Intrafirm Trade, Pay-Performance Sensitivity and Organizational Structure

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1 Introduction

Managers in divisionalized firms frequently engage in intrafirm trade in addition to their day-to-day operations. The incentives literature has, for the most part, looked separately at the problems of eliciting operating effort (the agency problem) and of guiding intrafirm trade and relationship-specific investments (the transfer pricing problem). The standard justification is that agency problems are addressed by calibrating managers' pay-performance sensitivity (PPS); at the same time, the relevant costs and revenues for intrafirm trade decisions are scaled uniformly by these bonus coefficients — they are not directly incurred by the managers but accrue to divisional P&Ls. The transfer pricing problem should therefore be unaffected by the managers' PPS.[1] In this paper we argue that this paradigmatic separation overlooks an important link between these two incentive problems. By analyzing this link we provide predictions for incentive contracts and the allocation of decision rights in vertically integrated firms.

A central observation underlying many of our findings is that intrafirm trade adds compensation risk. Investments in more efficient equipment result in greater trading volume which, in turn, translates into more trade-related uncertainty and thereby into compensation risk for managers. To study this link between compensation and investments in detail, we examine the contracting relationship between a principal and two division managers who exert personally costly effort and may trade an intermediate good. Division managers are risk-averse and compensated based on their own divisional profit, i.e., we ignore formal profit sharing contracts.[2] Transfer prices are negotiated.[3] Managers can make upfront

[1]Papers that have aimed at bridging this gap are Holmstrom and Tirole (1991), Anctil and Dutta (1999), Baldenius (2006), and Baiman and Baldenius (2008).

[2]Profit sharing arrangements are not frequently observed in practice, see Merchant (1989), Bushman, et al. (1995), Keating (1997), Abernethy, et al. (2004). Including them into our framework would not alter the qualitative conclusions.

[3]We ignore "administered" approaches to intrafirm trade, such as cost- or market-based.

specific investments that increase the gains from trade. We first study investments by one of the two divisions only and later turn to bilateral investments and the optimal assignment of authority over those.

A recurring theme in the literature is that incomplete contracting often results in underinvestment. A manager's P&L records all fixed costs from investing but only a share of the attendant contribution margin as the result of bargaining. While this hold-up problem is present also in our model, we show that a risk-averse manager's investment incentives are indeed affected by his PPS. A manager who operates under high-powered incentives—e.g., because he faces a rather stable operating environment—is very sensitive to additional noise introduced into his performance measure by interdivisional trade. As argued above, investments in fixed assets translate into greater trade-related risk. As a consequence, the higher the manager's PPS, the less he will invest.

While one might expect the incremental trade-related risk premium to exacerbate the underinvestment problem attributable to hold-up, the opposite is in fact the case. The investment level preferred by the principal, if she had enough information to take control of the decision, would take into account the incremental risk premium incurred by *both* division managers involved. With incomplete contracting and delegated investment choice, however, the investing manager takes *only his own* risk cost into consideration which, *c.p.*, leads him to overinvest. Which of the two forces dominates — hold-up or the externalized risk premium — determines whether ultimately there will be over- or underinvestment. A key factor moderating this tradeoff is the divisions' operating risk, because of its effect on the managers' PPS.

We benchmark equilibrium PPS and investments against a scenario of contractible investments. The division managers' optimal PPS with contractible investments is determined by two factors: their respective operating uncertainty (inversely related to PPS, as usual) and the centrally chosen investment level (also inversely related because of the additional risk premium). With incomplete contracting, the investment is chosen by a division manager in his own best interest; the principal's only control instrument is the compensation contract. If the divisions face severe operating uncertainty, all else equal, their PPS will be low. The externalized incremental risk premium associated with the investment then is small, whereas the hold-up problem remains unaffected. Underinvestment then obtains in equilibrium. As a silver lining, the principal can provide the manager who has no investment opportunity with higher-powered incentives. The investing manager, in contrast, may find his incentives muted as a way for the principal to mitigate the underinvestment problem.

The reverse logic applies to settings of high trade-related uncertainty. The externalized risk premium problem then weighs heavily, eventually giving rise to overinvestment. In response, the non-investing manager's PPS should be lowered, relative to the contractible investments benchmark, whereas the investing manager may face higher-powered incentives so as to curb his overinvestment tendency.

The preceding arguments illustrate that contractual incompleteness may introduce a wedge into the incentive power for managers facing identical operating environments except for their scope to engage in specific investments. The PPS for non-investing managers (more generally, for managers with less essential investment opportunities) is dictated solely by the anticipated equilibrium investment level: the adjustment in PPS relative to the contractible investment benchmark is inversely related to the equilibrium investment distortion. In contrast, the PPS for the investing manager should be chosen with an eye to his investment incentives. Here, the association between equilibrium investment distortion and PPS adjustment may be positive: to combat under- (over-) investment, the principal may want to lower (raise) this manager's PPS.

We then consider investments in both up- and downstream divisions and ask how the authority to choose those should be allocated between the division managers. This question lies at the heart of the classic organization design issue whether to organize divisions as profit or investment centers. We run a horse race between two organizational modes:

1. Each division is designated as an investment center, choosing its own investment and bearing the attendant fixed cost (*IC-IC*).

2. One division — the investment center — chooses both up- and downstream investments and is charged for all attendant fixed costs, whereas the other division is run as a profit center (*PC-IC*).

We conduct the horserace in a setting where each divisional investment is binary, 0 or 1, and where the pressing investment distortion is underinvestment (say, due to high operating noise). The downside of concentrating all investment authority in one hand (*PC-IC*), as compared with allocating it evenly (*IC-IC*), is that it exacerbates the hold-up problem. Under *IC-IC*, the incentive contract only needs to ensure that investing by each manager is a best response to the respective other manager also investing. The investment game between the two managers is characterized by strategic complementarity and therefore may have multiple equilibria: the desired one, (1,1), and an undesired one, (0,0). But we show below that whenever multiple equilibria exist, the desired one Pareto-dominates the undesired one (from the viewpoint of the division managers). The key here is that each manager's P&L only gets charged for his own investment. Under *PC-IC*, in contrast, the investment center manager gets charged for both investments (the "double whammy" effect), but still has to split the investment returns with the other manager. We identify parameter values for which the investment center manager prefers to forego both investments, even if the Pareto-dominant equilibrium under *IC-IC* is (1,1). If this is the case, the principal prefers to allocate authority evenly — i.e., the *IC-IC* mode.

The upside of concentrating all investment authority in one hand, *PC-IC*, is that it allows the principal to take advantage of the link between PPS and induced investments highlighted above. Whenever the two divisions differ in their operating risks, then the underinvestment problem can be alleviated by assigning investment authority to the manager facing the more volatile environment. All else equal, this manager will have lower-powered incentives and therefore is willing to invest more. If the differential in operating noise across divisions is substantial, then this risk effect can outweigh the double whammy effect, making *PC-IC* the preferred mode. The link between investments and compensation risk, highlighted in this paper, is thus important not just for calibrating incentive contracts but also for organizational design.

Earlier studies to link transfer pricing with divisional incentives (and organizational issues) are Holmstrom and Tirole (1991), Anctil and Dutta (1999). In both papers, risk averse managers invest more in response to profit sharing; but with pure divisional performance measurement the managers' PPS would not affect equilibrium investments. Anctil and Dutta (1999) highlight the risk sharing benefits associated with negotiated transfer pricing and show that this may give rise to relative performance evaluation. While risk sharing via bargaining features also in our model, we establish a direct link between PPS and investments even absent profit sharing.

The paper proceeds as follows. Section 2 describes the model. Section 3 describes benchmark with contractible investments, Section 4 addresses optimal contracts for non-contractible investments. Section 5 embeds bilateral investment choice in the larger context of organizational design. Section 6 concludes.

2 The Model

The model entails a principal contracting with two division managers. Division 1 produces an intermediate good and sells it to Division 2. Division 2 further processes it and sells a final good. We assume there is no outside market for the intermediate good. Each division is run by a manager who exerts unobservable effort $a_i \in \mathbb{R}_+$ with marginal productivity normalized to one.

The sequence of events is as follows. At Date 1, the principal contracts with the division managers. At Date 2, the managers choose their effort levels a_i. Efforts are "general purpose" in that they are unrelated to the transaction opportunity that arises between the two divisions. To increase the gains from intrafirm trade, the manager of the upstream division (Manager 1) can make a relationship-specific investment I. At Date 3, the managers, but not the principal, jointly observe the realization of the random variables $\theta_i,\ i = 1, 2$, drawn from (commonly known) distribution functions over supports $[\underline{\theta}_i, \bar{\theta}_i]$, respectively. Let $\sigma^2_{\theta_i}$ denote the variance of θ_i, and $\theta = (\theta_1, \theta_2)$.

At Date 4, the managers negotiate over the quantity of the intermediate good to be traded, $q \in \mathbb{R}_+$, and the (per-unit) transfer price $t \in T$. Finally, divisional profits are realized as follows:

$$\begin{aligned}\pi_1 &= a_1 + \widetilde{\varepsilon}_1 + tq - C(q, \theta_1, I) - F(I),\\ \pi_2 &= a_2 + \widetilde{\varepsilon}_2 + R(q, \theta_2) - tq,\end{aligned}$$

with

$$C(q, \theta_1, I) = (c - I - \theta_1)q \quad \text{and} \quad R(q, \theta_2) = r(q) + \theta_2 q. \tag{1}$$

Here, $C(q, \theta_1, I)$ is Division 1's (linear) variable cost associated with the trade and $F(I)$ is its fixed cost from investing, with $F(\cdot)$ increasing and sufficiently convex to ensure interior solutions; $R(q, \theta_2)$ is the (net) revenue realized by Division 2, with $r(q)$ an increasing and concave function. This formulation implies

Figure 1: Timeline

that Division 1's marginal production cost is decreasing in the investment, i.e., $\frac{\partial^2}{\partial q \partial I} C(q, \theta_1, I) < 0$. Let $M(q, \theta, I) \equiv R(q, \theta_2) - C(q, \theta_1, I)$ denote firmwide contribution margin. The ongoing operations of the division are risky, as expressed by normally distributed error terms $\widetilde{\varepsilon}_i$ with mean zero and variance $\sigma^2_{\varepsilon_i}$. All noise terms, $\varepsilon_i, \theta_i,\ i = 1, 2$, are statistically independent. In the following, we refer to $\sigma^2_{\varepsilon_i}$ as (general) *operating uncertainty*, associated with each division's stand-alone business, and to $\sigma^2_{\theta_i}$ as *trade-related uncertainty*, pertaining to the transaction between the two divisions.

We assume that the managers are compensated with linear contracts based solely on their own divisional profits, i.e., we ignore profit-sharing contracts (as studied in Anctil and Dutta, 1999).[4] We denote Manager i's compensation by $S_i = \alpha_i + \beta_i \pi_i$. The managers are assumed to have mean-variance preferences:

$$EU_i = E(S_i) - V_i(a_i) - \frac{\rho_i}{2} Var(S_i),$$

where $V_i(a_i)$ denotes disutility of effort and ρ_i is Manager i's coefficient of risk aversion. For convenience, we assume throughout that

$$V_i(a_i) = \frac{v_i}{2} a_i^2, \quad v_i > 0.$$

[4] Allowing for profit sharing would dampen the effects present in our model but not eliminate them, as long as more compensation weight is put on own-division profit for each manager.

To ensure the agents find it worthwhile to participate in the contract, the individual rationality condition

$$EU_i \geq 0 \tag{2}$$

has to hold throughout.

Before characterizing the optimal contract, we briefly describe two useful benchmarks. The *first-best* solution would obtain if the managers' effort choices a_i and the investment I were contractible, and the realization of θ were observable to the principal. The managers would be compensated with a fixed salary only, to avoid any risk premium, and the principal would essentially instruct them as to the desired effort and investment choices at Date 2. The managers would be indifferent as to the level of trade and, at Date 4, choose the *ex-post* efficient trading quantity, given θ and I:

$$q^*(\theta, I) \in \arg\max_q \; M(q, \theta, I).$$

The (necessary and sufficient) first-order condition reads $R_q(q^*(\cdot), \theta_2) = C_q(q^*(\cdot), \theta_1, I)$. Define $M(\theta, I) \equiv M(q^*(\theta, I), \theta, I)$ as the first-best contribution margin conditional on θ and I. The first-best effort and investment choices at Date 2 are given by $V_i'(a_i^*) = 1$ and $E_\theta[M_I(\theta, I^*)] = E_\theta[q^*(\theta, I^*)] = F'(I^*)$, applying the Envelope Theorem.

As a second benchmark, suppose that the divisions do not transact with each other ($q = 0$), but a moral hazard problem exists, because efforts are chosen privately. Divisional profits then simplify to $\pi_i = a_i + \widetilde{\varepsilon}_i$. As usual, the individual rationality condition (2) will be binding at the optimal solution (this will be true throughout this paper). Substituting for the fixed salaries, α_i, the agents' effort incentive constraints read, for $i = 1, 2$,

$$a_i(\beta_i) \in \arg\max_{a_i} \; \beta_i a_i - V_i(a_i) - \frac{\rho_i}{2}\beta_i^2 \sigma_{\varepsilon_i}^2. \tag{3}$$

We denote the component of the principal's payoff (equivalently, of total surplus) that is unrelated to intrafirm trade by

$$\Phi_i(\beta_i) \equiv a_i(\beta_i) - V_i(a_i(\beta_i)) - \frac{\rho_i}{2}\beta_i^2\sigma_{\varepsilon_i}^2, \tag{4}$$

subject to (3). Note that $\Phi_i'(0) > 0 > \Phi_i'(1)$ and $\Phi_i''(\beta_i) \leq 0$ for every $\beta_i \in [0,1]$. The principal's optimization problem then reads:

$$\max_{\beta} \sum_i \Phi_i(\beta_i).$$

Invoking our maintained assumption that $V_i(a_i) = \frac{v_i}{2}a_i^2$, the optimal effort of Manager i simplifies to $a_i(\beta_i) = \beta_i/v_i$ and the optimal pay-performance sensitivity (PPS) is given by $\beta_i^{MH} = \frac{1}{1+\rho_i v_i \sigma_{\varepsilon_i}^2}$. Without intrafirm trade, the model thus collapses to two completely separable standard moral hazard models.[5]

3 Contractible Investment

Now consider the full-fledged model with moral hazard, investments, and the prospect of intrafirm trade. Suppose for now that the investment I is contractible, e.g., it may entail specialized equipment used exclusively (in a verifiable manner) to make a particular product. At Date 4, the managers negotiate under symmetric information about the quantity to be traded, q, and the transfer price, t. For any realized θ and I, they agree on the conditionally efficient quantity $q^*(\theta, I)$ and, in the course of bargaining, maximize the firmwide *ex-post* surplus. In particular, the outcome of bargaining is unaffected by the managers' respective PPS. Assuming equal bargaining strength, the transfer price simply splits the attendant contribution margin:

$$t(\theta, I) = \frac{R(q^*(\theta, I), \theta_2) + C(q^*(\theta, I), \theta_1, I)}{2q^*(\theta, I)},$$

[5]In the pure moral hazard case, as usual, mean-variance preferences could be derived endogenously from CARA utility functions, given the standard "LEN" structure. Once we add interdivisional trade, the performance measures will no longer be normally distributed — hence our assumption of mean-variance preferences.

resulting in divisional profits of

$$\begin{aligned}\pi_1 &= a_1 + \widetilde{\varepsilon}_1 + \frac{1}{2}M(\theta, I) - F(I),\\ \pi_2 &= a_2 + \widetilde{\varepsilon}_2 + \frac{1}{2}M(\theta, I).\end{aligned}$$

At Date 1 the principal instructs the upstream manager as to the investment level. In choosing compensation contracts, the principal needs to observe the individual rationality constraint and the revised effort incentives constraint

$$a_i(\beta_i) \in \arg\max_{a_i} \ \beta_i a_i - V_i(a_i) - \frac{\rho_i}{2}\beta_i^2 \left[\sigma_{\varepsilon_i}^2 + \frac{Var(M(\theta, I))}{4}\right]. \quad (5)$$

Note that, given the managers' mean-variance preferences, the additional trade-related variance does not affect their equilibrium effort choice, holding constant the PPS. Hence, the effort-related surplus terms, $\Phi_i(\beta_i)$, remain as stated in (4). The principal thus solves the following optimization program (the superscript "*" indicates *contractible* investments))

Program $\mathcal{P}^*$:

$$\max_{\beta, I \in \mathbb{R}_+} \ \Pi^*(\beta, I) \equiv E_\theta[M(\theta, I)] - F(I) + \sum_i \left\{\Phi_i(\beta_i) - \frac{\rho_i}{8}\beta_i^2 Var(M(\theta, I))\right\},$$

where $Var(M(\theta, I))$ is the variance of the firmwide contribution margin from trading. The corresponding first-order conditions read:

$$\begin{aligned}\frac{\partial \Pi^*}{\partial I} &= E[q^*(\theta, I^*)] - F'(I^*) - \sum_i \frac{\rho_i}{8}(\beta_i^*)^2 \frac{\partial Var(M(\theta, I^*))}{\partial I} = 0, \quad (6)\\ \frac{\partial \Pi^*}{\partial \beta_i} &= \Phi_i'(\beta_i) - \frac{\rho_i \beta_i^*}{4} Var(M(\theta, I^*)) = 0. \quad (7)\end{aligned}$$

As we show in the Appendix, the higher the investment I, the greater the trade-related risk premium. More fixed assets in place translate into lower marginal production costs and thereby greater trading volume for any realization of θ. Higher expected trade volume in turn implies more trade-related risk, i.e.,

$\frac{\partial}{\partial I} Var(M(\theta, I)) > 0$. This link between investments and risk premia will play a crucial role in many of the arguments to follow. Applying standard comparative statics techniques, we find:

Lemma 1 *With contractible investment, I^* is decreasing in each manager's coefficient of risk aversion, ρ_i. Furthermore, $\beta_i^* < \beta_i^{MH}$.*

All proofs are found in the Appendix. Trade-related risk translates into lower-powered incentives. At the same time, risk aversion on the part of the agents reduces the optimal investment, as compared with a risk-neutral setting. At the heart of this finding lies the observation that the principal's objective function $\Pi^*(\beta, I)$ has weakly increasing differences in $(\beta_1, \beta_2, -I)$ because greater investment increases the marginal risk premium associated with an increase in the PPS. For future reference, the optimal PPS with contractible investment is

$$\beta_i^* = \frac{1}{1 + \rho_i v_i \left[\sigma_{\varepsilon_i}^2 + \frac{Var(M(\theta, I^*))}{4} \right]}. \tag{8}$$

We now turn to the often more descriptive case of incomplete contracting.

4 Non-Contractible Investment

In many instances divisional investments are neither observable nor contractible. While *aggregate* PP&E expenditures are routinely monitored and verifiable, it is often difficult to trace *individual* pieces of equipment (or personnel training expenditures) to particular transactions. We now ask how non-contractibility of investments affects the optimal incentive contracts and resulting investment levels. A standard finding in the earlier literature is that incomplete contracting in conjunction with *ex-post* negotiations over the realizable contribution margin leads to underinvestment because of hold-up. As we will show now, this finding does not always hold when intrafirm trade adds to compensation risk.

For given investments, the Date 4 bargaining game unfolds as before, resulting again in the *ex-post* efficient quantity, $q^*(\theta, I)$, and a corresponding transfer price, $t(\theta, I)$. With non-contractible investments (superscript "**"), however, the principal's optimization program now entails an additional constraint:

Program $\mathcal{P}^{**}$:

$$\max_{\beta} \; \Pi^{**}(\beta) \equiv E_\theta[M(\theta, I(\beta_1))] - F(I(\beta_1)) + \sum_i \left\{ \Phi_i(\beta_i) - \frac{\rho_i}{8}\beta_i^2 Var(M(\theta, I)) \right\},$$

subject to

$$I(\beta_1) \;\; \in \;\; \arg\max_{I \in \mathbb{R}_+} \; f(I \mid \beta_1) - \beta_1 F(I). \tag{9}$$

Here,

$$f(I \mid \beta_1) \equiv \beta_1 \left(\frac{1}{2} E_\theta[M(\theta, I)] - \frac{\rho_1}{8}\beta_1 Var(M(\theta, I)) \right)$$

is the investing manager's trade-related certainty equivalent. Denote by β^{**} the solution to this program.

The new constraint (9) reflects the fact that the investment is now chosen noncooperatively by Manager 1 so as to maximize his expected payoff at Date 2. It is straightforward to see that the seller's investment incentives are unaffected by the buying manager's pay-performance sensitivity, β_2. The key question is how the seller's own PPS affects his incentives to invest.

Lemma 2 *If the investment is non-contractible, the equilibrium investment $I(\beta_1)$ is decreasing in the investing manager's PPS, β_1.*

Stronger effort incentives subject Manager 1 to greater trade-related risk. As a result, he will invest less the higher his PPS. More formally, his objective function at Date 2, as expressed by (9), exhibits decreasing differences in (β_1, I).

To dissect Manager 1's investment incentives in more detail, note that equilibrium investments under non-contractibility deviate from those under contractibility for two distinct reasons. First, the profit measure of Division 1 reflects only half the benefit of the investment but all fixed PP&E-related costs. This is the classic hold-up problem. At the same time, the investing manager takes into account only his own incremental risk premium when investing and ignores that of the respective other manager. That is, the selling manager externalizes part of the trade-related risk cost. This works against the hold-up effect. Proposition 1 below identifies sufficient conditions that allow us to evaluate this tradeoff:

Condition IMH *(Identical Moral Hazard Problems):* $v_i = v$, $\rho_i = \rho$ and $\sigma_{\varepsilon_i} = \sigma_\varepsilon$, for $i = 1, 2$.

Proposition 1

(i) If $\sigma^2_{\varepsilon_i}$ is sufficiently large for $i = 1, 2$, then $I(\beta_1^{**}) \leq I^*$ and $\beta_2^{**} \geq \beta_2^*$.

(ii) If IMH holds, with v sufficiently small, and $\sum_i \sigma^2_{\theta_i}$ is sufficiently large, then $I(\beta_1^{**}) \geq I^*$ and $\beta_2^{**} \leq \beta_2^*$.

Part (i) of the result provides sufficient conditions for underinvestment to obtain in equilibrium. High operating risk calls for low-powered incentives. For muted incentives, the externalized incremental risk premium attributable to the investment becomes negligible, but the hold-up problem remains unabated. As a result, if the principal were to set $\beta_1 = \beta_1^*$, the selling manager would underinvest relative to the contractible investment benchmark level. To provide additional investment stimulus, by Lemma 2, the principal would need to lower β_1. However, if the moral hazard problem is sufficiently severe (as measured by high operating risk $\sigma^2_{\varepsilon_i}$), then the scope for muting incentives is small, because the PPS will be low to begin with, even without any trade-related risk. As a result, the hold-up problem then dominates the externalized risk premium.

On the other hand, if the trade-related uncertainty is very high, then the externalized incremental risk cost weighs heavily. As a result, if the principal were to set the investing manager's PPS equal to β_1^*, the latter would overinvest. To mitigate this problem, the principal can increase β_1, by Lemma 2. However, if effort is "cheap" (v is small), then the managers' PPS will be high to begin with. In that case, the scope for raising β_1, as a way to curb the selling manager's overinvestment tendency, is limited. As we show in the proof, even if he were residual claimant at the margin, i.e., $\beta_1 = 1$, Manager 1 would still overinvest — the externalized risk premium effect then dominates the hold-up problem.[6]

The preceding discussion has immediate consequences for the optimal PPS. Lower (higher) equilibrium investment implies a smaller (larger) expected trading quantity as compared with contractible investments. The attendant reduction (increase) in trade-related risk allows the principal to expose the non-investing manager (Manager 2) to stronger (weaker) incentives.

Note however that Proposition 1 is silent as to the optimal PPS for Manager 1. The reason is that, even though we can unambiguously rank the equilibrium investments and incentive contracts for Manager 2 between contractible and non-contractible investments (Proposition 1), the same is not possible for β_1. Take the case of underinvestment — Case (i) of Proposition 1. As argued above, underinvestment reduces the trade-related risk premium which, at the margin, calls for higher-powered incentives. At the same time, by Lemma 2 there is a countervailing force for Manager 1: to alleviate the underinvestment problem, β_1 should be lowered. The net effect cannot be evaluated cleanly with continuous investments (except for special cases where the managers differ systematically in their risk tolerance). We therefore now turn to the simpler case where the investment is chosen from a discrete set.

For the remainder of the paper we assume that the investment decision is binary, i.e., $I \in \{0, 1\}$. The fixed investment cost is described by $F(I) = FI$ for

[6] Note that IMH is a sufficient condition in that it simplifies evaluating the tradeoff without the need to add cumbersome notation.

some scalar $F > 0$. That is, the investment is of fixed size and can either be undertaken in its entirety or foregone completely. We denote by

$$\Delta M \equiv E_\theta[M(\theta,1) - M(\theta,0)]$$

the incremental expected contribution margin attributable to the upfront investment and by

$$\Delta Var \equiv Var(M(\theta,1)) - Var(M(\theta,0))$$

the incremental trade-related variance.

We briefly review the benchmark model with *contractible* investment for $I \in \{0,1\}$. The principal's optimization problem remains as stated in Program $\mathcal{P}^*$ except that now $I \in \{0,1\}$ and $F(I) = FI$. We denote by $\beta_i^*(I)$ the solution to the first-order condition (7) for $I \in \{0,1\}$ and adopt the vector notation $\beta^*(I) = (\beta_1^*(I), \beta_2^*(I))$. Since investment adds to trade-related risk, it immediately follows that $\beta_i^*(1) < \beta_i^*(0)$. The principal will contract on $I = 1$ whenever

$$\Pi^*(\beta^*(1), I = 1 \mid F) \;\geq\; \Pi^*(\beta^*(0), I = 0). \tag{10}$$

The investment condition (10) can be restated as: $I^*(F) = 1$ if and only if

$$\begin{aligned} F \;&\leq\; \Delta M - \sum_i \left\{ \frac{\rho_i}{8} \left[(\beta_i^*(1))^2 \cdot Var(M(\theta,1)) - (\beta_i^*(0))^2 \cdot Var(M(\theta,0)) \right] - \Delta\Phi_i \right\} \\ &\equiv\; F^*, \end{aligned} \tag{11}$$

where $\Delta\Phi_i \equiv \Phi_i(\beta_i^*(1)) - \Phi_i(\beta_i^*(0)) < 0$ captures the loss in effort-related surplus as a result of the investment being made. For fixed costs low enough, the contractible investment will be made ($I^* = 1$), in which case $\beta^* = \beta^*(1)$. On the other hand, $I^* = 0$ and $\beta^* = \beta^*(0)$ for $F > F^*$.

We now return to our maintained assumption in this section that the investment choice is *non-contractible*. With binary investment, the principal's optimization problem is again given by $\mathcal{P}^{**}$ except for $I \in \{0,1\}$ and $F(I) = FI$.

Manager 1's investment incentive-compatibility constraint (9) then can be restated as:

$$\begin{aligned} I^{**}(\beta_1 \mid F) = 1 \iff F &\leq \frac{f(1 \mid \beta_1) - f(0 \mid \beta_1)}{\beta_1} \\ &= \frac{\Delta M}{2} - \frac{\rho_1}{8}\beta_1 \cdot \Delta Var \\ &\equiv F^{**}(\beta_1). \end{aligned} \tag{12}$$

On occasion, it will be convenient to invert (12) as follows:

$$I^{**}(\beta_1 \mid F) = 1 \iff \beta_1 \leq \beta_1^{**}(F), \tag{13}$$

where $\beta_1^{**}(F)$ is a decreasing function, by Lemma 2. The higher the fixed cost, the lower-powered the selling manager's incentives must be to elicit the investment.

For continuous investments, Proposition 1 has provided sufficient conditions for underinvestment (overinvestment) as a result of lack of contractibility. Proposition 1′ states the analogous result for binary investments:

Proposition 1′ *Suppose the investment choice is binary.*

(i) For $\sigma^2_{\varepsilon_i}$ sufficiently large, $i = 1, 2$, $F^* > F^{**}(\beta_1^*(1))$ – underinvestment.

(ii) If IMH holds, with v sufficiently low, and $\sum_i \sigma^2_{\theta_i}$ is sufficiently large, then $F^* < F^{**}(\beta_1^*(0))$ – overinvestment.

In the following, we address separately the two cases described in Proposition 1′ which endogenously give rise to under- and overinvestment, respectively, and discuss the implications for the managers' optimal PPS.

4.1 Optimal PPS with High Operating Uncertainty (Underinvestment)

For the case of high operating uncertainty Proposition 1′ has shown that, if the principal were to set the selling manager's PPS equal to the optimal level conditional on $I = 1$, the manager would underinvest. Put differently, there exist fixed cost parameters F for which contractibility is a necessary condition for the investment to be undertaken.

For fixed cost values at which the investment incentive constraint (13) is binding, the principal has to reduce Manager 1's PPS below $\beta_1^*(1)$ in order to induce the selling manager to undertake the investment. If F drops low enough, however, incomplete contracting comes at no cost: for any $F \leq F^{**}(\beta_1^*(1))$ the investment-incentive constraint (13) is satisfied at $\beta_1 = \beta_1^*(1)$. For intermediate fixed cost values, we have: (i) $I^* = 1$ and (ii) a strict efficiency loss due to incomplete contracting. Specifically, this will be the case for $F \in (F^{**}(\beta_1^*(1)), F^*]$. The open question is whether the principal then finds it optimal to induce the investment, even if doing so requires compromising on general-purpose effort so as to satisfy the investment incentive condition (13).

More generally, we aim to compare the equilibrium incentive contracts with those for contractible investments. The optimal contract is the one that maximizes the value of the following (mutually exclusive) two optimization programs:

$\mathcal{P}^{**(1)}$ *(Induce investment):* for any $F \in (F^{**}(\beta_1^*(1)), F^*]$:

$$\max_{\beta} \ \Pi(\beta \mid I = 1),$$
$$\text{subject to} \ \ \beta_1 \leq \beta_1^{**}(F).$$

$\mathcal{P}^{**(0)}$ *(No investment):* for any $F \in (F^{**}(\beta_1^*(1)), F^*]$:

$$\max_{\beta} \ \Pi(\beta \mid I = 0),$$
$$\text{subject to} \ \ \beta_1 > \beta_1^{**}(F).$$

Under program $\mathcal{P}^{**(1)}$ the principal optimally sets $\beta_2^{**} = \beta_2^*(1)$ and $\beta_1^{**} = \beta_1^{**}(F)$ such that the investment constraint (13) is just binding. Note that

$\beta_1^{**}(F) < \beta_1^*(1)$ for any $F > F^{**}(\beta_1^*(1))$. Hence, in order to elicit the investment, the Manager 1 must be given *lower-powered* incentives as compared with the contractible investment setting. Under the no-investment program $\mathcal{P}^{**(0)}$, $\beta_i^{**} = \beta_i^*(0)$ holds for each manager $i = 1, 2$. That is, incentives are *higher-powered* for both managers because of the reduced trade-related risk premium.

Our next result provides a complete characterization of the optimal contract for the case of non-contractible discrete investments in the presence of severe moral hazard (resulting in underinvestment):

Proposition 2 *Suppose the investment choice is binary and non-contractible, and $\sigma_{\varepsilon_i}^2$ is sufficiently high for $i = 1, 2$, so that $F^* > F^{**}(\beta_1^*(1))$.*

(i) If $F \le F^{**}(\beta_1^*(1))$, then $\beta_i^{**} = \beta_i^* = \beta_i^*(1),\ i = 1, 2$. Manager 1 invests and non-contractibility does not impose additional costs on the principal.

(ii) There exists a unique $\hat{F} \in [F^{**}(\beta_1^*(1)), F^*]$ such that for any $F \in (F^{**}(\beta_1^*(1)), \hat{F}]$, $\beta_1^{**} = \beta_1^{**}(F) < \beta_1^* = \beta_1^*(1)$ and $\beta_2^{**} = \beta_2^* = \beta_2^*(1)$. Manager 1 invests but the principal's payoff is lower than with contractible investment.

(iii) If $F \in (\hat{F}, F^*]$, then $\beta_i^{**} = \beta_i^*(0) > \beta_i^* = \beta_1^*(1),\ i = 1, 2$. Manager 1 does not invest. The principal's payoff is lower than with contractible investment.

(iv) If $F > F^*$, then $\beta_i^{**} = \beta_i^* = \beta_i^*(0),\ i = 1, 2$, and Manager 1 does not invest. The principal's payoff is the same as with contractible investment.

Figure 2 compares the optimal PPS with the contractible investment benchmark for the special case of IMH (so that $\beta_1^* = \beta_2^*$). For extreme values of fixed costs the investing manager's PPS coincides with the contractible investment benchmark, and so does the resulting payoff to the principal. However, the investing manager's PPS differs from the benchmark solution for intermediate values of F, for which the principal essentially trades off investment and managerial effort distortions. For F above, but close to, $F^{**}(\beta_1^*(1))$, the investing manager's PPS should be muted, because the investment is sufficiently valuable to warrant compromises on managerial effort resulting from a reduction in β_1. As fixed costs increase and reach a critical level, denoted $\hat{F}$, the principal becomes indifferent between inducing the investment—at the expense of reduced effort incentives for Manager 1—and foregoing it. For $F \in (\hat{F}, F^*]$ the PPS of each manager is higher than in the benchmark case. In this region, incomplete contracting would make it too costly for the principal to induce the investment. This underinvestment outcome in turn reduces the risk premium.

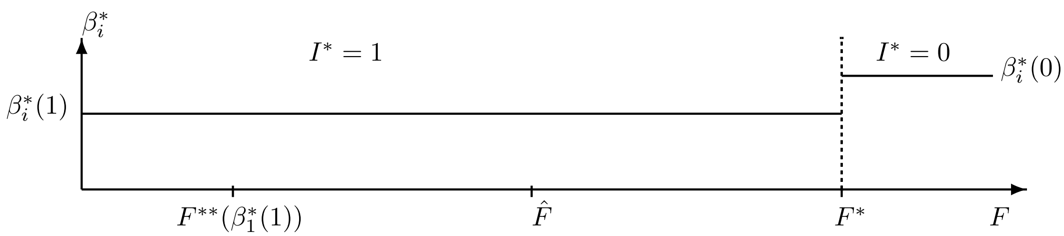

Fig. 2a: Contractible Investment

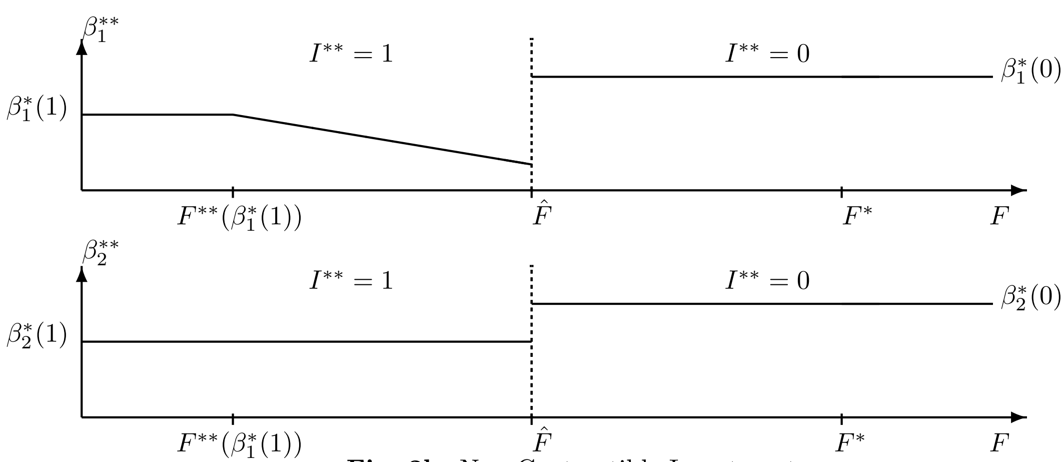

Fig. 2b: Non-Contractible Investment

Figure 2: Optimal Contracts and Resulting Investments

Proposition 2 demonstrates that, with severe divisional agency problems, non-contractibility of investments results in (weakly) higher-powered incentives for the non-investing manager, whereas the effect on the investing manager's PPS is ambiguous: higher-powered incentives for relatively high levels of fixed costs and muted incentives for low values of F.

4.2 Optimal PPS with High Trade-Related Uncertainty (Overinvestment)

As shown in Proposition $1'$, when the trade-related uncertainty is high (and effort is "cheap"), the investment may *only* be undertaken if it is non contractible (i.e., if $F > F^*$), because the investing manager does not internalize the entire incremental trade-related risk premium. The principal can mitigate this overinvestment problem by adjusting the investing manager's PPS. The question, again, is whether it is optimal to do so. As before, this boils down to a discrete comparison of the respective values of the two discrete optimization problems:

$\mathcal{P}^{**(1)}$ *(Allow investment):* for any $F \in [F^*, F^{**}(\beta_1^*(0))]$:

$$\max_{\beta} \; \Pi(\beta \mid I = 1),$$
$$\text{subject to} \;\; \beta_1 \le \beta_1^{**}(F).$$

$\mathcal{P}^{**(0)}$ *(Prevent investment):* for any $F \in [F^*, F^{**}(\beta_1^*(0))]$:

$$\max_{\beta} \; \Pi(\beta \mid I = 0),$$
$$\text{subject to} \;\; \beta_1 > \beta_1^{**}(F).$$

In order to curb Manager 1's overinvestment tendency (Program $\mathcal{P}^{**(0)}$), the principal needs to give the investing manager *higher-powered* incentives: $\beta_1^{**} = \beta_1^{**}(F) + \delta, \; \delta \to 0$.[7] Alternatively, the principal can acquiesce to the overinvestment problem (Program $\mathcal{P}^{**(1)}$) by setting $\beta_i^{**} = \beta_i^*(1)$ for each manager $i = 1, 2$. As compared with the case of contractible investment, where no

[7] Note that $\beta_1^{**}(F) > \beta_1^*(0)$ for any $F < F^{**}(\beta_1^*(0))$.

investment would be made, this calls for *lower-powered* incentives for both agents because of the additional trade-related risk premium.

The following result characterizes the optimal contract for non-contractible discrete investments in case of high trade-related uncertainty.

Proposition 3 *Suppose the investment choice is binary and non-contractible, IMH holds with v sufficiently low, and $\sum_i \sigma^2_{\theta_i}$ is sufficiently high, so that $F^* < F^{**}(\beta_1^*(0))$.*

(i) If $F \geq F^{**}(\beta_1^*(0))$, then $\beta_i^{**} = \beta_i^* = \beta_i^*(0)$, $i = 1, 2$. Manager 1 does not invest and non-contractibility does not impose additional costs on the principal.

(ii) There exists a unique $\hat{F} \in [F^*, F^{**}(\beta_1^*(0))]$ such that for any $F \in [\hat{F}, F^{**}(\beta_1^*(0))]$, $\beta_1^{**} = \beta_1^{**}(F) + \varepsilon > \beta_1^* = \beta_1^*(0)$ and $\beta_2^{**} = \beta_2^* = \beta_2^*(0)$. Manager 1 does not invest but the principal's payoff is lower than with contractible investment.

(iii) If $F \in [F^*, \hat{F})$, then $\beta_i^{**} = \beta_i^*(1) < \beta_i^* = \beta_1^*(0)$, $i = 1, 2$. Manager 1 invests. The principal's payoff is lower than with contractible investment.

(iv) If $F < F^*$, then $\beta_i^{**} = \beta_i^* = \beta_i^*(1)$, $i = 1, 2$, and Manager 1 invests. The principal's payoff is the same as with contractible investment.

Figure 3 is a close analogue to Figure 2, except now the principal trades off the optimal amount of excess investment against managerial effort distortions. Again, for extreme values of fixed costs, incomplete contracting does not affect the contractual outcome, but it does for intermediate values of F. For F larger than some critical value $\hat{F}$, but less than $F^{**}(\beta_1^*(0))$, the investing manager's PPS is increased so as to curb his overinvestment tendency. For $F \in [F^*, \hat{F})$, it is optimal for the principal to let the investment go acquiesce to overinvestment.

In summary, when trade-related uncertainty is high and eliciting managerial effort is cheap, incomplete contracting leads to overinvestment and (weakly) lower-powered incentives for managers without investment opportunities. An ambiguous result obtains for the manager who can invest: his PPS will be weakly greater (lower) than in the contractible benchmark setting if the fixed cost is sufficiently high (low) so that in equilibrium the principal wants to curb (acquiesce to) the manager's overinvestment tendency.

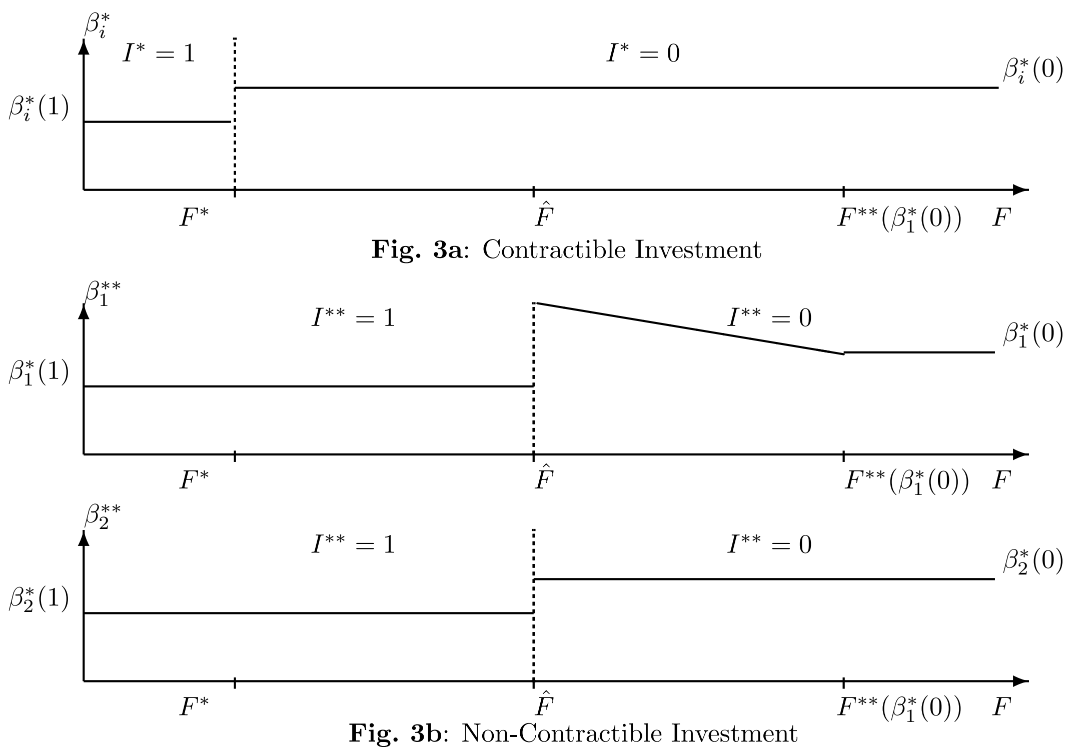

Figure 3: Optimal Contracts and Resulting Investments

A key tenet of prior work on incomplete contracting in divisionalized firms is that equilibrium investment levels are independent of managers' incentive contracts, absent profit sharing. According to this conventional wisdom, if two divisions operate in comparable operating environments but may have different scope to make specific investments, the managers should face similar PPS. We show that this logic breaks down once trade-related risk is accounted for. Proposition 2 and 3 demonstrate that the optimal response to *ex-ante* incentive problems often is to introduce a wedge between managers' PPS, i.e., to treat them differentially. Moreover, these results establish a non-monotonic relation between the *ex-ante* profitability of investment opportunities (measures here by fixed cost) and PPS for managers whose investments can add value to intrafirm trade (see Figures 2 and 3).

5 Assigning Investment Responsibility

A simplifying assumption in our analysis up to this point was that only one division had an investment opportunity. We now extend the analysis to bilateral investments and then employ the model to address an important organizational design question. Should both divisions be organized as *investment centers* — each manager choosing his "own" investment — or should authority for both up- and downstream investments instead be bundled in the hands of one divisional manager, thereby treating the respective other manager as a *profit center* manager to be evaluated based on contribution margin only?

We slightly modify the notation to accommodate bilateral investments. Let $I_i \in \{0, 1\}$ denote the (discrete) relationship-specific investment undertaken in Division $i = 1, 2$, at respective fixed costs of F_i, and $\mathbf{I} \equiv (I_1, I_2)$. For instance, I_2 could describe the demand-enhancing effect of sales promotions downstream. Normalizing the marginal return of I_1 and I_2 to unity, each, the divisional relevant costs and revenues from the transaction generalize those in (1) as follows:

$$C(q, \theta_1, I_1) = (c - I_1 - \theta_1)q \quad \text{and} \quad R(q, \theta_2, I_2) = r(q) + (I_2 + \theta_2)q. \tag{14}$$

The efficient quantity for given investments is $q^*(\theta, I_1, I_2) \in \arg\max_q \; R(q, \theta_2, I_2) - C(q, \theta_1, I_1)$. Let $M(\theta, \mathbf{I}) = M(q^*(\cdot), \theta, \mathbf{I})$.

At Date 4, the managers again negotiate over the transaction and split the attainable contribution margin $M(\theta, \mathbf{I})$ equally. In the following, we invoke a weaker form of our earlier "identical moral hazard problems" condition, now allowing for operating risks, σ_{ε_i}, to differ across divisions:

Condition IMH′: $v_i = v$ and $\rho_i = \rho$, for $i = 1, 2$.

We further modify the manager's trade-related certainty equivalent to accommodate bilateral investments,

$$f(\mathbf{I} \mid \beta_i) \equiv \beta_i \frac{E[M(\theta, \mathbf{I})]}{2} - \beta_i^2 \frac{\rho}{8} Var(M(\theta, \mathbf{I})). \tag{15}$$

To streamline the exposition, and to highlight the effects of assigning authority across divisions, we make a number of simplifying assumptions. First, the investments are equally costly, $F_1 = F_2 \equiv F$, which, together with (14), implies that they are equally productive. None of the results to follow hinge on this restriction. Second, we focus here on the case where both investments are sufficiently productive such that the principal would choose $\mathbf{I}^* = (1, 1)$ if investments were contractible. Third, the divisional agency problems are assumed sufficiently severe so that, as in Section 4.1, the managers tend to underinvest with incomplete contracting (i.e., the hold-up problem dominates the externalized risk premium). We then ask which organizational mode yields the higher expected surplus for the principal.

With contractible investments, the principal will induce an investment profile $\mathbf{I}^* = (1, 1)$ if and only if:[8]

$$\Pi^*(1,1) \geq \max\{\Pi^*(1,0),\ \Pi^*(0,0)\}, \tag{16}$$

[8] Given our maintained assumption that investments are of equal productivity, we have $\beta_i^*(1,0) \equiv \beta_i^*(0,1)$, $i = 1, 2$, $f(1, 0 \mid \beta) \equiv f(0, 1 \mid \beta)$, for any β, and $\Pi^*(1,0) = \Pi^*(0,1)$.

where

$$\begin{aligned}\Pi^*(\mathbf{I}) &\equiv E_\theta[M(\theta,\mathbf{I})] - (I_1+I_2)F \\ &\quad + \sum_{i=1}^{2}\left[\Phi_i(\beta_i^*(\mathbf{I})) - \frac{\rho_i}{8}(\beta_i^*(\mathbf{I}))^2 \cdot Var(M(\theta,\mathbf{I}))\right]\end{aligned}$$

describes the principal's expected payoff, with

$$\beta_i^*(\mathbf{I}) \in \arg\max_{\beta_i} \left\{\Phi_i(\beta_i) - \frac{\rho_i}{8}\beta_i^2 \cdot Var(M(\theta,\mathbf{I}))\right\}$$

as the optimal PPS conditional on a given (contracted) investment profile.

We now return to the setting with non-contractible investments and consider two organizational modes. Under the "*IC-IC*" mode, each division is organized as an investment center and each manager's performance is measured on the basis of profit net of divisional fixed costs (superscript "I" for *IC-IC*):

$$\pi_i^I = a_i + \widetilde{\varepsilon}_i + \frac{M(\theta,\mathbf{I})}{2} - FI_i, \quad i = 1,2.$$

Under the "*PC-IC*" mode, one division manager, say, k, is given authority to choose both I_1 and I_2. The other manager, $l \neq k$, is evaluated solely based on contribution margin. The respective performance measures thus read (superscript "P" for *PC-IC*):

$$\pi_i^{P\_k} = a_i + \widetilde{\varepsilon}_i + \frac{M(\theta,\mathbf{I})}{2} - (I_1+I_2)F\mathbb{1}_{ki},$$

where $\mathbb{1}_{ki} \in \{0,1\}$ is an indicator that takes the value of 1 if and only if $i = k$.

In the following, for each organizational mode, we describe how the optimal PPS and equilibrium investments vary in the *ex-ante* profitability of investments, measured by F. We then conduct the the performance comparison.

5.1 Both Divisions are Investment Centers (*IC-IC* mode)

Consider first the *IC-IC* mode, i.e., each division chooses (and gets charged for in its P&L) its own investment. The investment choices must form a (pure-strategy)

Nash equilibrium. In prior incomplete-contracting transfer pricing models, divisional investments are strategic complements. Ignoring any transfer-related risk, if by investing more the seller lowers his variable cost, the expected trading quantity will increase, which in turn raises the marginal return to the buyer's investment in revenue enhancement. On the other hand, the trade-related variance (and thus the risk premium) in (15) also has increasing differences in I_1 and I_2. This effect in isolation would make divisional investments strategic substitutes. To evaluate this tradeoff, denote $\overline{\sigma}_\varepsilon \equiv \max_i \sigma_{\varepsilon_i}$, $\underline{\sigma}_\varepsilon \equiv \min_i \sigma_{\varepsilon_i}$ and the corresponding PPS under contractible investments, respectively, by $\underline{\beta}^*(\mathbf{I}) \equiv \min_i \beta_i^*(\mathbf{I})$ and $\bar{\beta}^*(\mathbf{I}) \equiv \max_i \beta_i^*(\mathbf{I})$, for any $\mathbf{I}$. (Given IMH$'$, the division manager facing greater operating uncertainty will have the lower PPS.)

Lemma 3 *For $\underline{\sigma}_{\varepsilon_i}$ sufficiently high, the function $f(I_1, I_2 \mid \beta_i)$, as defined in (15), has strictly increasing differences in I_1 and I_2, for $\beta_i = \bar{\beta}_i^*(1,1)$.*

The proof follows immediately from inspection of (15).[9] Lemma 3 relies on the fact that, for high levels of operating uncertainty, the equilibrium PPS will be small. The strategic complementarity of the expected contribution margin then is the dominant force. Lemma 3 readily extends to values of $\beta < \bar{\beta}^*(1,1)$. Note that in any Nash equilibrium involving $\mathbf{I} = (1,1)$, the optimal PPS's are bounded from above by $\bar{\beta}^*(1,1)$.

We now turn to the simultaneous move investment game between the managers, for any given PPS $\beta_i = \bar{\beta}$ and $\beta_j = \underline{\beta} \leq \bar{\beta}$. The investment profile

[9] As β_i in (15) becomes small, the strict supermodularity in I_1 and I_2 that is inherent in the shared contribution margin dominates the decreasing differences inherent in the risk premium term. At the same time, in any Nash equilibrium under the IC-IC mode that supports $\mathbf{I} = (1,1)$ it is never optimal for the principal to set $\beta_i > \beta^*(1,1)$ where (given IMH$'$):

$$\beta_i^*(1,1) = \frac{1}{1 + \rho v \left[\sigma_{\varepsilon_i}^2 + \frac{Var(M(\theta,1,1))}{4}\right]}.$$

Since $\beta_i^*(1,1)$ is decreasing in σ_{ε_i}, Lemma 3 follows.

$\mathbf{I} = (1, 1)$ then constitutes a Nash equilibrium, holding fixed the benchmark PPS, whenever the fixed investment cost is low enough such that

$$f(1, 1 \mid \bar{\beta}) - \bar{\beta}F \geq f(0, 1 \mid \bar{\beta}). \tag{17}$$

At the same time, $(0, 0)$ is a Nash equilibrium if F is high enough such that

$$f(1, 0 \mid \underline{\beta}) - \underline{\beta}F \leq f(0, 0 \mid \underline{\beta}). \tag{18}$$

Note that (17) is cast in terms of the manager with the higher PPS, $\bar{\beta}$, as he will internalize a greater share of the incremental trade-related risk premium. Conversely, (18) is cast in terms of the manager with the lower PPS, $\underline{\beta}$, as it is this manager who is more eager to break away from $\mathbf{I} = (0, 0)$.

Games of strategic complementarity are routinely afflicted by multiple equilibria, and this is true also for the *IC-IC* mode. The following result however shows that, whenever multiple equilibria exist, the desired one that has both managers investing Pareto-dominates (for the two division managers) the equilibrium that entails no investments.

Lemma 4 *Consider the* IC-IC *mode under Condition IMH′ with PPS* $\underline{\beta}$ *and* $\bar{\beta}$*. For* $\bar{\beta} - \underline{\beta}$ *sufficiently small, there exist fixed cost parameters,* F*, for which* $\mathbf{I} = (1, 1)$ *and* $\mathbf{I} = (0, 0)$ *each are Nash equilibrium investment profiles, but then* $(1, 1)$ *is the Pareto-dominant one. For* $\bar{\beta} - \underline{\beta}$ *large, if* $(1, 1)$ *is a Nash equilibrium (i.e., (17) holds), then it is the unique one.*

Lemma 4 allows us to disregard the no-investment equilibrium as long as $\mathbf{I} = (1, 1)$ constitutes an equilibrium. It is straightforward then to generalize Proposition 2 to the bilateral investment case under the *IC-IC* mode (in the presence of severe moral hazard). First, given the supermodularity property of the investment game (by Lemma 3), Lemma 2 carries over directly to bilateral investments: each manager's investment incentives are decreasing in his PPS, *c.p.*. Denote by F^I the threshold fixed cost level up to which the benchmark solution of $\mathbf{I} = \mathbf{I}^* = (1,1)$ can be implemented without having to deviate from the benchmark PPS (superscript "I" for *IC-IC*):

$$F^I \equiv \frac{f(1,1 \mid \bar{\beta}) - f(0,1 \mid \bar{\beta})}{\bar{\beta}}. \tag{19}$$

Put differently, (17) becomes binding at F^I for the manager with the higher PPS.

By Lemma 2, to induce bilateral investments for $F > F^I$, the principal needs to reduce the higher PPS just enough to make (17) binding. Denote the resulting PPS by $\bar{\beta}^I(F)$. By Lemma 2, $\bar{\beta}^I(F)$ is a decreasing function. At fixed cost levels for which $\bar{\beta}^I(F) = \underline{\beta}^*(1,1)$, the principal needs to start also reducing the lower PPS for the manager who initially faced a lower PPS in lockstep, i.e., $\underline{\beta}^I(F) \equiv \bar{\beta}^I(F) \equiv \beta^I(F)$. At the critical level $\hat{F}^I$, the PPS distortions required to ensure $\mathbf{I} = (1,1)$ is an equilibrium investment profile become too costly and the principal instead induces $\mathbf{I} = (0,0)$ by raising each manager's PPS to $\beta_i^*(0,0)$.

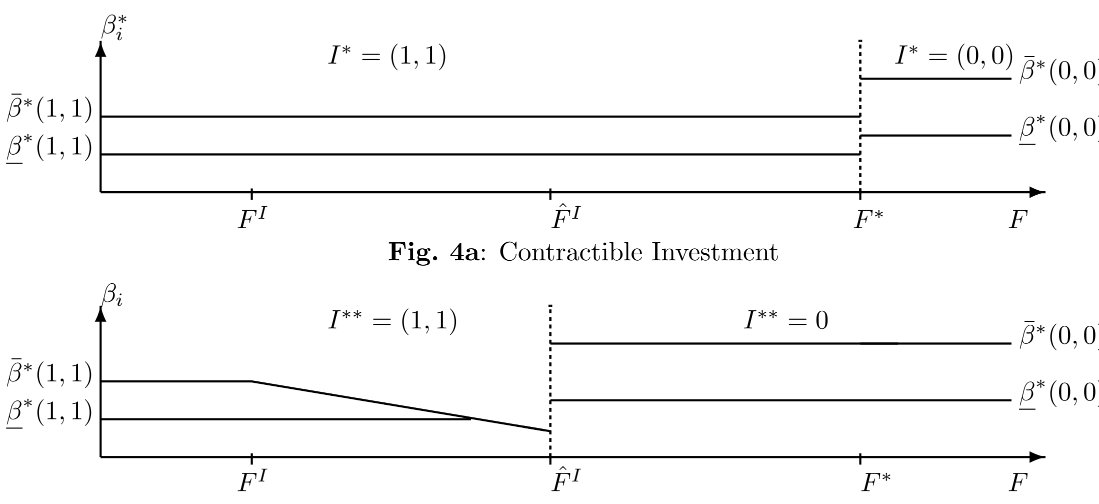

Fig. 4a: Contractible Investment

Fig. 4b: Non-Contractible Investment

Figure 4: Optimal PPS and Induced Investments Under *IC-IC*

Rather than formally stating a version of Proposition 2 modified for bilateral investments under the *IC-IC* mode, we depict the optimal PPS and resulting investments graphically in Figure 4.

5.2 Concentrating Investment Authority Within One Division (*PC-IC* mode)

We now turn to the *PC-IC* mode in which the managers are treated asymmetrically: one manager — the "investment center manager" — is given authority to choose both up- and downstream investments. Before addressing the issue of optimal incentives, the principal needs to decide which of the two division managers to grant this authority. Given that the pressing investment distortion in this section is, by assumption, underinvestment (due to high operating uncertainty), authority should be assigned to the manager whose division has greater operating uncertainty. By Lemma 2, this manager is more willing to invest because, all else equal, he faces a lower PPS and is therefore less sensitive to the incremental trade-related risk premium as a result of investing.

The investment center manager, facing a PPS of $\underline{\beta}$, will choose investments according to (superscript "P" for *PC-IC*):

$$\mathbf{I}^P(\underline{\beta} \mid F) \in \arg\max_{\mathbf{I}\in\{0,1\}^2} \left\{ f(\mathbf{I} \mid \underline{\beta}) - \sum_i I_i F \right\}. \tag{20}$$

By Lemma 3, only $\mathbf{I} = (0,0)$ or $\mathbf{I} = (1,1)$ are candidates for optimality.[10] It is also immediate to see that condition (17), by which $(1,1)$ was a Nash equilibrium under the *IC-IC* mode, is sufficient for the investment center manager under the

[10] To see that a "mixed" investment profile, (1,0) or (0,1) (they are economically identical in this setting), can never be optimal under *PC-IC*, suppose to the contrary that it is. Then:

$$\begin{aligned} f(1,0 \mid \underline{\beta}) - \underline{\beta}F \geq f(0,0 \mid \underline{\beta}) \quad &\iff \quad F \leq [f(1,0 \mid \underline{\beta}) - f(0,0 \mid \underline{\beta})]/\underline{\beta}, \\ f(1,0 \mid \underline{\beta}) - \underline{\beta}F \geq f(1,1 \mid \underline{\beta}) - 2\underline{\beta}F \quad &\iff \quad F \geq [f(1,1 \mid \underline{\beta}) - f(1,0 \mid \underline{\beta})]/\underline{\beta}. \end{aligned}$$

By Lemma 3, however, $f(1,1 \mid \underline{\beta}) - f(1,0 \mid \underline{\beta}) > f(1,0 \mid \underline{\beta}) - f(0,0 \mid \underline{\beta})$, a contradiction.

PC-IC to prefer $(1,1)$ over $(1,0)$, as he incurs a lower trade-related risk premium. More generally, the *PC-IC* mode benefits from the fact that it is the manager facing lower-powered incentives who makes all investment decisions. On the other hand, this manager's P&L now will be charged for the entire fixed costs of $\sum_i I_i F$, as opposed to only his own divisional fixed cost under *IC-IC*. This effect, which we label the "double whammy" effect, exacerbates the hold-up problem.

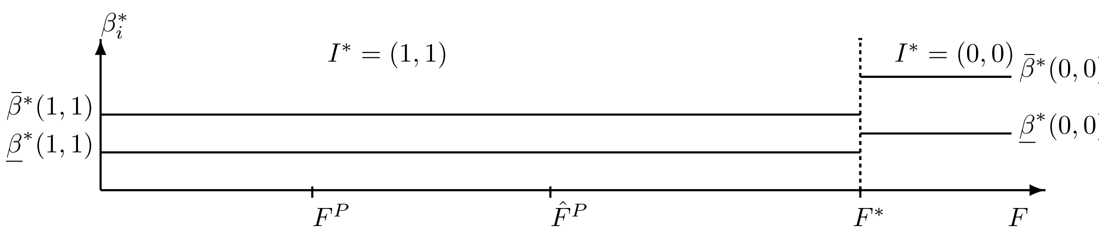

Fig. 5a: Contractible Investment

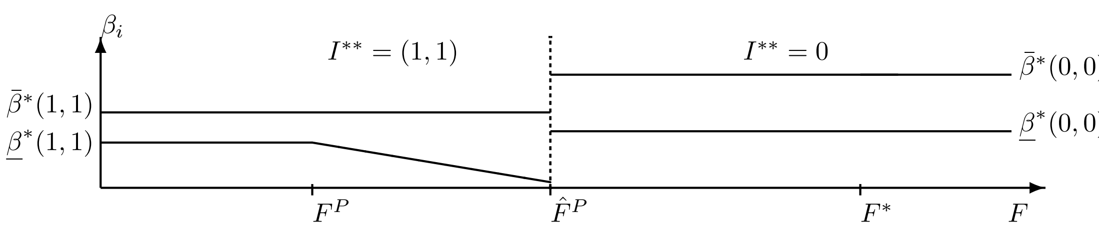

Fig. 5b: Non-Contractible Investment

Figure 5: Optimal PPS and Induced Investments Under *PC-IC*

As in Section 5.1, we describe how investments and PPS vary in F. Since this is a direct generalization of Proposition 2, we summarize the findings in Figure 5 and omit a formal result. For fixed costs below a threshold F^P:

$$F^P \equiv \frac{f(1,1 \mid \underline{\beta}) - f(0,0 \mid \underline{\beta})}{2\underline{\beta}}, \tag{21}$$

the contractible benchmark solution is achieved without adjustment to PPS, i.e., the investment center manager chooses $\mathbf{I}^* = (1,1)$ facing $\underline{\beta}^*(1,1)$. Again, generalizing Lemma 2 to the *PC-IC* mode, shows that to induce investment for

$F > F^P$, the principal needs to reduce the investing manager's PPS. Denote the resulting PPS by $\underline{\beta}^P(F)$. By Lemma 2, $\underline{\beta}^P(F)$ is a decreasing function. As long as $\mathbf{I} = (1,1)$ is induced, the profit center manager's PPS remains equal to $\bar{\beta}^*(1,1)$. Once fixed costs reach the critical level $\hat{F}^P$, the distortion in PPS for the investing manager necessary to induce $\mathbf{I} = (1,1)$ becomes too costly and the principal is better off foregoing the investment. In that case, each division manager's PPS jumps up to $\beta_i = \beta_i^*(0,0),\ i = 1,2$.

5.3 Comparing the Organizational Modes

We now turn to a performance comparison of the two alternative ways of allocating investment authority within the firm, *IC-IC* versus *PC-IC*. Evaluating the principal's expected payoff under these organizational modes, we find:

Proposition 4 *Given Condition IMH', σ_{ε_i} sufficiently large, $i = 1,2$, and (16) holds (i.e., $F \leq F^*$):*

(i) For $\Delta\sigma_\varepsilon \equiv |\sigma_{\varepsilon_i} - \sigma_{\varepsilon_j}|$ sufficiently small, the principal prefers IC-IC*.*

(ii) If $\Delta\sigma_\varepsilon$ is sufficiently large and $R(\cdot)$ sufficiently concave in q, then the principal prefers PC-IC*.*

The intuition for this result can best be grasped by directly contrasting the relevant investment incentive constraints under the two modes, holding constant the PPS. We refer back to two key earlier observations. As argued above, if $\mathbf{I} = (1,1)$ is a Nash equilibrium under *IC-IC*, then: (i) the principal can rest assured the agents will play this equilibrium (even if is not unique); and (ii) the investment center manager under *PC-IC* also will prefer $\mathbf{I} = (1,1)$ to investing in only one of the two divisions. However, because of the double whammy effect, the investment center manager under *PC-IC* may actually prefer $\mathbf{I} = (0,0)$. Now, the constraint ensuring $(1,1)$ is a Nash equilibrium under *IC-IC* reads

$$\begin{aligned} F &\leq \frac{f(1,1 \mid \bar{\beta}) - f(1,0 \mid \bar{\beta})}{\bar{\beta}} \\ &\equiv F^I(\bar{\beta}), \end{aligned} \tag{22}$$

whereas that ensuring the investment center manager under *PC-IC* prefers $\mathbf{I} = (1,1)$ to $\mathbf{I} = (0,0)$:

$$\begin{aligned} F &\leq \frac{f(1,1 \mid \underline{\beta}) - f(0,0 \mid \underline{\beta})}{2\underline{\beta}} \\ &= \frac{f(1,1 \mid \underline{\beta}) - f(1,0 \mid \underline{\beta})}{2\underline{\beta}} + \frac{f(1,0 \mid \underline{\beta}) - f(0,0 \mid \underline{\beta})}{2\underline{\beta}} \\ &= F^P(\underline{\beta}). \end{aligned} \tag{23}$$

By strategic complementarity of investments (Lemma 3), $F^P(\beta) < F^I(\beta)$, for given β. That is, if the managers face (roughly) similar PPS, because their divisions operate under similar operating risks, then the *IC-IC* mode generates stronger investments incentives in aggregate. The reason is that ensuring high levels of complementary inputs in form of a Nash equilibrium is "cheap" and the principal need not worry about the undesired $(0,0)$ equilibrium. In contrast, the "double whammy" effect under the *PC-IC* makes the investment center manager less willing to invest. As we show in the proof, the logic extends beyond a simple ranking of F^I and F^P. Hence, if manager face (roughly) similar operating risks, *IC-IC* is the preferred mode.

On the other hand, the benefit of concentrating investment authority in the hand of one division manager is that the principal can take advantage of the PPS/investments link highlighted above. If $\Delta\sigma_\varepsilon$ is high, then by assigning investment authority to the manager whose division is more volatile (who hence has a lower PPS, *c.p.*), the underinvestment problem can be mitigated. If at the same time the degree of strategic complementarity is limited, then the risk effect will weigh more heavily. An important determinant of the strategic complementarity between investments is the curvature of the firm's revenue function, $R(\cdot)$. The more concave revenues are in q, the smaller is

$$(f(1,1 \mid \beta) - f(1,0 \mid \beta)) - \left(f(1,0 \mid \underline{\beta}) - f(0,0 \mid \underline{\beta})\right),$$

for any β. Therefore, if the divisions face sufficiently different volatility and, at the same time, the revenue function for the final product is highly concave, then the risk effect outweighs the double whammy effect and makes *PC-IC* the preferred mode for the principal (part (ii) of Proposition 4).

In sum, the horse race between an "equitable" organizational mode and one that concentrates all investment decision authority in the hands of one division is jointly decided by two forces: the relative volatility in the divisions' operating environments and the demand (or market) structure in the final product market.

6 Concluding Remarks

This paper has studied how the PPS for divisional managers affects their incentives to invest and engage in intrafirm trade. We find that introducing a wedge between division managers' PPS can increase shareholder value. The reason is that investments in fixed assets add compensation risk by increasing the expected level of trade. Therefore, muting (increasing) the incentives of the manager who has an investment opportunity, relative to that of other managers in otherwise comparable business units, will raise (decrease) his willingness to invest. Hence, the need to elicit relationship-specific investments to foster intrafirm trade may introduce a wedge in the PPS of managers of otherwise similar divisions.

The PPS/investment link also has implications for organizational design. Some business units in practice are designated as profit centers, without investment authority, others as investment centers with PP&E charges netted against contribution margin. We show that treating divisions asymmetrically — i.e., by designating one as an investment center in charge of choosing investments for both divisions engaged in a transaction and the other as a profit center — may be beneficial, in particular in cases where the divisions face significantly different operating risks. In that case, the manager facing greater operating risk should be given investment authority as for him the incremental trade-related risk premium will be relatively small (because of low-powered incentives). On the other hand, if the divisions face rather similar operating risks, then it is preferable to allocate investment authority evenly between the managers, i.e., designating each division as its own investment center, because that way the overall hold-up problem is reduced.

Appendix

Proof of Lemma 1: Follows from (6) and (7), together with the fact that $Var(M(\theta, I))$ is increasing in I. To see that $Var(M(\theta, I))$ is increasing in I start by noting that $Var(M(\theta, I)) = E_\theta[M(\theta, I)^2] - (E_\theta[M(\theta, I)])^2$, by definnition. Therefore,

$$\begin{aligned}\frac{\partial Var(M(\theta, I))}{\partial I} &= E_\theta\left[2M(\theta, I) \cdot M_I(\theta, I)\right] - 2E_\theta[M(\theta, I)] \cdot E_\theta\left[M_I(\theta, I)\right] \\ &= 2Cov\left(M(\theta, I), M_I(\theta, I)\right) \qquad (24)\end{aligned}$$

Applying the Envelope Theorem we note that $M_I(\theta, I) = M_{\theta_1}(\theta, I) = M_{\theta_2}(\theta, I) = E_\theta[q^*(\theta, I)] > 0$ and further observe that all second and third partial and cross-partial derivatives of $M(\theta, I)$ are non-negative and conclude that $M(\theta, I)$, $M_I(\theta, I)$ and $M_{II}(\theta, I)$ are monotonic increasing functions. It follows that for any $\theta^o < \theta^{oo}$, $M(\theta^o, I) < M(\theta^{oo}, I)$ and $M_I(\theta^o, I) < M_I(\theta^{oo}, I)$. This in turn demonstrates that $Cov(M(\theta, I), M_I(\theta, I)) > 0$ and therefore, $\frac{\partial Var(M(\theta, I))}{\partial I} > 0$. This completes the proof of Lemma 1.

Proof of Lemma 2: Denoting the (non-effort related part of the) selling agent's objective in (9) by $\hat{\pi}_1$, the necessary first-order condition associated with (9) reads:

$$\frac{\partial \hat{\pi}_1}{\partial I} = \beta_1 \left(\frac{E_\theta[q^*(\theta, I)]}{2} - F'(I) - \frac{\rho_1}{8}\beta_1 \frac{\partial Var(M(\theta, I))}{\partial I} \right) = 0. \qquad (25)$$

The second derivative of $\hat{\pi}_1$ with respect to I at the optimal investment choice $I(\beta_1)$ and the optimal quantity $q^*(\theta, I(\beta_1))$ is given by:

$$\frac{\partial^2 \hat{\pi}_1}{\partial I^2} = \beta_1 \left(\frac{1}{2} \frac{\partial E_\theta[q^*(\theta, I(\beta_1))]}{\partial I} - F''(I(\beta_1)) - \frac{\rho_1}{8}\beta_1 \frac{\partial^2 Var(M(\theta, I(\beta_1)))}{\partial I^2} \right). \qquad (26)$$

Since $F(\cdot)$ is assumed sufficiently convex, to show that (26) is negative we need to verify that the second derivative with respect to I of the variance of the trade surplus at the optimal quantity is positive. This is indeed true for any I because:

$$\begin{aligned}
\frac{\partial^2 Var(M(\theta,I))}{\partial I^2} &= 2E_\theta\left[M_I(\theta,I)\cdot M_I(\theta,I)\right] + 2E_\theta\left[M(\theta,I)\cdot M_{II}(\theta,I)\right] \\
&\quad -2E_\theta\left[M_I(\theta,I)\right]\cdot E_\theta\left[M_I(\theta,I)\right] - 2E_\theta\left[M(\theta,I)\right]\cdot E_\theta\left[M_{II}(\theta,I)\right] \\
&= 2Cov\left(M(\theta,I), M_{II}(\theta,I)\right) + 2Var(M_I(\theta,I)). \qquad (27)
\end{aligned}$$

To see that $\frac{\partial^2 Var(M(\theta,I))}{\partial I^2} > 0$ note that $Var(M_I(\theta,I)) > 0$, because $M_{I\theta}(\theta,I) > 0$ and that $Cov(M(\theta,I), M_{II}(\theta,I)) \geq 0$, because both $M(\theta,I)$ and $M_{II}(\theta,I)$ are monotonic increasing functions as shown in the Proof of Lemma 1. It follows that $\frac{\partial^2 \hat{\pi}_1}{\partial I^2} < 0$

By the Implicit Function Theorem, to show that the optimal investment choice $I(\beta_1)$ is non-increasing in β_1, we need to verify that $\frac{\partial^2 \hat{\pi}_1}{\partial I \partial \beta_1} \leq 0$ at the optimal investment choice $I(\beta_1)$ and the optimal quantity $q^*(\theta, I(\beta_1))$. That however follows immediately from (25) because

$$\frac{\partial^2 \hat{\pi}_1}{\partial I \partial \beta_1} = \frac{E_\theta[q^*(\theta, I(\beta_1))]}{2} - F'(I(\beta_1)) - \frac{\rho_1}{4}\beta_1 \frac{\partial Var(M(\theta, I(\beta_1)))}{\partial I},$$

together with β_1, ρ_1, and $\frac{\partial Var(M(\theta,I(\beta_1)))}{\partial I}$ all being positive.

Proof of Proposition 1:

Part (i) If the investment is contractible, the first-order derivative of the principal's objective function with respect to investment I^* is

$$g^*(I \mid \beta_i^*) \equiv E_\theta[q^*(\theta,I)] - F'(I) - \frac{\sum_i \rho_i (\beta_i^*)^2}{8} \cdot \frac{\partial Var(M(\theta,I))}{\partial I}. \qquad (28)$$

With non-contractible investments, Manager 1's marginal benefit from increasing I equals

$$\begin{aligned}
g^{**}(I \mid \beta_1) &\equiv f'(I \mid \beta_1) - \beta_1 F'(I) \\
&= \beta_1 \left(\frac{E_\theta[q^*(\theta,I)]}{2} - F'(I) - \frac{\rho_1 \beta_1}{8} \cdot \frac{\partial Var(M(\theta,I))}{\partial I} \right). \qquad (29)
\end{aligned}$$

A sufficient condition for $I^* \geq I^{**}(\beta_1)$ is that the expression in (28) exceeds that in (29) for any β_1. Since $\beta_1 \leq 1$,

$$\begin{aligned} g^*(I \mid \beta_i^*) - g^{**}(I \mid \beta_1) &\geq \frac{E_\theta[q^*(\theta, I)]}{2} - \frac{1}{8} \cdot \frac{\partial Var(M(\theta, I))}{\partial I} \left(\sum_i \rho_i (\beta_i^*)^2 - \beta_1 \right) \\ &\geq \frac{E_\theta[q^*(\theta, I)]}{2} - \frac{1}{8} \cdot \frac{\partial Var(M(\theta, I))}{\partial I} \sum_i \rho_i (\beta_i^*)^2 \\ &\geq 0, \end{aligned}$$

for $\sigma^2_{\varepsilon_i}$ sufficiently high, as then $\beta_i^* \to 0$ while $q^*(\cdot)$ remains bounded away from zero.

It remains to rank the bonus coefficients for Manger 2. Given that $I^* \geq I^{**}(\beta_1)$, for any β_1, and that $\frac{\partial Var(M(\theta,I))}{\partial I} > 0$, it is straightforward to see that $\beta_2^{**} \geq \beta_2^*$.

Part (ii) A sufficient condition for $I^* \leq I(\beta_1)$ is that the expression in (28) is lower than that in (29) for any β_1. Since $\beta_1 \leq 1$ and since under IMH assumption, $\beta_1^* = \beta_2^* = \beta^*$,

$$\begin{aligned} g^*(I \mid \beta^*) - g^{**}(I \mid \beta_1) &= \left(1 - \frac{\beta_1}{2}\right) E_\theta[q^*(\theta, I)] - (1 - \beta_1) F'(I) \\ &\quad - \frac{1}{8} \cdot \frac{\partial Var(M(\theta, I))}{\partial I} \left(\sum_i \rho_i (\beta_i^*)^2 - \rho_1 \beta_1^2 \right) \\ &\leq \frac{E_\theta[q^*(\theta, I)]}{2} - \frac{\rho}{8} \cdot \frac{\partial Var(M(\theta, I))}{\partial I} \left(2(\beta^*)^2 - 1 \right). \end{aligned}$$

Next, we use Taylor expansions to express $Var(M(\theta, I))$ and find

$$\frac{\partial Var(M(\theta, I))}{\partial I} \approx 2q^*(\mu, I) \frac{\partial q^*(\mu, I)}{\partial I} \sum_i \sigma^2_{\theta_i}.$$

Now note that $\frac{\partial q^*(\mu,I)}{\partial I} > 0$ and $E_\theta[q^*(\theta, I)] = q^*(\mu, I)$ Hence,

$$\begin{aligned} g^*(I \mid \beta^*) - g^{**}(I \mid \beta_1) &\leq \frac{E_\theta[q^*(\cdot)]}{2} \left[1 - \frac{\rho}{2} \frac{\partial q^*(\cdot)}{\partial I} \sum_i \sigma^2_{\theta_i} \left(2(\beta^*)^2 - 1 \right) \right] \quad (30) \\ &\leq 0, \end{aligned}$$

if v sufficiently low and $\sum_i \sigma^2_{\theta_i}$ sufficiently high. To see this note that as $v \to 0$, $\beta^* \to 1$ and $g^*(I \mid \beta^*) - g^{**}(I \mid \beta_1) \to \frac{E_\theta[q^*(\theta,I)]}{2}\left[1 - \frac{\rho}{2}\frac{\partial q^*(\mu,I)}{\partial I}\sum_i \sigma^2_{\theta_i}\right] \le 0$ for $\sum_i \sigma^2_{\theta_i}$ sufficiently high.

Given that $I^* \le I^{**}(\beta_1)$, for any β_1, and that $\frac{\partial Var(M(\theta,I))}{\partial I} > 0$, it is straightforward to see that $\beta_2^{**} \le \beta_2^*$.

Alternatively:

A necessary condition for (31) to be negative is that $2(\beta^*)^2 - 1 > 0$. This is satisfied when $v < v^*(\rho, \sigma_\varepsilon, \sigma_\theta)$, where $v^*(.)$ satisfies:

$$\frac{1}{(1 + \rho v^*(\sigma^2_\varepsilon + (q^*(\mu, I)^2)\sum_i \sigma^2_{\theta_i}))^2} = \frac{1}{2}$$

A sufficient condition for (31) to be negative is that $1 - \frac{\rho}{2}\frac{\partial q^*(\mu,I)}{\partial I}\sum_i \sigma^2_{\theta_i}(2(\beta^*)^2 - 1) < 0$, which is true for $\sum_i \sigma^2_{\theta_i} > s^*(\rho, \sigma_\varepsilon, v)$, where $s^*(.)$ satisfies:

$$\frac{\rho}{2}\frac{\partial q^*(\mu, I)}{\partial I}s^*\left(2\left[\frac{1}{(1 + \rho v(\sigma^2_\varepsilon + (q^*(\mu, I)^2)s^*))^2}\right] - 1\right) = 1$$

Note that $\frac{\partial v^*(.)}{\partial \sum_i \sigma^2_{\theta_i}} < 0$ and $\frac{\partial s^*(.)}{\partial v} > 0$.

Proof of Proposition 1′:

Part (i) We need to show that $D \equiv F^* - F^{**}(\beta_1^*(1)) > 0$.

$$
\begin{aligned}
D &= \frac{\Delta M}{2} + \sum_i \Delta\Phi_i \\
&\quad -\frac{\rho_1}{8}\left[\left((\beta_1^*(1))^2 - \beta_1^*(1)\right)\cdot Var(M(\theta,1)) - \left((\beta_1^*(0))^2 - \beta_1^*(1)\right)\cdot Var(M(\theta,0))\right] \\
&\quad -\frac{\rho_2}{8}\left[(\beta_2^*(1))^2\cdot Var(M(\theta,1)) - (\beta_2^*(0))^2\cdot Var(M(\theta,0))\right] \\
&> \frac{\Delta M}{2} + \sum_i \Delta\Phi_i \\
&\quad -\frac{\rho_1}{8}\left[\left((\beta_1^*(1))^2 - \beta_1^*(1)\right)\cdot Var(M(\theta,1)) - \left((\beta_1^*(1))^2 - \beta_1^*(1)\right)\cdot Var(M(\theta,0))\right] \\
&\quad -\frac{\rho_2}{8}\left[(\beta_2^*(1))^2\cdot Var(M(\theta,1)) - (\beta_2^*(1))^2\cdot Var(M(\theta,0))\right] \\
&= \frac{\Delta M}{2} + \sum_i \Delta\Phi_i \\
&\quad -\frac{\rho_1}{8}\left((\beta_1^*(1))^2 - \beta_1^*(1)\right)\left[Var(M(\theta,1)) - Var(M(\theta,0))\right] \\
&\quad -\frac{\rho_2}{8}(\beta_2^*(1))^2\left[Var(M(\theta,1)) - Var(M(\theta,0))\right] \\
&> 0,
\end{aligned}
$$

for $\sigma^2_{\varepsilon_i}$ sufficiently high, as then $\beta_i^*(I) \to 0,\ I = 0, 1$, and $\Delta\Phi_i \to 0$ while ΔM remains bounded away from zero.

Part (ii) We need to show that $D \equiv F^* - F^{**}(\beta_1^*(0)) < 0$. Using Taylor expansions to express $Var(M(\theta, I))$ and the fact that, given IMH, $\beta_1^*(I) = \beta_2^*(I) =$

$\beta^*(I)$,

$$\begin{aligned} D &= \frac{\Delta M}{2} + \sum_i \Delta\Phi_i \\ &\quad -\frac{\rho}{8} Var(M(\theta,1)) \left[\sum_i \beta_i^*(1))^2 - \beta_1^*(0) \right] \\ &\quad +\frac{\rho}{8} Var(M(\theta,0)) \left[\sum_i \beta_i^*(0))^2 - \beta_1^*(0) \right] \\ &= \frac{\Delta M}{2} + \sum_i \Delta\Phi_i \\ &\quad -\frac{\rho}{8} \sum_i \sigma_{\theta_i}^2 \left[(q^*(\mu,1))^2 \left(2\beta^*(1))^2 - \beta^*(0) \right) \right] \\ &\quad +\frac{\rho}{8} \sum_i \sigma_{\theta_i}^2 \left[(q^*(\mu,0))^2 \left(2\beta^*(0))^2 - \beta^*(0) \right) \right] \\ &< 0, \end{aligned}$$

if v sufficiently low and $\sum_i \sigma_{\theta_i}^2$ sufficiently high.

To see this note that as $v \to 0$, $\beta_i(I) \to 1$ and hence, $D \to \frac{\Delta M}{2} + \sum_i \Delta\Phi_i - \frac{\rho}{8} \sum_i \sigma_{\theta_i}^2 \left[(q^*(\mu,1))^2 - (q^*(\mu,0))^2 \right] < 0$ for $\sigma_{\theta_i}^2$ sufficiently high. Note also that $\Delta\Phi_i < 0$. This completes the proof of Proposition 1′.

Proof of Proposition 2: Parts (i) and (iv) are obvious, so we only prove here parts (ii) and (iii). Denote by Ψ^* the value of program $\mathcal{P}^*$ and by $\Psi^{**(I)}$ the value of program $\mathcal{P}^{**(I)}$, for $I \in \{0,1\}$ and $F(I) = FI$. Thus, for any $F \in [F^{**}(\beta_1^*(1)), F^*]$:

$$\begin{aligned} \Psi^*(F) &= \Pi(\beta^*(1), I = 1 \mid F) \\ \Psi^{**(1)}(F) &= \Pi(\beta_1^{**}(F), \beta_2^*(1), I = 1 \mid F) \\ \Psi^{**(0)} &= \Pi(\beta^*(0), I = 0) \end{aligned}$$

Begin by considering fixed cost values $F = F^{**}(\beta_1^*(1)) + \varepsilon$ for $\varepsilon \to 0$. Then, $\Psi^{**(1)}(F) = \Psi^*(F) - \delta,\ \delta \to 0$, because $\beta_1^{**}(F)$ is a continuous function of F and

$\Pi(\beta_1^{**}(F), \beta_2^*(1), I = 1 \mid F)$ is continuous in both β_1 and F. That is, the value of program $\mathcal{P}^{**(1)}$ converges to that of the benchmark program $\mathcal{P}^*$ (with contractible investment) as ε becomes small. At the same time, $\Psi^{**(0)}$ is bounded away from $\Psi^*(F)$ for F close to $F^{**}(\beta_1^*(1))$. This holds because, by Proposition 1′, $F^{**}(\beta_1^*(1))$ is strictly less than F^*, together with the observations that at F^* we have $\Psi^{**(0)} = \Psi^*(F)$ (by definition of F^*, $\Pi(\beta^*(1), I = 1 \mid F) = \Pi(\beta^*(0), I = 0)$) and that $\Psi^*(F)$ is monotonically decreasing in F. Thus, we have shown that for $F \downarrow F^{**}(\beta_1^*(1))$, $\Psi^{**(1)}(F) > \Psi^{**(0)}$, whereas for $F \uparrow F^*$, $\Psi^{**(1)}(F) < \Psi^{**(0)}$.

Lastly, since $\Psi^{**(1)}(F)$ is monotonically decreasing in F whereas $\Psi^{**(0)}$ is independent of F, it follows that there exists a unique indifference value $\hat{F}$ at which $\Psi^{**(1)}(\hat{F}) = \Psi^{**(0)}$. This completes the proof of parts (ii) and (iii).

Proof of Proposition 3: The proof follows a logic similar to that of Proposition 2. Again, we only prove parts (ii) and (iii), because parts (i) and (iv) are obvious. Denote by Υ^* the value of program $\mathcal{P}^*$ and by $\Upsilon^{**(I)}$ the value of program $\mathcal{P}^{**(I)}$, for $I \in \{0, 1\}$ and $F(I) = FI$. For any $F \in [F^*, F^{**}(\beta_1^*(0))]$:

$$\begin{aligned}
\Upsilon^* &= \Pi(\beta^*(0), I = 0) \\
\Upsilon^{**(0)}(F) &= \Pi(\beta_1^{**}(F) + \varepsilon, \beta_2^*(0), I = 0) \\
\Upsilon^{**(1)}(F) &= \Pi(\beta^*(1), I = 1 \mid F)
\end{aligned}$$

Consider $F = F^{**}(\beta_1^*(0)) - \eta$ for $\eta \to 0$. Then, $\Upsilon^{**(0)}(F) = \Upsilon^* - \delta,\ \delta \to 0$, because $\beta_1^{**}(F)$ is continuous in F and $\Upsilon^{**(0)}(F)$ is continuous in β_1. Note that $\Upsilon^{**(1)}(F)$ is lower than Υ^* for F close to $F^{**}(\beta_1^*(0))$ and hence, $F \uparrow F^{**}(\beta_1^*(0))$, $\Upsilon^{**(1)}(F) < \Upsilon^{**(0)}(F)$.

Now note that at F^* we have $\Upsilon^{**(1)}(F) = \Upsilon^*$, because by definition of F^*, $\Pi(\beta^*(1), I = 1 \mid F) = \Pi(\beta^*(0), I = 0)$. However, $\Upsilon^{**(0)}(F)$ is lower than Υ^*. To see this note that $\Pi(\beta, I = 0)$ is concave and is maximized at $\beta(0)$. Further, recal that $\beta_1(F)$ is decreasing in F and that by Proposition 1′, $F^{**}(\beta_1^*(0))$ is strictly higher than F^*. Hence, $\beta_1(0) = \beta_1(F^{**}) < \beta_1(F^*)$. It is now obvious that for $F \downarrow F^*$, $\Upsilon^{**(1)}(F) > \Upsilon^{**(0)}(F)$. Lastly, there exists a unique indifference value $\hat{F}$ at which $\Upsilon^{**(1)}(\hat{F}) = \Upsilon^{**(0)}$.

Proof of Lemma 4: Rewriting (17), the investment profile (1,1) is a Nash equilibrium investment profile whenever $F \leq F_{11}(\bar{\beta})$, where $F_{11}(\beta_i) \equiv \frac{f(1,1 \ |\beta_i) - f(1,0 \ |\beta_i)}{\beta_i}$. By (18), the no-investment profile (0,0) constitutes a Nash equilibrium whenever $F > F_{00}(\underline{\beta})$, where $F_{00}(\beta_i) \equiv \frac{f(1,0 \ |\beta_i) - f(0,0 \ |\beta_i)}{\beta_i}$.

Recall that $f(I_1, I_2|\beta)$ has increasing differences in (I_1, I_2), as shown in Lemma 3. Hence, as $\bar{\beta}$ and $\underline{\beta}$ converge, so will $F_{11}(\bar{\beta}) \geq F_{00}(\underline{\beta})$. In this case, for any $F \in [F_{11}(\bar{\beta}), F_{00}(\underline{\beta}))$, there exist two pure strategy equilibria, $(0,0)$ and $(1,1)$. However, since $\frac{f(I_1, I_2 \ |\beta_i)}{\beta_i}$ is decreasing in β_i, $(F_{11}(\bar{\beta}) - F_{00}(\underline{\beta}))$ is decreasing in $(\bar{\beta} - \underline{\beta})$. For $(\bar{\beta} - \underline{\beta})$ sufficiently large, $F_{11}(\bar{\beta}) \leq F_{00}(\underline{\beta})$ holds, with the consequence that whenever $(1,1)$ is a Nash equilibrium, it is the unique one.

We now show that if $F_{11}(\bar{\beta}) > F_{00}(\underline{\beta})$, then (1,1) Pareto dominates (0,0) for any $F \in [F_{00}(\underline{\beta}), F_{11}(\bar{\beta})]$. Pareto dominance of (1,1) over (0,0) requires that $f(1,1 \ |\beta_i) - \beta_i F \geq f(0,0 \ |\beta_i)$ for any $\beta_i \in \{\underline{\beta}, \bar{\beta}\}$. Rearranging, we get for the division manager with lower-powered incentives, i.e., $\beta_i = \underline{\beta}$:

$$\begin{aligned}
\frac{f(1,1 \ |\underline{\beta}) - f(0,0 \ |\underline{\beta})}{\underline{\beta}} - F &= \frac{f(1,1 \ |\underline{\beta}) - f(1,0 \ |\underline{\beta})}{\underline{\beta}} + \frac{f(1,0 \ |\underline{\beta}) - f(0,0 \ |\underline{\beta})}{\underline{\beta}} - F \\
&= F_{11}(\underline{\beta}) + F_{00}(\underline{\beta}) - F \\
&> F_{11}(\bar{\beta}) + F_{00}(\underline{\beta}) - F \\
&> 0, \quad \forall \ F \in [F_{00}(\underline{\beta}), F_{11}(\bar{\beta})].
\end{aligned}$$

For the manager with higher-powered incentives, i.e., $\beta_i = \bar{\beta}$:

$$\begin{aligned}
\frac{f(1,1 \ |\bar{\beta}) - f(0,0 \ |\bar{\beta})}{\bar{\beta}} - F &= \frac{f(1,1 \ |\bar{\beta}) - f(1,0 \ |\bar{\beta})}{\bar{\beta}} + \frac{f(1,0 \ |\bar{\beta}) - f(0,0 \ |\bar{\beta})}{\bar{\beta}} - F \\
&= F_{11}(\bar{\beta}) + F_{00}(\bar{\beta}) - F \\
&> 0, \quad \forall \ F \in [F_{00}(\underline{\beta}), F_{11}(\bar{\beta})].
\end{aligned}$$

Each manager is better off under (1,1) than under (0,0), if both investment profiles are Nash equilibria. This completes the proof.

Proof of Proposition 4:

Part (i): To show that IC-IC dominates PC-IC we need to verify the following conditions:

1. $F^P \leq F^I$.

2. $\hat{F}^P \leq \hat{F}^I$.

3. $\Pi^I(\beta, \mathbf{I}) \geq \Pi^P(\beta, \mathbf{I})$ for any $F \in (F^I, \hat{F}^P]$.

Start with Condition 1. In the limit,

$$\begin{aligned}
\lim_{\Delta\sigma_\varepsilon \to 0} F^P &= \lim_{\Delta\sigma_\varepsilon \to 0} \frac{[f(1,1 \mid \underline{\beta}) - f(0,0 \mid \underline{\beta})]}{2\underline{\beta}} \\
&= \frac{[f(1,1 \mid \bar{\beta}) - f(0,0, \mid \bar{\beta})]}{2\bar{\beta}} \\
&= \frac{[f(1,1 \mid \bar{\beta}) - f(1,0 \mid \bar{\beta})]}{2\bar{\beta}} + \frac{[f(1,0 \mid \bar{\beta}) - f(0,0 \mid \bar{\beta})]}{2\bar{\beta}} \\
&\leq F^I, \qquad (31)
\end{aligned}$$

using (19), (21) and Lemma 3.

Now continue with Condition 2. Note that $\hat{F}^P$ is defined by:

$$\Pi^P((1,1), \bar{\beta}^*(1,1), \underline{\beta}^P(F) | \hat{F}^P) = \Pi^P((0,0), \bar{\beta}^*(0,0), \underline{\beta}^*(0,0)) \qquad (32)$$

Denote by

$$\bar{\Phi}(\bar{\beta}) \equiv a(\bar{\beta}) - V(a(\bar{\beta})) - \frac{\rho}{2}\bar{\beta}^2 \underline{\sigma}^2_{\varepsilon_i}$$

the effort-related payoff to the principal from the division facing lower operating uncertainty, and define $\bar{\Phi}(\underline{\beta})$ correspondingly. We can now simplify (32):

$$\begin{aligned}
\hat{F}^P &= E[M(\theta, (1,1)] + \bar{\Phi}(\bar{\beta}^*(1,1)) + \underline{\Phi}(\underline{\beta}^P(F)) \\
&\quad - \frac{\rho}{8}[(\bar{\beta}^*(1,1))^2 + (\underline{\beta}^P(F))^2] Var(M(\theta, (1,1)) \\
&\quad - \Pi^P((0,0), \bar{\beta}^*(0,0), \underline{\beta}^*(0,0)) \qquad (33)
\end{aligned}$$

Note that $\hat{F}^I$ is defined by:

$$\Pi^I((1,1), \bar{\beta}^I(F), \underline{\beta}^I(F) | \hat{F}^I) = \Pi^I((0,0), \bar{\beta}^*(0,0), \underline{\beta}^*(0,0)). \quad (34)$$

This, again, can be simplified to:

$$\begin{aligned} \hat{F}^I &= E[M(\theta,(1,1)] + \bar{\Phi}(\bar{\beta}^I(F)) + \underline{\Phi}(\underline{\beta}^I(F)) \\ &\quad -\frac{\rho}{8}[(\bar{\beta}^I(F))^2 + (\underline{\beta}^I(F))^2]Var(M(\theta,(1,1)) \\ &\quad -\Pi^I((0,0), \bar{\beta}^*(0,0), \underline{\beta}^*(0,0)) \end{aligned} \quad (35)$$

Given that:

$$\begin{aligned} \Pi^P((0,0), \bar{\beta}^*(0,0), \underline{\beta}^*(0,0)) &= \Pi^I((0,0), \bar{\beta}^*(0,0), \underline{\beta}^*(0,0)) \\ &= \Pi^*((0,0), \bar{\beta}^*(0,0), \underline{\beta}^*(0,0)), \end{aligned}$$

to show that $\hat{F}^P \leq \hat{F}^I$, we need to verify that

$$\begin{aligned} &\bar{\Phi}(\bar{\beta}^*(1,1)) + \underline{\Phi}(\underline{\beta}^P) - \frac{\rho}{8}[(\bar{\beta}^*(1,1))^2 + (\underline{\beta}^P)^2]Var(M(\theta,(1,1)) \leq \\ &\bar{\Phi}(\bar{\beta}^I) + \underline{\Phi}(\underline{\beta}^I) - \frac{\rho}{8}[(\bar{\beta}^I)^2 + (\underline{\beta}^I)^2]Var(M(\theta,(1,1)) \end{aligned} \quad (36)$$

Taking into account the optimal effort choices yields $\bar{\Phi}(\beta) = \frac{\beta}{v} - \frac{\beta^2}{2v} - \frac{\rho}{2}\beta^2\underline{\sigma}^2$ and $\underline{\Phi}(\beta) = \frac{\beta}{v} - \frac{\beta^2}{2v} - \frac{\rho}{2}\beta^2\overline{\sigma}^2$, $\forall\beta$. Substituting and simplifying:

$$\begin{aligned} & [\bar{\beta}^*(1,1) - \bar{\beta}^I(F)]\left[\frac{1}{v} - \left(\frac{\rho\underline{\sigma}^2}{2} + \frac{1}{2v} + \frac{\rho}{8}Var(M(\theta,(1,1))\right)(\bar{\beta}^*(1,1) + \bar{\beta}^I(F))\right] \\ + \quad & [\underline{\beta}^P(F) - \underline{\beta}^I(F)]\left[\frac{1}{v} - \left(\frac{\rho\bar{\sigma}^2}{2} + \frac{1}{2v} + \frac{\rho}{8}Var(M(\theta,(1,1))\right)(\underline{\beta}^P(F) + \underline{\beta}^I(F))\right] \\ \leq \quad & 0 \end{aligned} \quad (37)$$

Now note that:

$$\bar{\beta}^*(1,1) \geq \bar{\beta}^I(F) \quad (38)$$

$$\underline{\beta}^I(F) \geq \underline{\beta}^P(F) \quad (39)$$

The first inequality follows from Lemma 2. For the second inequality, from Lemma 3:

$$\begin{aligned}
\underline{\beta}^P(F) &= \frac{[f(1,1\mid\underline{\beta})-f(0,0\mid\underline{\beta})]}{2F} \\
&= \frac{[f(1,1\mid\underline{\beta})-f(1,0\mid\underline{\beta})]}{2F}+\frac{[f(1,0\mid\underline{\beta})-f(0,0\mid\underline{\beta})]}{2F} \\
&\leq \frac{[f(1,1\mid\underline{\beta})-f(1,0\mid\underline{\beta})]}{2F}+\frac{[f(1,1\mid\underline{\beta})-f(1,0\mid\underline{\beta})]}{2F} \\
&= \underline{\beta}^I(F)
\end{aligned}$$

From (38) and (39) it follows that $\bar{\beta}^*(1,1)-\bar{\beta}^I(F)>0$ and $\underline{\beta}^P(F)-\underline{\beta}^I(F)<0$. Therefore, a sufficient condition for (37) is that:

$$\left|\begin{array}{l}
\frac{1}{v}-\left[\frac{\rho\underline{\sigma}^2}{2}+\frac{1}{2v}+\frac{\rho}{8}Var(M(\theta,(1,1))\right](\bar{\beta}^*(1,1)+\bar{\beta}^I)\leq 0 \\
\frac{1}{v}-\left[\frac{\rho\bar{\sigma}^2}{2}+\frac{1}{2v}+\frac{\rho}{8}Var(M(\theta,(1,1))\right](\underline{\beta}^P+\underline{\beta}^I)\geq 0
\end{array}\right. \tag{40}$$

Summing the two inequalities in (40) and rearranging:

$$\Delta\sigma=\bar{\sigma}^2-\underline{\sigma}^2\leq\frac{2}{\rho v}\left[\frac{1}{\underline{\beta}^P(F)+\underline{\beta}^I(F)}-\frac{1}{\bar{\beta}^*(1,1)+\bar{\beta}^I(F)}\right] \tag{41}$$

Given (38) and (39) the expression on the RHS is positive. Hence, for sufficiently small diferential between the divisional operating uncertainties $\hat{F}^P\leq\hat{F}^I$.

Now, note that for any $F<F^P$ under both regimes, IC-IC and PC-IC, the investment profile $(1,1)$ is achieved with the contractible benchmark PPS, $\beta_i^*(1,1)$. Hence, the principal's payoff is equal under both regimes. For any $F\in[F^P,F^I]$ the principal's payoff under PC-IC regime is lower than his respective payoff under IC-IC regime, because under the former regime by Lemma 2 he needs to reduce the investing manager's PPS to achieve the investing profile $(1,1)$, whereas under the latter regime the same investing profile is still achieved with the contractible benchmark PPS. For any $F\in[\hat{F^P},\hat{F^I}]$ it follows by revealed preference that $\Pi^P(\beta,\mathbf{I})\leq\Pi^I(\beta,\mathbf{I})$. For any $F>\hat{F^I}$ under both regimes the principal finds it very costly to induce investment profile $(1,1)$ and switches to $(0,0)$ and gives the division managers the contractible benchmark PPS $\beta_i^*(0,0)$. Hence, for $F > \hat{F}^I$ the principal's payoff is equal under both regimes.

We cannot rank unambiguously F^I and $\hat{F^P}$. However, as long as $\hat{F^P} \leq F^I$, the IC-IC mode is preferred and no further proof rather than Condition 1 and Condition 2 is needed. If $\hat{F^P} > F^I$, then we need to verify Condition 3. For any $F \in [F^I, \hat{F^P}]$:

$$\begin{aligned}
\Pi^P(\beta, \mathbf{I}) &= E[M(\theta,(1,1)] - F + \bar{\Phi}(\bar{\beta}^*(1,1)) + \underline{\Phi}(\underline{\beta}^P) \\
&\quad -\frac{\rho}{8}[(\bar{\beta}^*(1,1))^2 + (\underline{\beta}^P)^2]1))Var(M(\theta,(1,1)) \qquad (42) \\
\Pi^I(\beta, \mathbf{I}) &= E[M(\theta,(1,1)] - F + \bar{\Phi}(\bar{\beta}^I) + \underline{\Phi}(\underline{\beta}^I) \\
&\quad -\frac{\rho}{8}[(\bar{\beta}^I)^2 + (\underline{\beta}^I)^2]Var(M(\theta,(1,1)) \qquad (43)
\end{aligned}$$

To show that $\Pi^P(\beta, \mathbf{I}) \geq \Pi^I(\beta, \mathbf{I})$ we need to verify that:

$$\begin{aligned}
&\bar{\Phi}(\bar{\beta}^*(1,1)) + \underline{\Phi}(\underline{\beta}^P) - \frac{\rho}{8}[(\bar{\beta}^*(1,1))^2 + (\underline{\beta}^P)^2]Var(M(\theta,(1,1)) \leq \\
&\bar{\Phi}(\bar{\beta}^I) + \underline{\Phi}(\underline{\beta}^I) - \frac{\rho}{8}[(\bar{\beta}^I)^2 + (\underline{\beta}^I)^2]Var(M(\theta,(1,1))
\end{aligned}$$

This, however is the same inequality as (36) which, as shown above, holds for sufficiently small differential between the divisional operating uncertainty. This completes the proof of *Part (i)*.

Part (ii): Analogically to *Part (i)* in order to show that PC-IC dominates IC-IC, we need to verify the following conditions:

1. $F^P \geq F^I$.

2. $\hat{F}^P \geq \hat{F}^I$.

3. $\Pi^P(\beta, \mathbf{I}) \geq \Pi^I(\beta, \mathbf{I})$ for any $F \in (F^I, \hat{F}^I]$.

Start with Condition 1. To see that $F^P \geq F^I$ note that:

$$\begin{aligned} F^P &\equiv \frac{f(1,1 \mid \underline{\beta}) - f(0,0 \mid \underline{\beta})}{2\underline{\beta}} \\ &= \frac{[f(1,1 \mid \underline{\beta}) - f(1,0 \mid \underline{\beta})]}{2\underline{\beta}} + \frac{[f(1,0 \mid \underline{\beta}) - f(0,0 \mid \underline{\beta})]}{2\underline{\beta}} \\ &> \frac{[f(1,1 \mid \bar{\beta}) - f(1,0 \mid \bar{\beta})]}{2\bar{\beta}} + \frac{[f(1,0 \mid \bar{\beta}) - f(0,0 \mid \bar{\beta})]}{2\bar{\beta}} \\ &\approx F^I, \end{aligned}$$

if $r(q)$ is sufficiently concave, so that $\frac{\partial^2}{\partial I_1 \partial I_2} f(\cdot)$ becomes small for any β.

Continue with Condition 2. Analogically to to *Part (i)* we have to verify that:

$$\begin{aligned} &\bar{\Phi}(\bar{\beta}^*(1,1)) + \underline{\Phi}(\underline{\beta}^P) - \frac{\rho}{8}[(\bar{\beta}^*(1,1))^2 + (\underline{\beta}^P)^2]Var(M(\theta,(1,1)) \geq \\ &\bar{\Phi}(\bar{\beta}^I) + \underline{\Phi}(\underline{\beta}^I) - \frac{\rho}{8}[(\bar{\beta}^I)^2 + (\underline{\beta}^I)^2]Var(M(\theta,(1,1)) \end{aligned} \tag{44}$$

A sufficient condition for (44) to hold is:

$$\Delta\sigma \leq \frac{2}{\rho v}\left[\frac{1}{\underline{\beta}^P(F) + \underline{\beta}^I(F)} - \frac{1}{\bar{\beta}^*(1,1) + \bar{\beta}^I(F)}\right] \tag{45}$$

$$\leq \frac{1}{\rho v}\left[\frac{1}{\underline{\beta}^P(F)} - 1\right], \tag{46}$$

using (38) and (39). Hence, for sufficiently high differential between the general operating uncertainties Condition 2 holds.

Again, note that for any $F < F^I$ the principal's payoff is equal under both regimes, because the investment profile $(1,1)$ is achieved with the contractible benchmark PPS, $\beta_i^*(1,1)$. For any $F \in [F^I, F^P]$ the principal's payoff under IC-IC regime is lower than his payoff under PC-IC regime, because under the former regime by Lemma 2 he needs to reduce the investing manager's PPS to achieve the investing profile $(1,1)$, whereas under the latter regime the same investing profile is still achieved with the contractible benchmark PPS. For any $F \in [\hat{F^I}, \hat{F^P}]$, $\Pi^P(\beta, \mathbf{I}) \leq \Pi^I(\beta, \mathbf{I})$ by revealed preference. For any $F > \hat{F^I}$ the principal's payoff is equal under both regimes, because he finds it very costly to induce investment and switches to $(0, 0)$ with the contractible benchmark PPS $\beta_i^*(0, 0)$.

We cannot rank unambiguously F^P and $\hat{F^I}$. However, as long as $\hat{F^I} \leq F^P$, the PC-IC mode is preferred and no further proof rather than Condition 1 and Condition 2 is needed. If $\hat{F^I} > F^P$, then we need to verify Condition 3. For any $F \in [F^P, \hat{F^I}]$:

$$\begin{aligned}
\Pi^P(\beta, I) &= E[M(\theta, (1,1)] - F + \bar{\Phi}(\bar{\beta}^*(1,1)) + \underline{\Phi}(\underline{\beta}^P) \\
&\quad -\frac{\rho}{8}[(\bar{\beta}^*(1,1))^2 + (\underline{\beta}^P)^2]1))Var(M(\theta, (1,1)) \qquad (47) \\
\Pi^I(\beta, I) &= E[M(\theta, (1,1)] - F + \bar{\Phi}(\bar{\beta}^I) + \underline{\Phi}(\underline{\beta}^I) \\
&\quad -\frac{\rho}{8}[(\bar{\beta}^I)^2 + (\underline{\beta}^I)^2]Var(M(\theta, (1,1)) \qquad (48)
\end{aligned}$$

To show that $\Pi^P(\beta, I) \geq \Pi^I(\beta, I)$ for any $F \in (F^I, \hat{F}^I]$ we need to verify that:

$$\begin{aligned}
&\bar{\Phi}(\bar{\beta}^*(1,1)) + \underline{\Phi}(\underline{\beta}^P) - \frac{\rho}{8}[(\bar{\beta}^*(1,1))^2 + (\underline{\beta}^P)^2]Var(M(\theta, (1,1)) \geq \\
&\bar{\Phi}(\bar{\beta}^I) + \underline{\Phi}(\underline{\beta}^I) - \frac{\rho}{8}[(\bar{\beta}^I)^2 + (\underline{\beta}^I)^2]Var(M(\theta, (1,1))
\end{aligned}$$

This, however is the same inequality as (44) which, as shown above, holds for sufficiently high differential between the divisional operating uncertainty. This completes the proof.

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