the financial structures of the firms being studied or, if there was such debt, it was ignored. The goal of the present paper is to study a firm’s manager’s voluntary disclosure decisions when her firm has publicly traded equity and debt, and the manager takes into account both when making her disclosure decisions. The motivation for the study is threefold: first, many firms do have publicly traded debt. Second, since managers are stewards of the whole firm they manage, managers of firms with traded debt may feel some obligation to take actions that affect the market value of that debt, in addition to taking actions that affect the market value of their firms’ equity. Those actions may include the disclosure choices the managers make. Third, equityholders’ perceptions of the value of their residual claims in a firm are directly affected by both the debt the firm has and its manager’s disclosure choices. So, even if the managers of a firm make their disclosure decisions so as to maximize only the market price of their firm’s equity, we would expect there to be some sort of interaction between the managers’ disclosure policies and the level of their firms’ debt.

Motivated by the preceding, we examine how a manager’s voluntary disclosure decisions depend on the market prices of her firm’s equity and debt. We suppose the firm is financed by common equity shares and zero coupon bonds, with the bonds having priority over the equity claims in the event the firm turns out to be incapable of repaying the face value of the debt in full. We consider compensation contracts that reward the manager based on a weighted average $\theta P_E + (1 - \theta) P_D$ of the market prices of the firm’s traded equity $P_E$ and debt $P_D$, where these market prices are established on a securities market that operates at some interim point between the time the firm originally issued the securities and the time its bonds mature. (We call these "D&E (debt and equity) contracts" in the following.) Throughout, we impose the natural restriction that at least as much weight is placed on the price of equity as on the price of debt in the manager’s contract, i.e., $\theta \geq .5$.

The model yields a variety of new and empirically testable results. For example, we show that there is a fundamental nonmonotonicity in the relationship between the extent of a firm’s indebtedness and its manager’s optimal disclosure policy. Specifically, as long as $.5 < \theta < 1$, i.e., as long as the manager’s D&E contract places strictly more weight on equity than debt but places positive weight on debt, there is a critical threshold level of the firm’s indebtedness (that varies with $\theta$) such that, if the firm’s debt remains below the threshold, then increases in the firm’s..
debt induce the manager to disclose less of her private information. But, if the firm’s level of debt starts out above this critical threshold, then increases in the firm’s debt level induce the manager to disclose more of her private information. This nonmonotonicity property may be important for empirical researchers to recognize when trying to document a statistical relationship between debt and disclosure. While there are obvious statistical advantages to selecting firms that exhibit wide variations in their debt levels when trying to establish an empirical relationship between debt and disclosure, this new finding demonstrates that there may also be a cost to selecting too wide a range of firms’ indebtedness for empirical examination. If a researcher’s sample includes both firms whose debt levels are relatively low - and therefore fall into the region where there is a monotone decreasing relationship between debt and disclosure - and firms whose debt levels are relatively high - and therefore fall into the region where there is a monotone increasing relationship between debt and disclosure - then the empirical findings from a linear regression analysis relating debt to disclosure are likely to be spurious.

As another example of our results, we show that the "equally weighted" D&E contract, i.e., \( \theta = .5 \), maximizes the probability the manager discloses the private information she receives among all D&E contracts we consider, i.e., among all contracts with \( \theta \geq .5 \). We also show that this equally weighted D&E contract yields a "Modigliani Miller theorem" for disclosure: this contract has the property that the manager’s preferred disclosure policy is independent of the level of the firm’s debt. Further, we show that only the "equally weighted" D&E contract has this capital irrelevance feature: every other D&E contract induces that manager to change her preferred disclosure policy as the level of the firm’s indebtedness changes.

As a third example of our results, we show that whenever \( \theta \) is increased, i.e., whenever more weight is placed on the price of firm’s equity (and corresponding less weight is placed on the price of the firm’s debt) in the manager’s D&E contract, this always strictly reduces the propensity of the firm’s manager to disclose the private information she receives. This implies, perhaps surprisingly, that "all equity" management contracts - which reward the manager based only on the market price of her firm’s equity (and thus ignore the market price of her firm’s debt) - lead the manager to minimize the probability the manager discloses her private information among all D&E contracts with \( \theta \geq .5 \).

As far as we are aware, all these results are new to the disclosure literature.

These results are established in a model where the examined firm’s net cash flows are normally distributed, there is risk-neutral pricing of all securities, and information arrives to the manager of the firm probabilistically, as in the Dye [1985] and Jung and Kwon [1988] models. We focus on
how a firm’s manager’s optimal disclosure policy is determined given the firm’s capital structure; we make no attempt to derive the firm’s optimal capital structure. We see three benefits to this approach. First, for many firms, their capital structures are legacy choices that were made long before the firm’s current managers were hired. Such situations fit perfectly with our model. Second, we want managers’ disclosure decisions to be the focal point of our study. Were we to develop a model of voluntary disclosure in which the firm’s capital structure is also endogenous, then in order to generate a realistic portrayal of capital structure, we would be obliged to take into account several of the many known forces that affect and/or are affected by a firm’s capital structure. E.g., the ability of capital structure to: affect agency problems at a firm, the amount of risk the firm’s shareholders are subject to; the tax payments the firm has to make, etc. Incorporating these forces into the model runs the risk of having disclosure-related issues get lost in the sea of complexity required to have a realistic capital structure emerge from the model. Third and related, by taking debt levels as exogenous, when we generate comparative statics involving debt levels and disclosure, we can learn exactly how the debt level change itself affects the manager’s disclosure policy. Were we to develop a model where both a firm’s capital structure and the manager’s disclosure policy arise endogenously, a change in the firm’s indebtedness can arise only as a result of a change in some exogenous parameter of the model, in which case we could not tell to what extent a change in debt levels affected a change in disclosure policies independently of the parameter change.

Related theory literature: As we noted at the outset, all of the early papers on voluntary disclosure and, in fact, most of the successor papers as well, concentrate on all-equity firms. Three disclosure-related papers not focused just on effects of disclosure by all equity firms are those by Bertomeu, Beyer, and Dye [2011] (“BBD”), Fischer and Verrecchia [CAR 1997] (“FV”), and Arya and Glover [1998] (“AG”). BBD study a model that is both broader and narrower than our model. BBD’s model is broader than the present model insofar as it does incorporate both a firm’s capital structure and its design of financial securities as endogenous variables. BBD is narrower than our model in so far as it is a three state model, whereas in our model, the set of states is infinite (since as we noted above, the distribution of the firm’s net cash flows in our model is normally distributed). FV study the structure of the price reactions of equity and debt to exogenous disclosures by the firm. AG study a model of debt and disclosure where the emphasis is on how disclosure affects the value of the firm’s debt by influencing the firm’s decision to replace or retain its managers.

Related empirical literature. Empirical evidence on the relation between voluntary disclosure and leverage is mixed: Eng and Mak (2003) finds that for a sample of public firms in Singapore, lower leverage is related to greater disclosure (Eng and Mak 2003). Meek et al. (1995) find a similar
correlation among a sample of US and European firms. In contrast, Hossain et al. (1995) find a positive relation between leverage and voluntary disclosure in annual reports in a sample of firms in New Zealand. Chow and Wong-Boren (1987) finds that among a sample of 52 Mexican publicly listed firms, there is no relation between voluntary disclosure and financial leverage. Similarly, Depoers (2000) finds that among a sample of 102 French firms, leverage and voluntary disclosure in annual reports are not significantly correlated. This mixed empirical evidence is consistent with our finding above that, if one includes firms with a wide range of debt levels, no monotone relationship between debt and disclosure will be found because there is mixing of firms for which there is a monotone increasing relation between debt and disclosure and firms for which there is a monotone decreasing relation between debt and disclosure.

The paper proceeds as follows. The next section lays out the model and contains the main analysis and results. Conclusions follow. The appendix contains detailed proofs of all the main results in the paper.

2 The Model

We study a two period model, with periods labeled 1 and 2, whose time line runs as follows. At some point in time before period 1, a firm obtained capital by issuing bonds and selling equity shares. The bonds are zero coupon bonds with face value $F$ that mature at the end of period 2. The equity securities are ordinary equity.\(^2\) At the start of period 1, the firm hires a manager. To simplify the model, we suppose that the manager’s duties are all disclosure-related.\(^3\) As in Dye [1985] and Jung and Kwon [1988], we posit that the manager privately receives information during period 1 with probability $p \in (0, 1)$. If she receives information, she must decide whether to disclose it. At the end of the first period - after the manager has made her disclosure decision - a securities market opens on which both the firm’s equity and its bonds are traded. The market prices of the firm’s equity and bonds on this securities market are generically denoted by $P_E$ and $P_D$ respectively. All that happens in period 2 is that at the end of that period, the firm is liquidated and all securities are settled.

We represent the liquidation value of the firm at the end of period 2 net of any nontraded liabilities it has but gross of any payments to its bondholders by the realization $x$ of the random variable $\tilde{x}$. Given realization $x$, we take the payoff to all equityholders to be $\max\{x - F, 0\}$, and we

\(^2\)For present purposes, it is not necessary to specify the fraction of the firm’s equity that was sold.

\(^3\)We could expand the model so that the manager’s duties include influencing the distribution of the firm’s cash flows.
take the payoff to all debtholders to be $\min\{x, F\}$. Note that the sum of the payoffs to debtholders and equityholders satisfies:
\[
\max\{x - F, 0\} + \min\{x, F\} = x,
\]
that is, the firm’s liquidation value is fully distributed to bondholders and equityholders. We take the ex ante (as of period 1) distribution of $\tilde{x}$ to be normal with prior mean $m$ and prior variance $\sigma^2$, henceforth written as $\tilde{x} \sim N(m, \sigma^2)$.

The sequence of events is summarized in Figure 1.

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Figure 1: Timeline

It is worth emphasizing an implication of the preceding discussion of the setup: the firm we study may have both traded debt and nontraded debt. The nontraded debt is incorporated in $x$, and the traded debt is the bond issue with face value $F$. For expositional convenience, though (because we never single out the nontraded debt from the other components of $x$, nor do we single out those who supply the nontraded debt), in the following we sometimes say "debtholders" when we mean more narrowly "bondholders." Also for expositional convenience, from now on we refer to $x$ as the firm’s "realized cash flows" rather than by the more cumbersome, but more accurate phrase: "the liquidation value of the firm at the end of period 2 net of any nontraded liabilities it has but gross of any payments to its bondholders."

We suppose that the contract the owner of the firm gives the manager at the time of hire in period 1, what we called the "D&E contract" in the Introduction, takes the form:
\[
w + s \times (\theta \times P_E + (1 - \theta) \times P_D),
\]
\footnote{With this depiction, it is possible for the firm’s liquidation value, as we have defined it, to be negative. Were that to happen, as we have represented the payoffs to security holders, equityholders are protected by limited liability, but debtholders are not. There are circumstances in practice under which lenders may not have limited liability, consistent with this depiction. See, e.g., King [1988], in her discussion of environmental liabilities. In any event, we remark (as is often done in accounting models where one represents an economic random variable that can only assume nonnegative values by a normal random variable) that if the mean $m$ of the distribution of $\tilde{x}$ is sufficiently high, then it becomes increasingly unlikely that $\tilde{x}$ will be negative, and therefore increasingly unlikely that debtholders could assume obligations related to their lending.}
where $w$ is the manager’s base salary, $s \times \theta$ is a bonus coefficient attached to the market price of the firm’s equity and $s \times (1 - \theta)$ is a bonus coefficient attached to the market price of the firm’s debt, where $w, s > 0$.\footnote{Our emphasis throughout on the manager’s contract will be on the contract’s relative weighting $\theta$ and $1 - \theta$, and not on the parameters $w$ and $s$, as it will be clear that these parameters will have no effect on the manager’s disclosure choices.} Essentially, the bonus is a convex combination of the prices of the firm’s equity and traded debt. Throughout the analysis, we maintain as an exogenous assumption that $\theta \in [0.5, 1]$, i.e., that the weight attached to the price of the firm’s equity in the manager’s contract is always at least as high as the weight attached to the price of the firm’s traded debt. Throughout, we also assume this contract, or at least the weight $\theta$ attached to the price of equity, is publicly observable, and in particular, is observable to investors participating in the end-of-period 1 securities market.

This compensation scheme is natural if the cash flows generated by the manager’s effort do not realize their value until far in the future; in that case, it is natural that managers get compensated based on variables that realize their values while the manager is working for the firm, such as the market prices of the firm’s debt and equity.

All traders are all taken to be homogeneously informed, with their (common) exclusive source of information about the firm being the manager’s disclosures (or lack of disclosures).\footnote{We presume that the manager of the firm, discussed next, does not participate in either the purchase or sale of either of the securities on this market, as we are not interested in issues involving inside trading in this paper.} These traders are further assumed to be risk-neutral and (only for notational convenience) do not discount the firm’s future cash flows.

To keep the analysis tractable, we shall suppose that the information the manager receives, if she receives any information, is perfect advanced knowledge of the actual realization $x$ of $\tilde{x}$.

As is usual in the voluntary disclosure literature, we shall assume that the manager’s disclosure decisions consists of disclosing $x$ when she receives $x$ or disclosing nothing; that is, truth-telling is required; partial disclosures are not considered; and if the manager gets no information, she necessarily makes no disclosure (in particular, the manager cannot credibly claim not to have received any information). We further assume, as is also conventional in this literature, that the probability the manager receives information is independent of the information received.

We sometimes write

$$P_E^d(x)(= \max\{x - F, 0\}) \text{ and } P_D^d(x)(= \min\{x, F\})$$

\hspace{1cm} (3)

for the market values of the firm’s equity and debt, respectively, when the manager receives and discloses $x$.\footnote{Note that by writing $P_E(x) = \max\{x - F, 0\}$ and $P_D(x) = \min\{x, F\}$ for the prices/payoffs to equity and debt,} When the manager makes no disclosure, we write the firm’s equity and debt prices as

$P_E^{nd}$ and $P_D^{nd}$ (or some variant on this notation, as discussed further below) respectively.
In view of (2), the manager’s compensation if she makes no disclosure is the constant
\[ w + s \times [\theta P_{nd}^E + (1 - \theta) P_{nd}^D]. \] (4)

If the manager discloses \( x \), the manager’s compensation is given by:
\[ w + s \times [\theta P_{d}^E(x) + (1 - \theta) P_{d}^D(x)]. \] (5)

Obviously, if \( x < F \), then \( \theta P_{d}^E(x) + (1 - \theta) P_{d}^D(x) = (1 - \theta) x \), and if \( x \geq F \), then \( \theta P_{d}^E(x) + (1 - \theta) P_{d}^D(x) = \theta \times (x - F) + (1 - \theta) \times F \). It follows that the manager’s D&E contract is strictly increasing in the information \( x \) she discloses. It follows from this last observation and from (4) and (5) that the manager’s preferred disclosure policy is always right-tailed: if she chooses to disclose \( x \) when received, she will also choose to disclose any \( x' > x \) she receives too. Hence, the manager’s disclosure policy will be characterized by some cutoff \( x^c \), with disclosure taking place for, and only for, those realizations \( x \geq x^c \) the manager receives. Using this observation, we modify the notation for the nondisclosure prices of equity and debt to \( P_{nd}^E(x^c) \) and \( P_{nd}^D(x^c) \) to take into account how investors’ perceptions of the manager’s disclosure policy, as reflected in the cutoff \( x^c \), affects the nondisclosure prices of equity and debt.

We start off the formal computations with expressions for the unconditional expected values of the firm’s equity and debt calculated as of the start of period 1. Using well-known results about truncated normal distributions (the start of the Appendix reviews some of these results), we observe that the unconditional expected value of the firm’s equity (in the expressions here and elsewhere, \( h(x) \) and \( H(x) \) respectively refer to the density and cumulative distribution function (cdf) associated with the initial priors on \( \tilde{x} \) ) equals:
\[ E[\max\{\tilde{x} - F, 0\}] = (m - F)(1 - H(F)) + \sigma^2 h(F), \] (6)

and the unconditional expected value of the firm’s debt equals:
\[ E[\min\{\tilde{x}, F\}] = (m - F)H(F) - \sigma^2 h(F) + F. \] (7)

Note that the sum of the unconditional expected values of the payoffs to equity and debt equals \( m \), the unconditional expected value of the firm’s cash flows. This follows from (1). Since all of
the firm’s realized cash flows get split between equityholders and debtholders, the same also is true in expectation: the unconditional expected value of the firm’s cash flows get divided up between equityholders and debtholders.

To proceed, we break up the analysis into two cases, one involving those face values of the firm’s debt $F$ that fall below the disclosure cutoff $x_c$ and the other involving face values $F$ exceeding $x_c$. We refer to the former as the ”low debt case” and the latter as the ”high debt case.”

Prices of the two securities in the low debt case are described next.

**Lemma 1** In the low debt case (with $x_c \geq F$), if investors believe the manager has adopted a disclosure policy defined by the cutoff $x_c$, and the manager does not make a disclosure, investors will calculate the expected value of the firm’s equity and debt respectively to be as follows:

$$P^{nd}_E(x_c) = E[\max\{\tilde{x} - F, 0\}|nd] = m - F + \frac{-(m - F)H(F) + \sigma^2h(F) - \rho\sigma^2h(x_c)}{1 - p + pH(x_c)}; \quad (8)$$

$$P^{nd}_D(x_c) = E[\min\{\tilde{x}, F\}|nd] = \frac{(m - F)H(F) - \sigma^2h(F)}{1 - p + pH(x_c)} + F. \quad (9)$$

Note that when $P^{nd}_E(x_c|m)$ and $P^{nd}_D(x_c|m)$ are added together, one gets $m - \frac{\rho\sigma^2h(x_c)}{1 - p + pH(x_c)}$. The latter expression can be shown to be investors’ perceptions of the total expected value of the firm’s cash flows conditional on the manager’s nondisclosure and use of the cutoff $x_c$. This is yet another implication of (1).

In view of the form of the manager’s compensation contract in (2), it is clear that, in the low debt case, the manager will prefer to disclose any information $x$ she receives if and only if $	heta(x - F) + (1 - \theta)F \geq \theta P^{nd}_E(x_c) + (1 - \theta)P^{nd}_D(x_c)$. If $x_c$ is an equilibrium cutoff, then evaluated at $x = x_c$, this last inequality must be an equality, i.e., if the realized value of $\tilde{x}$ turns out to equal the cutoff, then the manager must be indifferent between disclosing it and making no disclosure. We state this formally as:

**Definition 2** An equilibrium disclosure cutoff in the low debt case is a value $x_c$ that satisfies:

(a) $F \leq x_c \quad (10)$

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These classifications are imprecise, because the manager’s equilibrium disclosure cutoff is endogenous and in particular, as we show below, may vary with $F$. We shall eventually precisely determine how small (big) $F$ can be to qualify as belonging to the low debt (resp., high debt) case based on exogenous variables (see Theorem 10 below), but for now, we proceed with the imprecise classifications just given.
and

\[
\theta P_E^d(x_c) + (1 - \theta) P_D^d(x_c) = \theta P_E^{nd}(x_c) + (1 - \theta) P_D^{nd}(x_c).
\]

(11)

Condition (a) must be satisfied in order for the cutoff to be consistent with the low debt circumstances under which the "no disclosure" equity and debt prices in Lemma 1 above were derived. Condition (b) is clear from the discussion above the definition’s statement, but for purposes of developing intuition for some of the results that follow, it is worth rewriting the equilibrium condition as

\[
\theta \Delta E(x_c) + (1 - \theta) \Delta D(x_c) = 0,
\]

(12)

where \( \Delta E(x_c) \equiv P_E^d(x_c) - P_E^{nd}(x_c) \) and \( \Delta D(x_c) \equiv P_D^d(x_c) - P_D^{nd}(x_c) \) refers to the (possibly negative) increment in a security’s value from the manager disclosing over not disclosing. Equation (12) asserts that evaluated at the equilibrium cutoff, the sum of the marginal benefits of disclosure over nondisclosure for the two securities multiplied by the weight attached to the price of the securities in the manager’s contract must be zero. Of course, (12) is equivalent to:

\[
\theta \Delta E(x_c) = -(1 - \theta) \Delta D(x_c),
\]

(13)

so if the marginal benefit of disclosure over nondisclosure for either of the securities is positive in equilibrium, then the marginal benefit of disclosure over nondisclosure for the other security must be negative.

It is convenient at this point to introduce the following additional definitions and notation. The "standardized" value of a cutoff \( x_c \) and the "standardized" face value of the firm’s debt \( F \) are respectively defined as:

\[
z_c \equiv \frac{x_c - m}{\sigma} \quad \text{and} \quad f \equiv \frac{F - m}{\sigma}.
\]

(14)

We use these new terms interchangeably with the original terms \( x_c \) and \( F \). Specifically, we consider the cutoff \( z_c \) and the cutoff \( x_c \) to refer to exactly the same disclosure policy. In this notation, the low debt case is described by the inequality \( f \leq z_c \). The notation \( \Phi(\cdot) \) and \( \phi(\cdot) \) refer to the cdf and density of the standard normal random variable. We also define the function:

\[
\Gamma(z) \equiv z \times \Phi(z) + \phi(z).
\]

(15)

This function \( \Gamma(\cdot) \) has many properties (e.g, \( \Gamma(z) > 0 \), \( \Gamma'(z) > 0 \), and \( \Gamma(z) > z \) for all \( z \))\(^{10} \) and many uses in the following. For example, using \( \Gamma(\cdot) \) in conjunction with (9) above, we can show formally

\(^9\)Of course, if we have identified a standardized equilibrium cutoff \( z_c \), then we can construct the corresponding "regular" equilibrium cutoff via the inverse transformation \( x_c = \sigma \times z_c + m \).

\(^{10}\)See (28) at the start of the Appendix for a description some of \( \Gamma(\cdot) \)'s properties.
the intuitively obvious assertion that in the low debt case the price of debt given nondisclosure is always strictly below the face value of debt for any cutoff $x^c$, equilibrium or otherwise:

$$F - P^\text{nd}_D(x^c) = -\left(\frac{m-F}{1-p+pH(x^c)}\right)H(F) - \sigma h(F) = \sigma \times \left(\frac{m-F}{1-p+pH(x^c)}\right)$$

$$= \sigma \times \left(\frac{f\Phi(f) + \phi(f)}{1-p+pH(x^c)}\right) = \sigma \times \frac{\Gamma(f)}{1-p+pH(x^c)} > 0.$$ (16)

A second example of $\Gamma(\cdot)$'s use is in the following lemma, where we simplify the equation a firm's equilibrium cutoff in the low debt case must satisfy.

**Lemma 3** When $f$ is small enough so that the low debt case applies, then for any $\theta \in [0.5, 1]$:

(a) equilibrium equation (11) is algebraically equivalent to the following equation:

$$z^c(1-p) + p\Gamma(z^c) = \frac{2\theta - 1}{\theta} \times \Gamma(f).$$ (17)

(b) there exists a unique standardized cutoff solving equation (17), which we denote by $z^c(f, \theta)$.

While this lemma does not prove the existence of an equilibrium cutoff (since it does not speak to the issue of whether $z^c(f, \theta)$ satisfies the low debt criterion in (10): $f \leq z^c(f, \theta)$), it produces testable comparative statics. Specifically, total differentiation of equation (17) immediately shows that:

**Proposition 4** When $f$ is small enough so that the low debt case applies, then:

(a) for any $\theta > 0.5$, $z^c(f, \theta)$ is strictly increasing in $f$, i.e., the probability the manager makes a voluntary disclosure is strictly decreasing in the amount of the firm’s debt financing;

(b) $z^c(f, \theta)$ is strictly increasing in $\theta$, i.e., the probability the manager makes a voluntary disclosure is strictly decreasing in the weight attached to the price of equity in the manager’s contract;

(c) $z^c(f, \theta)$ is strictly decreasing in $p$.

Part (a) shows that increasing the face value of the firm’s debt in the low debt case strictly reduces the propensity of the manager to disclose the information she receives. The left part of Figure 2 illustrates how the standardized disclosure threshold in the low debt case varies with the debt level. Part (b) shows that increasing the emphasis on the price of equity in the manager’s contract (i.e., increasing $\theta$) strictly reduces the propensity of the manager to disclose her information. This is illustrated in the right part of Figure 3. Part (c) extends the well-known result from previous

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11 The perhaps surprising feature of (16) is this last observation before it: not only is the price of debt less than the face value of debt when evaluated at the equilibrium cutoff, the price of debt is less than the face value of debt even when the cutoff is evaluated at arbitrary values.
Figure 2: Graphs of the standardized disclosure thresholds for the low debt case $\bar{z}(\theta = .7, f)$ and high debt case $\bar{z}(\theta = .7, f)$ as the standardized debt level $f$ varies over the interval [-1,1], along with the identity function $i(f) = f$ and the constant $f^*(\theta = .7) = -0.1422$. The parameters held fixed in addition to $\theta = .7$ are $p = .7$, $m = 2$, and $\sigma^2 = 1$. The low debt cutoff $\bar{z}(\theta = .7, f)$ constitutes an equilibrium cutoff for those $f$ for which $\bar{z}(\theta = .7, f) \geq f$, i.e., for those $f \leq f^*(\theta = .7)$. The high debt cutoff $\bar{z}(\theta = .7, f)$ constitutes an equilibrium cutoff for those $f$ for which $\bar{z}(\theta = .7, f) < f$, i.e., for those $f > f^*(\theta = .7)$. The function $f^*(\theta)$ is defined in Theorem 10 below.

research (e.g., Dye [1985] and Jung and Kwon [1988]) for all equity firms that, as the probability the manager receives information increases, she discloses the information she receives more often, to the present circumstances where firms have both debt and equity.

To obtain intuition for Proposition 4(a), start with an equilibrium cutoff $x^*$ for a given face value $F$ of the firm’s debt, and then increase $F$ slightly. Note that, regardless of whether the manager makes a disclosure or stays silent, the sum of the prices of the firm’s debt and equity always equals the expected value of the firm’s cash flows conditional on the information available to investors. This is another manifestation of (1) above. Thus, increasing $F$ always decreases the price of equity by the exact same amount it increases the price of debt. Since more weight is attached to the price of equity than to the price of debt in the manager’s contract (since $\theta > .5$), any decline in the price of equity due to an increase in $F$ will have a larger effect on the manager’s compensation.
than will the increase in the price of debt due to the increase in $F$. So, the net effect of increasing $F$ is always to (at least weakly) reduce the manager’s compensation, whether or not the manager makes a disclosure.

In the low debt case, we assert that the compensation of the manager will decrease more with an increase in $F$ for those $x'$s the manager discloses in equilibrium than for those $x'$s the manager makes no disclosure. To see this, just note that any $x$ the manager discloses in the low debt case is larger than the face value of debt, so the price of equity decreases one-for-one in $F$ and the price of debt increases one-for-one in $F$. In contrast, were $x$ small enough so the manager makes no disclosure, the "no disclosure" prices of equity and debt change less than one-for-one with the
increase in $F$. This can be seen by using (8) to get

$$\frac{\partial}{\partial F} P_E^n(x^c) = -1 + \frac{H(F)}{1-p + pH(x^c)} = -\frac{\partial}{\partial F} P_D^n(x^c).$$

(18)

Thus, as $F$ increases, there is a smaller change in the prices of (both) debt and equity when the manager makes no disclosure than when the manager discloses. Thus, as $F$ increases, disclosure becomes relatively less attractive than nondisclosure. The only way to restore the manager’s indifference at the margin (i.e., at the equilibrium cutoff) is to increase the cutoff, that is, to disclose the private information the manager receives less often, consistent with the assertion in Proposition 4(a). This is illustrated in the left part of Figure 2 labeled “Low debt.” The equilibrium standardized disclosure threshold $z^c(\theta)$ is increasing in the weight $\theta$ placed on equity and exceeds the standardized debt level $f$ as required by definition of the low debt case.

To get intuition for Proposition 4(b), begin with an equilibrium cutoff $x^c = x^c(\theta)$ for a given $\theta$ and then increase $\theta$ slightly. Note that the marginal effect on the manager’s compensation associated with increasing $\theta$ slightly is given by the difference between the price of equity and the price of debt, both when the manager engages in disclosure and when the manager makes no disclosure, i.e., $\frac{\partial}{\partial \theta} [\theta P_E^d(x) + (1-\theta) P_D^d(x)] = P_E^d(x) - P_D^d(x)$ and $\frac{\partial}{\partial \theta} [\theta P_E^n(x^c) + (1-\theta) P_D^n(x^c)] = P_E^n(x^c) - P_D^n(x^c)$. If we knew that the inequality

$$P_E^d(x^c) - P_D^d(x^c) < P_E^n(x^c) - P_D^n(x^c)$$

(19)

held, then we would know that increasing $\theta$ slightly while holding the disclosure cutoff fixed has decreased the marginal benefit of disclosure (over nondisclosure). If that is the case, then to restore the equilibrium - so that at the equilibrium cutoff the manager is indifferent between disclosure and nondisclosure - we would also know that the equilibrium disclosure cutoff must be increased somewhat. This is what Proposition 4 (b) claims occurs. Let’s see why that is the case, i.e., why inequality (19) must hold.

Recalling the $\Delta_E(x^c)$, $\Delta_D(x^c)$ notation introduced at (12) above, inequality (19) is obviously equivalent to the inequality:

$$\Delta_E(x^c) < \Delta_D(x^c).$$

(20)

So, if we knew why inequality (20) holds, we would have an explanation for Proposition 4(b). But, we do know why inequality (20) holds: we know $\Delta_D(x^c)$ is positive in the low debt case because in that case disclosure only occurs when the realized $x$ is high enough so that the debt is fully repaid, which yields $\Delta_D(x^c) = F - P_D^n(x^c) > 0$ (recall (16)). And, we know that $\Delta_E(x^c)$ is negative, as we know from (13) that $\Delta_E(x^c)$ and $\Delta_D(x^c)$ are of opposite signs.
In Theorem 10 below, we identify exactly for what values of $f$ (and hence for what values of $F = \sigma f + m$) the solution $z^c(f, \theta)$ to equation (17) constitutes an equilibrium in the low debt case. But, before getting to that theorem, we first produce the counterparts to Lemma 3 and Proposition 4 for the high debt case. The following lemma contains a preparatory step, by producing expressions corresponding to $P^E(x^c)$ and $P^D(x^c)$ in (8) and (9) above for the high debt case.

**Lemma 5** In the high debt case (where $F > x^c$), if investors believe the manager has adopted a disclosure policy defined by the cutoff $x^c$, and the manager has not made a disclosure, investors will calculate the expected value of the firm’s equity and debt respectively to be as follows:

\[
P^E(x^c) = \frac{(1 - p)\{(m - F)(1 - H(F)) + \sigma^2 h(F)\}}{1 - p + pH(x^c)}.
\]

(21)

and

\[
P^D(x^c) = \frac{p\{mH(x^c) - \sigma^2 h(x^c)\} + (1 - p)\{(m - F)H(F) - \sigma^2 h(F) + F\}}{1 - p + pH(x^c)}.
\]

(22)

For further reference, it is useful to note that $P^E(x^c)$ in (21) implies, since $\Gamma(f) - f > 0$ for all $f$:

\[
P^E(x^c) = \frac{(1 - p)\{\frac{m - F}{\sigma}(1 - H(F)) + \sigma h(F)\}}{1 - p + pH(x^c)}.
\]

(23)

With expressions (21) and (22) replacing expressions (8) and (9), the equation for an equilibrium cutoff in the high debt case is also given by equation (11) above. A cutoff $x^c$ (or $z^c$) is an equilibrium cutoff in the high debt case if it solves this equation and also satisfies the requirement $x^c < F$ (or $z^c < f$), this last inequality being the natural counterpart to the corresponding inequality in the low debt case. Upon performing some additional algebra on equation (11) with these substitutions, we get the following counterpart to Lemma 3.

**Lemma 6** If $f$ is big enough so that the high debt case applies, then for any $\theta \in [0.5, 1)$:

(a) equation (11) is algebraically equivalent to the following equation:

\[
z^c(1 - p) + p\Gamma(z^c) = \frac{2\theta - 1}{1 - \theta} \times (1 - p) \times (\Gamma(f) - f);
\]

(24)

(b) there exists a unique cutoff $z^c$ solving equation (24), which we denote by $z^c(f, \theta)$.
While this lemma, like Lemma 3 above, does not prove the existence of an equilibrium cutoff, it does yield additional testable comparative statics for the high debt case. Total differentiation of equation (24) shows that:

**Proposition 7** When \( f \) is large enough so that the high debt case applies, then:

(a) for any \( \theta < 1 \), \( z^c(f, \theta) \) is strictly decreasing in \( f \), i.e., the probability the manager makes a voluntary disclosure is strictly increasing in the amount of the firm’s debt financing;

(b) \( z^c(f, \theta) \) is strictly increasing in \( \theta \), i.e., the probability the manager makes a voluntary disclosure is strictly decreasing in the weight attached to the price of equity in the manager’s contract;

(c) \( z^c(f, \theta) \) is strictly decreasing in \( p \).

Part (a) asserts that increases in the face value of the firm’s debt in the high debt case strictly increases the propensity of the manager to disclose her information. Note that this is opposite of the corresponding prediction in Proposition 4 in the low debt case (see also Figure 2). Thus, the relationship between the face value of firms’ debt levels and the amount managers disclose is not monotonic if one considers face values of firms’ debt levels that span both the low debt and high debt cases. As was noted in the Introduction, this result may pose a challenge for empiricists trying to determine the relationship between debt and disclosure: any attempt to document a relationship between debt and disclosure using linear regressions will likely result in spurious results unless attention is confined to a narrow band of debt levels, since when the set of debt levels considered is broad enough to encompass both the low debt and high debt cases, this nonmonotonicity will not be accounted for using linear regressions.

Parts (b) and (c) provide further demonstrations of the robustness of the corresponding results in Proposition 4: both results hold independently of whether the low debt or the high debt cases applies.

To get intuition for Proposition 7(a) and why it differs from Proposition 4 (a), fix \( \theta > .5 \) and \( F \) and identify the equilibrium cutoff \( x^c = x^c(F) \), and then increase \( F \) slightly. Look at the effects on the margin of the manager disclosing or not disclosing the information \( \tilde{x} = x^c \). In this case, since the firm is in default when the manager discloses \( x = x^c \) (since \( x^c < F \) in the high debt case), there is no effect on either the price of equity or the price of debt when \( F \) is increased, that is: \( \frac{\partial}{\partial F} P^d_E(x^c) = \frac{\partial}{\partial F} 0 = 0 \) and \( \frac{\partial}{\partial F} P^d_D(x^c) = \frac{\partial}{\partial F} x^c = 0 \), so this slight increase in \( F \) has no effect on the manager’s compensation when the manager discloses \( x = x^c \).

Contrast this to the effect of increasing \( F \) slightly on the ”no disclosure” compensation of the manager. Using (21), we see that the slight increase in \( F \) reduces the price of equity and it increases
the price of debt by the positive amount $Y$, where:

\[
\frac{\partial}{\partial F} P^\text{nd}_E(x_c) = \frac{(1 - p)(1 - H(F))}{1 - p + pH(x_c)} = -Y \quad \text{and} \quad \frac{\partial}{\partial F} P^\text{nd}_D(x_c) = Y > 0.
\]

So, the effect of increasing $F$ slightly on the manager’s compensation when she makes no disclosure while retaining her original disclosure policy and $\theta > .5$ is $s \times$

\[
\frac{\partial}{\partial F} [\theta P^\text{nd}_E(x_c) + (1 - \theta) P^\text{nd}_D(x_c)] = \theta \times (-Y) + (1 - \theta) \times Y = (1 - 2\theta)Y < 0.
\]

Thus, increasing $F$ slightly reduces the manager’s compensation more when she makes no disclosure than when she discloses $x = x^c$. In order to restore an equilibrium, disclosure of more $x$’s must occur, i.e., the cutoff $x^c$ has to be reduced, consistent with Proposition 7 (a).

Let’s step back and identify why Propositions 4 (a) and 7 (a) reach different conclusions. Any explanation for the difference between these two propositions’ conclusions can be obtained by examining just one of (a) how the price of the firm’s equity changes or (b) how the price of the firm’s debt changes, when $F$ is increased slightly, because the price of equity has to decrease by the exact same amount as the price of debt increases as $F$ increases. Suppose we examine how the price of the firm’s equity changes with a small increase in $F$. Regardless of the situation - low debt case or high debt case, disclosure or no disclosure - a (slight) increase in $F$ always has to, at least weakly, reduce the value of the firm’s equity.

But, the size of the decline in the price of equity in response to a slight increase in $F$ changes case by case. When the manager makes no disclosure and $F$ increases slightly, the price of the firm’s equity always strictly declines, but it never declines one-for-one with the increase in $F$, both in the low debt and high debt cases. That is: $-1 < \frac{\partial}{\partial F} P^\text{nd}_E(x_c) < 0$. In contrast, when the manager discloses at the margin (i.e., at the equilibrium cutoff) in the low debt (resp., high debt) case, the price of equity decreases one-for-one in $F$ (resp., does not change at all in $F$). This asymmetric change in the price of equity due to the slight increase in $F$ in the low debt/high debt disclosure/no disclosure cases accounts for the difference between the two propositions’ conclusions: in the low debt case, when the manager discloses at the margin (i.e., at the equilibrium cutoff), the price of equity declines more with the slight increase in $F$ than when the manager makes no disclosure. In the high debt case, when the manager discloses at the margin, the price of equity declines less with the slight increase in $F$ than when the manager makes no disclosure. This asymmetric price response necessitates an asymmetric change in the equilibrium cutoff to restore the equilibrium: since disclosure becomes relatively less (more) advantageous at the margin than nondisclosure in the low debt (high debt) case, the equilibrium cutoff has to be raised (lowered) in that case. These conclusions are consistent with the findings we obtained in part (a) of these two propositions.
The intuition for Proposition 7 (b) can be obtained by piggybacking on the intuition we developed for Proposition 4 (b) above. Recall from that previous discussion that if we knew (and could explain why) inequality (20) holds, we will have captured the intuition underlying the conclusion that the equilibrium cutoff rises as $\theta$ is increased slightly. In the high debt case, the firm defaults when the manager discloses $\tilde{x} = x^c$, so the price of equity is zero then, and since $P^{nd}_E(x^c)$ is always positive for any cutoff (recall (23) above), it is clear that the increment in the value of equity with disclosure over no disclosure, $\Delta_E(x^c) = P^{d}_E(x^c) - P^{nd}_E(x^c)$, is negative. By (13), it follows that $\Delta_D(x^c)$ is positive, and so inequality (20) holds in the high debt case too.

The conclusions of Propositions 4 (b) and 7 (b) are the same: we get the conclusion that any increase in the weight $\theta$ attached to the price of equity in the manager’s contract, regardless of whether we are dealing with the high debt or low debt cases, and regardless of starting value of $\theta \in [0.5, 1]$, results in the manager being less inclined to disclose whatever private information she has received. Thus, we can conclude:

**Proposition 8** Among all $\theta \in [0.5, 1]$ and among all scaled face values $f$:

(a) the contract weighting scheme that maximizes the manager’s propensity to disclose her private information is $\theta = 0.5$. That is, the manager discloses the information she receives most often when the manager’s compensation contract encourages her to maximize the value of the sum of the firm’s debt and equity;

(b) the contract weighting scheme that minimizes the manager’s propensity to disclose her private information is $\theta = 1$. That is, the manager discloses the information she receives least often when the manager’s compensation contract encourages her to maximize the value of the firm’s equity.

The proposition is potentially surprising. Before learning of it, one might have expected that as $\theta$ increases, i.e., as more of a manager’s compensation depends on the price of her firm’s equity, then - since equity is considered to be a more “informationally sensitive” security (see e.g., Boot and Thakor [1993]) than is debt - the manager would disclose the information she receives more often. Notice that this last contention is the exact opposite of the conclusion of the proposition. The correct intuition for this result was given above in the “marginal” analysis associated with the discussion of Propositions 4 (b) and 7 (b) above. The marginal analysis described what is best from the standpoint of the manager’s incentives at the time she receives a particular piece of information.

Related, we usually think that a managerial contract that encourages the manager to disclose a lot of the private information she receives is a desirable contract, because a manager, anticipating
that much of her private information will be disclosed, will be encouraged to do a good job. At the
same time, we also tend to think that contracts that have lots of incentive effects, that is contracts
where there is high pay-for-performance sensitivity (PFPS), are desirable contracts. Since equity
prices fluctuate in value more than debt prices do, placing more weight on the price of equity would
seem to increase the contract’s PFPS. What the previous proposition demonstrates, though, is that
contracts that encourage lots of disclosure may be in conflict with contracts that have a high degree
of PFPS, i.e., contracts with high $\theta$.

Proposition 8 (a) makes clear that contracts with weight $\theta = .5 = 1 - \theta$ attached to both equity
and debt are special. There is another special feature of these contracts. If one examines either of
equations (17) or (24) at $\theta = .5$, one sees that RHS of the equation is independent of $f$, and hence
the equilibrium cutoff $\hat{z}(f, \theta = .5)(= \hat{z}^c(f, \theta = .5))$ too is independent of $f$. This independence of
the equilibrium cutoff from the face value of the firm’s debt $f$ is the property that we referred to
in the Introduction as the "Modigliani-Miller property" for disclosures when $\theta = .5$. We state it
formally as:

**Proposition 9** When $\theta = .5$, the manager’s preferred disclosure cutoff is independent of the face
value of the firm’s debt.

This proposition is intuitive: when the goal of the manager is to maximize the total value of the
firm, then the disclosure policy of the manager is independent of how much debt financing the firm
has because the manager does not care what the prices of the firm’s debt or equity are individually;
she only cares about the sum of the debt and equity prices. When calculating the total value of
these two financial securities, the face value of the firm’s debt "washes out." Further, it is easy
to see from equations (17) or (24) that for no weight $\theta \neq .5$ is the manager’s preferred disclosure
policy independent of the amount of debt the firm has. Figure 6 illustrates the sensitivity of the
manager’s disclosure cutoff to changes in the debt level for different $\theta$.

Our final theorem builds on Lemmas 3 and 6 above to determine precisely what debt levels $F$
correspond to the low and high debt cases.

**Theorem 10** Unless explicitly otherwise noted, all of the following results apply to any $\theta \in (.5, 1)$.
(a) The equation

$$ f(1 - p) + p\Gamma(f) = \frac{2\theta - 1}{\theta} \times \Gamma(f) $$

has a unique solution, which we denote by $f^*(\theta)$;
(b) the low debt case applies when $F$ is such that $\frac{F - m}{\sigma} \leq f^*(\theta)$;
Figure 4: Graphs of the regular equilibrium disclosure cutoff \( x^c(\theta, F) \) as the face value \( F \) varies over the interval \([1,3]\) for six different values of \( \theta \) along with the identity function \( i(F) = F \). The disclosure thresholds above the identity function are low debt cases and the disclosure thresholds below the identity function are high debt cases. The parameters held fixed are \( p = .7, m = 2, \) and \( \sigma^2 = 1 \).

(c) the high debt case applies when \( F \) is such that \( \frac{F-m}{\sigma} \geq f^*(\theta) \);

(d) when \( f = f^*(\theta) \), the two equations (17) and (24) are algebraically identical, both the low debt and high debt cases apply, and \( \bar{z}^c(f^*(\theta), \theta) = \bar{z}^c(f^*(\theta), \theta) \);

(e) \( f^*(\theta) \) is strictly increasing in \( \theta \);

(f) for \( \theta = 1 \), the low debt case applies for all \( f \in \mathbb{R} \);

(g) when \( \theta = 1 \), the high debt case never applies for any \( f \in \mathbb{R} \), i.e., there is never an equilibrium in the high debt case for any \( f \) when \( \theta = 1 \).

Said in different words, parts (a)-(d) of the theorem assert that for any fixed \( \theta \in (.5,1) \), the threshold level \( F^*(\theta) \equiv \sigma f^*(\theta) + m \) separates the low debt and high debt cases: if \( F \leq F^*(\theta) \), then the low-debt case applies and so the manager’s equilibrium disclosure policy is defined (in standardized terms) by the solution \( \bar{z}^c(f, \theta) \) to equation (17), and if \( F \geq F^*(\theta) \), the high debt case applies and so the manager’s equilibrium disclosure policy is defined (in standardized terms) by the
solution $z^c(f, \theta)$ to equation (24). When the face value of the firm’s debt $F$ is on the dividing line $F^*(\theta)$, the manager’s equilibrium disclosure policy can be described by either of the cutoffs $z^c(f, \theta)$ or $z^v(f, \theta)$ because in this knife-edge case, these two cutoffs can be shown to be one and the same.

As part (e) asserts, as more weight is placed on equity in the manager’s contract, the set of debt levels for which the low debt case applies expands.

Parts (f) and (g) cover the special case $\theta = 1$, where only equity prices affect the manager’s compensation. Part (f) asserts that "all equity" contracting is consistent with an equilibrium in the low debt case for every possible face value of the firm’s debt when $\theta = 1$, and part (g) asserts that no equilibrium is consistent with the high debt case for any face value of the firm’s debt when $\theta = 1$. This second result is not surprising. Since the high debt case is characterized by the face value of the firm’s debt exceeding the disclosure cutoff, this case entails that any $x$ in the interval $x \in [x^c,F]$ is to be disclosed in equilibrium, yet doing so would result in the price of equity being zero. In contrast, making no disclosure necessarily results in the price of the equity being positive, and so is necessarily preferred from the viewpoint of a manager compensated entirely on the price of her firm’s equity, i.e., when $\theta = 1$.12

Parts (f) and (g), when viewed in conjunction with both the other parts of this theorem and with Propositions 4 (a)and 7 (a), yields an important difference between the $\theta = 1$ case and all other cases. Since according to part (g), $\theta = 1$ is only consistent with the low debt case, it follows that the conclusions of Proposition 7 (a) never apply in this case, and hence the nonmonotone relationship between debt and disclosure displayed in all cases $\theta \in (.5,1)$ does not hold for the case $\theta = 1$. We state this as a proposition.

**Proposition 11** When there is "all equity" compensation, i.e., $\theta = 1$, there is always a monotone relationship between debt and disclosure: more debt always leads to strictly less disclosure by the manager.

We conclude with a brief discussion as to how an empiricist might try to distinguish between the low debt or high debt cases. One way of telling is an immediate consequence of the definitions of these two cases. In the high debt case when $\theta < 1$, there are some voluntary disclosures in which the market price of equity following the disclosure is as low as possible - zero in our specification. (This can happen in the high debt case because there are $x$’s in the interval $(x^c,F)$ that, if disclosed, would result in a zero price for equity.) In the low debt case, other than for the knife edge case

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12The price of equity given nondisclosure must be positive in view of (a) equity holders have limited liability (they ultimately receive payoff $\max\{x - F, 0\} \geq 0$ if the realized value of the firm’s cash flows are $x$) and (b) there is a positive probability $\tilde{x}$ will exceed $F$. 

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where \( x^c = F \), this cannot happen: disclosure always yields a positive price for equity.

Another way to get a sense as to which of the two cases applies is by explicitly computing a firm’s threshold level \( F^*(\theta) \). If we know, or can estimate, each of \( p, \theta, m, \sigma \) then we can calculate \( F^*(\theta) \). For example, if \( \theta = .5 \) and \( p = .1 \), we know according to (25) that \( f^*(.5) \) satisfies the equation \( f^*(.5) \times .9 + .1 \times \Gamma(f^*(.5)) = 0 \). It can be shown numerically that \( f^*(.5) \approx -.042 \). So, upon substituting estimated values for \( m \) and \( \sigma \), we can calculate \( F^*(.5) \) as \(-.042\sigma + m\). Then, one can compare a firm’s actual debt level to this debt threshold. If the actual debt levels of the sample firms studied sometimes falls below and sometimes falls above this liability threshold, then the nonmonotonicity issues we have discussed in the text must be addressed in an empirical analysis of the relationship between debt and disclosure for this sample.

But, we must acknowledge there are many practical difficulties in adapting our two period model to real data on firms with traded debt for a variety of reasons: as examples, firms may have traded debt with multiple distinct maturities and so the level of their indebtedness is not summarizable by the face value of the issued bonds. Also, bonds issued may have coupon payments which must be paid periodically. Also, bond issues may have more complicated payoff structures than the simple bonds we study - e.g., the bonds may have call provisions, they could be highbred securities such as mandatorily redeemable preferred stock; they could have performance pricing features that result in the interest rates on the bond issue varying over time. One could potentially address all of these issues by reinterpreting the parameter \( F \) of our model as the the sum total of all cash payments due all debtholders in the coming period.

Notwithstanding the additional complexities of bonds actually issued by firms, we conjecture that at least two of our central comparative statics will hold up in the face of these complexities: (1) there is not a monotone relation between a firm’s level of indebtedness appropriately measured and its managers’ preferred disclosure policy and (2) increasing the emphasize on the price of equity in the manager’s contract may have the surprising property of causing the manager to disclose the private information she receives less often than would a compensation contract that placed less emphasis on equity prices. The first assertion already has some, albeit limited, support in the empirical work on the relationship between debt and disclosure we cited in the Introduction, as one possible reason why the existing empirical results are mixed is that the data examined in these empirical studies may have commingled both low debt and high debt firms.
3 Conclusions

We have studied how the presence of debt in a public firm’s capital structure influences the firm’s manager’s optimal voluntary disclosure policy, when the manager is compensated based on a weighted average of the market prices of the firm’s debt and equity, and the weight attached to the price of equity in the manager’s compensation contract is at least 50%. In this setting, we have shown that:

- as long as the manager’s compensation puts positive weight on the price of the firm’s publicly traded debt, there is a threshold level for the amount of debt the firm has such that, if the debt remains below this threshold, increases in the firm’s debt leads the manager to voluntarily disclose less information, whereas if the debt level rises above the threshold, further increases in debt leads the manager to voluntarily disclose more information;

- if the manager is given an "equally weighted" contract, where the manager’s bonus equally weights the market values of the firm’s equity and debt, then there is a "Modigliani Miller theorem" for disclosures, i.e., the manager’s preferred disclosure policy is independent of the level of the firm’s debt financing. No other compensation scheme based on a weighted average of the price of the firm’s equity and debt yields such an irrelevance result;

- the "equally weighted" contract also maximizes the propensity of the manager to disclose the private information she receives;

- regardless of the firm’s level of indebtedness, increases in the weight attached to the price of equity (and hence decreases in the weight attached to the price of the firm’s debt) in the manager’s contract leads the manager to disclose the information she receives less often;

- if the manager’s compensation is "all-equity" based ($\theta = 1$), and so disregards the price of debt, then there is a monotone relationship between debt and disclosure: in that case, more debt always leads to less disclosure.

4 References


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5 Appendix

Before presenting the proofs of any lemmas, propositions, or theorems, we start by collecting a few observations that are used repeatedly in the following. When \( \tilde{x} \sim N(m, \sigma^2) \) has density \( h(\cdot) \) and cdf
\( H(\cdot), \) then for any constant \( A: \)
\[
\int_{-\infty}^{A} xh(x)dx = mH(A) - \sigma^2h(A), \tag{26}
\]
and
\[
\int_{A}^{\infty} xh(x)dx = m(1 - H(A)) + \sigma^2h(A). \tag{27}
\]
Also, recall the function \( \Gamma(z), \) defined in the text at (15). The following assertions hold for \( \Gamma(\cdot) \):\(^{13}\)
\[
\text{for all } z : \quad \Gamma(z) > 0; \quad \Gamma(z) > z; \quad \Gamma'(z) = \Phi(z); \tag{28}
\]
\( \Gamma(z) \) is strictly increasing; \( \Gamma(z) - z \) is strictly decreasing
\[
\lim_{z \to -\infty} \Gamma(z) = 0; \quad \lim_{z \to \infty} \Gamma(z) = \infty; \quad \lim_{z \to -\infty} \Gamma(z) - z = -\infty \quad \lim_{z \to \infty} \Gamma(z) - z = 0; \quad \lim_{z \to \infty} \frac{\Gamma(z)}{z} = 1.
\]

**Proof of Lemma 1**

In the low debt case, if the reason the manager made no disclosure was that she withheld information below the cutoff \( x^c, \) the expected value of the firm’s equity conditional on that knowledge is (using (26) twice):
\[
E[\max\{\bar{x} - F, 0\} | \bar{x}] < x^c = \frac{\int_{-\infty}^{x^c} (x - F)h(x)dx}{H(x^c)} = \frac{\int_{-\infty}^{x^c} xh(x)dx - \int_{-\infty}^{F} xh(x)dx - F(H(x^c) - H(F))}{H(x^c)}
\]
\[
= \frac{(m - F)(H(x^c) - H(F)) - \sigma^2(h(x^c) - h(F))}{H(x^c)}.
\tag{29}
\]
By Bayes rule, the probability the manager withheld information given she made no disclosure and used the cutoff \( x^c \) is \( \frac{\rho H(x^c)}{1-p + \rho H(x^c)} \), so it follows from combining (6) and (29) that investors’ perceptions of the expected value of equity conditional on (only) knowledge that the manager made

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\(^{13}\)All these assertions follow easily, most by using L’Hospital’s rule along with one or more of the following obvious facts: \( \phi'(z) = -z\phi(z); \quad \Gamma'(z) = \Phi(z); \) each of \( \phi(z), z\phi(z), \) and \( z^2\phi(z) \) approach zero as \( z \to \pm \infty. \) E.g., To see that \( \Gamma(z) > z \) holds everywhere, just note that \( \Gamma(z) - z \) is strictly decreasing in \( z \) throughout \( \mathbb{R}, \) since its derivative is \( \Phi(z) - 1 < 0. \) Thus, if \( \lim_{z \to \infty} \Gamma(z) - z \geq 0 \) it will follow that \( \Gamma(z) > z \) holds for all \( z. \) Now, observe \( \lim_{z \to \infty} \Gamma(z) - z = \lim_{z \to \infty} z(\Phi(z) - 1) + \phi(z) = \lim_{z \to \infty} \frac{\Phi(z) - 1}{z} = \lim_{z \to \infty} -z^2\phi(z) = 0. \) Thus, the claim follows. As another example, to see that \( \Gamma(z) > 0 \) everywhere, observe that \( \Gamma'(z) = \Phi(z), \) so it suffices to show \( \lim_{z \to -\infty} \Gamma(z) \geq 0. \) Observe \( \lim_{z \to -\infty} \Gamma(z) = \lim_{z \to -\infty} z\Phi(z) + \phi(z) = \lim_{z \to -\infty} \frac{\Phi(z)}{z} = \lim_{z \to -\infty} -z^2\phi(z) = 0. \) Since the preceding shows \( \lim_{z \to -\infty} \Gamma(z) = 0, \) it obviously follows that \( \lim_{z \to -\infty} \Gamma(z) - z = -\infty, etc. \)
no disclosure is given by:

\[
P_E^{nd}(x^c|m) = E[\max\{x - F, 0\}|nd] = \frac{p((m - F)(H(x^c) - H(F)) - \sigma^2(h(x^c) - h(F)))}{1 - p + pH(x^c)} + \frac{(1 - p)((m - F)(1 - H(F)) + \sigma^2 h(F))}{1 - p + pH(x^c)} \\
= \frac{p(m - F)H(x^c) - (m - F)H(F) - p\sigma^2 h(x^c) + (1 - p)(m - F) + \sigma^2 h(F)}{1 - p + pH(x^c)} \\
= m - F + \frac{(m - F)H(F) - p\sigma^2 h(x^c) + \sigma^2 h(F)}{1 - p + pH(x^c)}.
\]

This last expression is (8) in the text.

Next, we make corresponding computations regarding investors’ perceptions of the expected value of the firm’s debt in the low debt case. If the manager made no disclosure because she was withholding information below cutoff \(x^c\), the expected value of the firm’s debt is given by:

\[
E[\min\{\tilde{x}, F\}|\tilde{x} < x^e] = \frac{\int_{-\infty}^{F} xh(x)dx + \int_{F}^{x^e} FH(x)dx}{H(x^c)} = \frac{mH(F) - \sigma^2 h(F) + F(H(x^c) - H(F))}{H(x^c)}.
\]

Combining this last fact with the observation that the probability the manager withheld information given she makes no disclosure is \(\frac{pH(x^c)}{1 - p + pH(x^c)}\) together with (7), we conclude that the expected value of the firm’s debt conditional on (only) knowledge that the manager made no disclosure is given by:

\[
P_D^{nd}(x^c) = E[\min\{\tilde{x}, F\}|nd] = \frac{p(mH(F) - \sigma^2 h(F) + F(H(x^c) - H(F)))}{1 - p + pH(x^c)} + \frac{(1 - p)(mH(F) - \sigma^2 h(F) + F(1 - H(F)))}{1 - p + pH(x^c)} \\
= mH(F) - \sigma^2 h(F) + pH(F(x^c) - H(F)) + (1 - p)F(1 - H(F)) \\
= \frac{mH(F) - \sigma^2 h(F) + pH(F(x^c) - H(F)) + (1 - p)F - (1 - p)FH(F)}{1 - p + pH(x^c)} \\
= \frac{mH(F) - \sigma^2 h(F) - pFH(F) - (1 - p)FH(F) + F}{1 - p + pH(x^c)} \\
= \frac{(m - F)H(F) - \sigma^2 h(F)}{1 - p + pH(x^c)} + F.
\]

This last expression is (9) in the text. ■

Proof of Lemma 3

Part (a) Substituting the expressions for the no disclosure prices of equity and debt (8) and (9) into the equilibrium equation (11), we find that the equation can be rearranged into any of the
that the function \(\overline{z}(\sigma)\) is strictly continuously increasing in \(\overline{z}\) everywhere and has range \((0, \infty)\) (see (28)), and therefore LHS (17) strictly continuously increases and has range \((-\infty, +\infty)\). (Recall (28) above.) Therefore there exists a unique solution to the equation.

Part (b). This is easy: the function \(\Gamma(\overline{z})\) is strictly continuously increasing in \(\overline{z}\) everywhere and has range \((0, \infty)\) (see (28)), and therefore LHS (17) strictly continuously increases and has range \((-\infty, +\infty)\). (Recall (28) above.) Therefore there exists a unique solution to the equation.

For future reference, we also add to the statement of the lemma:

(c) \(\overline{z}(f, \theta)\) is a continuous function. To see this, for fixed \(f\) and \(\theta\), we know from part (b) there exists exactly one solution to equation (31), namely \(\overline{z}(f, \theta)\). For present purposes, let \([a, b]\) represent the fixed interval \([\overline{z}(f, \theta) - k, \overline{z}(f, \theta) + k]\) for some positive constant \(k\). Notice that, as the unique solution to (31), \(\overline{z}(f, \theta)\) can also be viewed as the unique solution to the minimization problem \(\min_{z \in [a, b]} (z(1 - p) + p\Gamma(z) - \frac{2\theta - 1}{\theta} \times \Gamma(f))^2\). (All we have done in the preceding is represent the solution of an equation as the solution to a minimization problem. The advantage of doing the latter is that it permits us to invoke Berge’s [1963] ”maximum principle” to conclude that the function \(\overline{z}(f, \theta)\) is a continuous function.)

**Proof of Proposition 4**

Recall from (28) that \(\Gamma'(\cdot) = \Phi(\cdot) > 0\) and \(\Gamma'(\overline{z}) > \overline{z}\) for all \(\overline{z}\), from which it follows immediately that: LHS(17) strictly increases in both \(\overline{z}\) and \(p\), and also RHS(17) is strictly increasing in \(f\) (the latter holding strictly only when \(\theta > .5\)). Since \(\Gamma(\cdot) > 0\), it further follows that RHS(17) is strictly increasing in \(\theta\). All claims in the proposition follow immediately from the preceding observations, e.g., (b) follows since as \(\theta\) goes up, RHS(17) must go up, which implies LHS(17) must go up, and hence \(\overline{z}\) increases in \(\theta\). The other comparative statics follow in a similarly straightforward way.

**Proof of Lemma 5**
Clearly, in the high debt case (where \( x^e < F \)), the expected value of equity, conditional on the manager receiving and withholding information, is given by:

\[
E[\max\{x - F, 0\}|x < x^e] = 0.
\]

Combining this with (6), we conclude that the expected value of equity, conditional on the manager making no disclosure, in this case is given by:

\[
P_E^{nd}(x^e) = E[\max\{\tilde{x} - F, 0\}|nd] = \frac{pH(x^e) \times 0 + (1 - p)\{(m - F)(1 - H(F)) + \sigma^2 h(F)\}}{1 - p + pH(x^e)}.
\]

This is (21).

The expected value of the firm’s debt, conditional on the manager receiving and withholding information, is given by:

\[
E[\min\{\tilde{x}, F\}|x < x^e] = \frac{\int_{-\infty}^{x^e} xh(x)dx}{H(x^e)} = \frac{mH(x^e) - \sigma^2 h(x^e)}{H(x^e)}.
\]

Combining this with (7), we conclude that the expected value of the firm’s debt, conditional on the manager making no disclosure, is given by:

\[
P_D^{nd}(x^e) = E[\min\{\tilde{x}, F\}|nd] = \frac{p\{mH(x^e) - \sigma^2 h(x^e)\} + (1 - p)\{(m - F)H(F) - \sigma^2 h(F) + F\}}{1 - p + pH(x^e)}.
\]

This is (22).

**Proof of Lemma 6**

In view of Lemma 6, it follows that for the high debt case, the convex combination of the equity and debt’s market value when the manager makes no disclosure and is perceived by investors to use the cutoff \( x^e \) is given by:

\[
\theta \times P_E^{nd}(x^e) + (1 - \theta) \times P_D^{nd}(x^e) = \theta \times \frac{(1 - p)\{(m - F)(1 - H(F)) + \sigma^2 h(F)\}}{1 - p + pH(x^e)} + (1 - \theta) \times \frac{p\{mH(x^e) - \sigma^2 h(x^e)\} + (1 - p)\{(m - F)H(F) - \sigma^2 h(F) + F\}}{1 - p + pH(x^e)}.
\]

Thus, since \( P_E^{nd}(x^e) = 0 \) and \( P_D^{nd}(x^e) = x^e \) in this case, the equilibrium equation for the high debt case must satisfy the equation (analogous to the equation (11)):

\[(1 - \theta)x^e = \theta \times P_E^{nd}(x^e) + (1 - \theta) \times P_D^{nd}(x^e).
\]

This last equation is algebraically equivalent to each of the following equations:

\[
(1 - \theta)x^e = \frac{\theta \times (1 - p)\{(m - F)(1 - H(F)) + \sigma^2 h(F)\}}{1 - p + pH(x^e)} + (1 - \theta) \times \frac{p\{mH(x^e) - \sigma^2 h(x^e)\} + (1 - p)\{(m - F)H(F) - \sigma^2 h(F) + F\}}{1 - p + pH(x^e)}
\]
\[(1 - \theta)x^c (1 - p + pH(x^c)) = \theta \times (1 - p)((m - F)(1 - H(F)) + \sigma^2 h(F))
\quad + (1 - \theta) \times [p \{mH(x^c) - \sigma^2 h(x^c)\}]
\quad + (1 - p) \{(m - F)H(F) - \sigma^2 h(F) + F\}\]

\[(1 - \theta)x^c (1 - p + pH(x^c)) = \theta \times (1 - p)((m - F)(1 - H(F)) + \sigma^2 h(F))
\quad + (1 - \theta) \times [p \{mH(x^c) - \sigma^2 h(x^c)\} + (1 - p)m - (1 - p)m +
\quad (1 - p)\{(m - F)H(F) - \sigma^2 h(F) + F\}\]

\[(1 - \theta)(x^c - m)(1 - p + pH(x^c)) = \theta \times (1 - p)((m - F)(1 - H(F)) + \sigma^2 h(F))
\quad + (1 - \theta) \times [-p\sigma^2 h(x^c) - (1 - p)m(1 - H(F))]
\quad + (1 - p)\{-FH(F) - \sigma^2 h(F) + F\}\]

\[(1 - \theta)(x^c - m)(1 - p + pH(x^c)) = \theta \times (1 - p)((m - F)(1 - H(F)) + \sigma^2 h(F))
\quad + (1 - \theta) \times [-p\sigma^2 h(x^c) - (1 - p)m(1 - H(F))]
\quad + (1 - p)F(1 - H(F)) - (1 - p)\sigma^2 h(F)\]

\[(1 - \theta)(x^c - m)(1 - p + pH(x^c)) = \theta \times (1 - p)((m - F)(1 - H(F)) + \sigma^2 h(F))
\quad + (1 - \theta) \times [-p\sigma^2 h(x^c) - (1 - p)(m - F)(1 - H(F))]
\quad - (1 - p)\sigma^2 h(F)\]

\[(1 - \theta)(x^c - m)(1 - p + pH(x^c)) = (\theta - (1 - \theta)) \times (1 - p)(m - F)(1 - H(F)) + \theta \times (1 - p)\sigma^2 h(F)
\quad + (1 - \theta) \times [-p\sigma^2 h(x^c) - (1 - p)\sigma^2 h(F)]\]

\[(1 - \theta)(\frac{x^c - m}{\sigma})(1 - p + pH(x^c)) = (\theta - (1 - \theta)) \times (1 - p)(\frac{m - F}{\sigma})(1 - H(F))
\quad + \theta \times (1 - p)\sigma h(F) + (1 - \theta) \times [-p\sigma h(x^c) - (1 - p)\sigma h(F)]\]

\[(1 - \theta)z^c (1 - p + p\Phi(z^c)) = -(2\theta - 1) \times (1 - p)f(1 - \Phi(f)) + \theta \times (1 - p)\phi(f)
\quad - (1 - \theta) \times [p\phi(z^c) + (1 - p)\phi(f)]\]

\[(1 - \theta)z^c (1 - p + p\Phi(z^c)) = -(2\theta - 1) \times (1 - p)f(1 - \Phi(f)) + (2\theta - 1) \times (1 - p)\phi(f) - (1 - \theta) \times p\phi(z^c)\]
\[(1 - \theta)\{z^c(1 - p + p\Phi(z^c)) + p\phi(z^c)\} = (2\theta - 1)(1 - p) \times \{-f(1 - \Phi(f)) + \phi(f)\}\]
\[(1 - \theta)\{z^c(1 - p) + p\Gamma(z^c)\} = (2\theta - 1)(1 - p) \times \{\Gamma(f) - f\}. \tag{32}\]

This proves the first part of Lemma 6. The other parts of the lemma, including the suppressed in the text part (c) claim that \(z^c(f, \theta)\) is continuous, are proven similarly to the corresponding proofs in Lemma 3 and are omitted.■

**Proof of Proposition 7**

The proof is almost the same as the proof of Proposition 4. The main difference is that \(\Gamma(f) - f\) is decreasing in \(f\), which follows since its derivative is \(\Phi(f) - 1\).■

**Proof of Theorem 10**

This proof is simple, but it has a lot of parts to it.

**Low Debt Components of the Proof**

We start by considering the special case \(\theta = .5\). In that case, equation (17) becomes, for all \(f\):

\[z^c(1 - p) + p\Gamma(z^c) = 0 \tag{33}\]

Let the solution to equation (33) (which we know from Lemma 3 part (b) exists and is unique) be denoted by any of the following four distinct symbols

\[\bar{z}^c(f, .5), z^c_0, f^*(.5), \text{ or } \bar{f} \tag{34}\]

(the advantage of this seemingly redundant notation will be clear below). If we restrict \(f\) to those \(f \leq \bar{f}\), then the solution \(\bar{z}^c(f, .5)\) to (17) when \(\theta = .5\) is an equilibrium of the model (since it solves (17) by definition of \(\bar{z}^c(f, .5)\), and since it satisfies \(\bar{z}^c(f, .5) \geq f\) for all \(f \leq \bar{f}\), by definition of \(\bar{f}\).

Now appealing to the notation \(f^*(.5)\), it is clear that \(f^*(.5)\) solves equation (25) when \(\theta = .5\), since that equation, written in terms of \(f\), is identical to equation (33), written in terms of \(z^c\). Finally, we note that the uniqueness of this solution, i.e., the uniqueness of \(f^*(.5)\), has already been established (again, in Lemma 3), as part of the more general claim there that the solution \(\bar{z}^c(f, \theta)\) is unique for any \(\theta\).

Next, we consider an arbitrary \(\theta \in (.5, 1)\). From Lemma 4 (b), we know that \(\bar{z}^c(f, \theta)\) is strictly increasing in \(\theta\) for any \(f\). This implies in particular that:

\[
\text{if } f \leq \bar{f}, \text{ then } \bar{z}^c(f, \theta) > \bar{z}^c(f, .5) = z^c_0 = \bar{f} \geq f, \tag{35}\]

and so \(\bar{z}^c(f, \theta)\) constitutes an equilibrium for any \(f \leq \bar{f}\).

Stated differently, but equivalently, (35) establishes that for any \(\theta \in (.5, 1)\):

\[
(-\infty, \bar{f}] \subset \{f | z^c(f, \theta) \geq f\}. \tag{36}\]
In fact, (35) establishes even more, since it shows for any \( \theta > .5 \) that \( \bar{z}(f, \theta) > \bar{f} \). So, by the continuity of \( \bar{z}(\cdot) \), there exists \( f' > \bar{f} \) such that \((-\infty, f'] \subset \{ f | \bar{z}(f, \theta) \geq f \} \). For any given \( \theta > .5 \), we let \( f^*(\theta) \) denote the largest such \( f' \), i.e., we set
\[
f^*(\theta) \equiv \sup \{ f' \mid (-\infty, f'] \subset \{ f | \bar{z}(f, \theta) \geq f \} \}.
\] (36)

If \( f^*(\theta) = \infty \), then we conclude that, for this \( \theta \), for all possible (scaled) face values \( f \in \mathbb{R} \) of the firm’s debt, \( \bar{z}(f, \theta) \geq f \).

If \( f^*(\theta) \) is finite, then it follows, again by the continuity of \( \bar{z}(\cdot) \), that
\[
\bar{z}(f^*(\theta), \theta) = f^*(\theta),
\] (37a)
in which case equation (17) for the special \( f = f^*(\theta) \) can be rewritten as (substituting all instances of \( z \) with \( f \) for this special \( f \)):
\[
f(1-p) + p \Gamma(f) = \frac{2\theta - 1}{\theta} \times \Gamma(f).
\] (38)

Once we prove that this last equation has a solution for every \( \theta \in (.5, 1) \), then in view of (36), we will have proven both parts (a) and (b) of Theorem 10.

**Lemma 12** For every \( \theta \in (.5, 1) \), equation (38) has a solution, and this solution is unique.

**Proof of Lemma 12** We rewrite equation (38) as
\[
f(1-p) + \beta \Gamma(f) = 0,
\] (39)
where we define
\[
\beta \equiv p - \frac{2\theta - 1}{\theta}.
\] (40)

When \( \beta \geq 0 \), then LHS(39) is continuous, strictly increasing, and ranges over \(-\infty \) to \( \infty \). Thus the equation has a unique solution. So, for the remainder of the proof, we consider only negative \( \beta \).

We first analyze the case where \( \beta \leq -(1-p) \). Then, since \( \Gamma(f) > f \) for all \( f \) (from (28)), we have
\[
f(1-p) + \beta \Gamma(f) \leq f(1-p) - (1-p)\Gamma(f) = (1-p)(f - \Gamma(f)) < 0 \text{ for all } f,
\]
so equation (39) has no solution for such \( \beta \).

Next, we consider those negative \( \beta \) where \( \beta > -(1-p) \). Since \( \lim_{f \to \infty} \frac{\Gamma(f)}{f} = 1 \) (from (28)), in this case, we see that:
\[
\begin{align*}
\lim_{f \to \infty} f(1-p) + \beta \Gamma(f) &= \lim_{f \to \infty} f(1-p + \beta \frac{\Gamma(f)}{f}) = \left( \lim_{f \to \infty} f \right) \times \lim_{f \to \infty} (1-p + \beta \frac{\Gamma(f)}{f}) \\
&= \left( \lim_{f \to \infty} f \right) \times (1-p + \beta \lim_{f \to \infty} \frac{\Gamma(f)}{f}) = \infty.
\end{align*}
\]
Thus, since LHS(39) is continuous in $f$ and equals $\beta \Gamma(0) < 0$ when evaluated at $f = 0$, it follows that there exists some (positive) $f$ that solves equation (39).

Next, we observe when (39) has a solution, i.e., when $\beta > -(1 - p)$, that: $\frac{d\text{LHS}(39)}{df} = 1 - p + \beta \Phi(f) > 1 - p - (1 - p)\Phi(f) = (1 - p)(1 - \Phi(f)) > 0$. This establishes that the solution to (39), when it exists, is unique.

Finally, we recall the definition of $\beta$: $\beta \equiv p - \frac{2\theta - 1}{\theta}$. The condition $\beta > -(1 - p)$ is the same as the condition $p - \frac{2\theta - 1}{\theta} > -(1 - p)$, i.e., is the same as $\frac{2\theta - 1}{\theta} < 1$, which holds for all $\theta < 1$. This completes the proof of Lemma 12. ■

Next, we consider the case where $\theta = 1$. In that case, equation (17) becomes

$$z^c(1 - p) + p \Gamma(z^c) = \Gamma(f).$$

(41)

Since $\Gamma(z^c) > z^c$ for all $z^c$, equation (41) implies

$$\Gamma(z^c) = \Gamma(z^c)(1 - p) + p \Gamma(z^c) > z^c(1 - p) + p \Gamma(z^c) = \Gamma(f).$$

(42)

Since $\Gamma(\cdot)$ is monotone increasing, inequality (42) implies $z^c > f$. Thus, when $\theta = 1$, the cutoff $z^c(f, 1)$ that solves (17) is an equilibrium for every $f \in \mathbb{R}$.

This completes the proof of those parts of Theorem 10 involving the low debt case.

**High Debt Components of the Proof**

We start with the special case $\theta = 1$. We observe that there is no solution to (32) in this case, since in this case, its LHS = 0 and its RHS > 0.

Next, we consider another special case, $\theta = .5$. In this case, equations (17) and (24) are identical; their common solution exists, is unique, is independent of $f$, and is given by $z^c_0$ as defined above in (34). This solution will be an equilibrium in the high debt case provided we restrict $f$ to $f \geq z^c_0$.

Now we consider any $\theta \in (.5, 1)$. We have already shown that for any such $\theta$, the solution $z^c(f, \theta)$ to (32) exists, is unique, and is continuously strictly decreasing in $f$. Since the identity function $i(f) \equiv f$ obviously strictly continuously increases in $f$ and ranges over all of $\mathbb{R}$, there exists a unique $f$, call it (for the time being)

$$\tilde{f}(\theta), \text{ for which } z^c(\tilde{f}(\theta), \theta) = \tilde{f}(\theta).$$

(43)

Since the function $z^c(f, \theta)$ is everywhere strictly decreasing in $f$, it lies strictly below the identity function $i(f)$ for all $f \geq \tilde{f}(\theta)$, i.e., for all such $f$, $z^c(f, \theta) \leq f$. Hence, $z^c(f, \theta)$ constitutes an equilibrium for all such $f$. 32
Also notice that in the special case where \( f = \bar{f}(\theta) \), equation (32) can be written, replacing all instances of \( z^c \) in equation (32) by this special \( f \), as:

\[
f(1 - p) + p \Gamma(f) = \frac{2\theta - 1}{1 - \theta} \times (1 - p) \times \{\Gamma(f) - f\}. \tag{44}
\]

Rewrite this as first as

\[
f(1 - p)(1 + \frac{2\theta - 1}{1 - \theta}) + (p + \frac{1 - 2\theta}{1 - \theta} \times (1 - p))\Gamma(f) = 0
\]

and then as:

\[
f(1 - p) + \frac{p + \frac{1 - 2\theta}{1 - \theta} \times (1 - p)}{1 + \frac{\theta - 1}{1 - \theta}} \times \Gamma(f) = 0.
\]

Observe that the coefficient of \( \Gamma(f) \) in this last equation can be written in any of the following ways:

\[
\frac{p(1 - \theta) + (1 - 2\theta) \times (1 - p)}{\theta} = \frac{p\theta + (1 - 2\theta)}{\theta} = p - \frac{2\theta - 1}{\theta}.
\]

Notice that this last expression is the expression \( \beta \) in (40) above. Hence, equation (44) is exactly the same as equation (38). Hence, equation (44) has a solution under exactly the same circumstances as equation (38) does, and that solution is exactly the same as the solution to equation (38), namely \( f^*(\theta) \). That is to say, what we recently labeled as \( \bar{f}(\theta) \) in (43) in the high debt case, is exactly the same as what we called \( f^*(\theta) \) in the low debt case, which was the solution to equation (38). Thus, in view of this observation and (43) and (37a), we have for any \( \theta \in (0.5, 1) \):

\[
z^c(\bar{f}(\theta), \theta) = \bar{f}(\theta) = f^*(\theta) = z^c(f^*(\theta), \theta). \tag{45}
\]

These results collectively prove the high debt components of the Theorem 10.

All that remains to complete the proof of Theorem 10 is the claim that \( f^*(\theta) \) is strictly increasing in \( \theta \). We prove this now. Write equation (39), with \( f^*(\theta) \) substituted for \( f \) and \( p + \frac{1}{\theta} - 2 \) substituted for \( \beta \), as:

\[
f^*(\theta)(1 - p) + (p + \frac{1}{\theta} - 2)\Gamma(f^*(\theta)) = 0.
\]

Totally differentiate this equation with respect to \( \theta \) to get:

\[
\{1 - p + (p + \frac{1}{\theta} - 2)\Phi(f^*(\theta))\} \frac{df^*(\theta)}{d\theta} - \frac{\Gamma(f^*(\theta))}{\theta^2} = 0,
\]

or upon rearrangement, as

\[
\{1 - p + (p - 1 + \frac{1}{\theta} - 1)\Phi(f^*(\theta))\} \frac{df^*(\theta)}{d\theta} - \frac{\Gamma(f^*(\theta))}{\theta^2} = 0,
\]

33
or as
\[
\{(1 - p)(1 - \Phi(f^*(\theta)) + \frac{1}{\theta} - 1)\Phi(f^*(\theta))\} \frac{df^*(\theta)}{d\theta} - \frac{\Gamma(f^*(\theta))}{\theta^2} = 0.
\]

From this last expression, it is clear that the coefficient of \( \frac{df^*(\theta)}{d\theta} \) in this last equation is positive, from which we that \( sgn \frac{df^*(\theta)}{d\theta} \) is positive, as claimed. ■