Monetary Policy and the Cyclicality of Risk^{*}

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Abstract

This paper develops a theoretical framework to study the relationship between monetary policy and movements in risk. As emphasized in the household finance literature, variation in risk arises in the model because households face fixed costs of transferring cash across financial accounts, implying that some (inattentive) households rebalance their portfolios infrequently. Accordingly, prices for risky assets respond sharply to aggregate shocks because only a relatively small subset of attentive consumers are available to absorb these shocks. We show that the model can account for both the mean and the volatility of returns on equity and the risk-free rate, and in line with empirical evidence generates a decline in the equity premium following an unanticipated easing of monetary policy. We also find that countercyclical monetary policy generates higher average welfare than constant money growth or zero inflation policies, and more broadly that changes in the systematic component of monetary policy have important consequences for the cyclicality of risk.

Keywords: segmented markets, equity premium, monetary policy rules

JEL classification: E32, E44

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1 Introduction

Monetary policy primarily affects the macroeconomy through its effect on financial markets. In standard monetary models, this interaction between the financial and real sides of the economy primarily occurs through short-term interest rates, as changes in monetary policy affect the conditional mean of the short-term interest rate which in turn affects macroeconomic variables such as output, employment, and inflation. Researchers using these models typically abstract from another channel through which monetary policy affects financial markets and the macroeconomy. In particular, they do not study how monetary policy affects the conditional variances of variables or the perceived riskiness of the economy.¹ Interestingly, recent evidence suggests that monetary policy does affect risk, implying that standard monetary models are potentially missing an important channel through which monetary shocks propagate from the financial to the real economy. For instance, empirical evidence suggests that while an unanticipated easing of monetary policy lowers real short-term interest rates, it also has a large quantitative effect on equity returns occurring through a reduction in the equity premium.²

In this paper, we develop a DSGE model in which monetary policy affects the economy through the standard interest rate channel and through its effect on economic risk. The key feature of our model is that asset and goods markets are segmented, because it is costly for households to transfer funds between these markets. Accordingly, households may only infrequently update their desired allocation of cash between a checking or liquid account devoted to purchasing goods and a brokerage or illiquid account used for financial transactions. The optimal decision by an individual household to rebalance their cash holdings is a state-dependent one, reflecting that doing so involves paying a fixed cost in the presence of uncertainty. Households are heterogenous in this fixed cost, and only those households that rebalance their portfolios during the current period matter for determining asset prices. Because the fraction of these

 $^{^1}$ See Alvarez, Atkeson, and Kehoe (2007) for an extended discussion of this point. See also Atkeson and Kehoe (2008) and Cochrane (2008).

 $^{^{2}}$ See Bernanke and Kuttner (2005). Additional evidence reinforcing the influence of monetary policy shocks on equity prices include Ehrmann and Fratzscher (2004), Ammer, Vega, and Wongswan (2008), and references therein.

household changes over time in response to both real and monetary shocks, risk in the economy endogenously varies over time.

Evidence from the household finance literature provides strong support for infrequent portfolio rebalancing. For instance, Brunnermeier and Nagel (2008) show that, while there is considerable heterogeneity across households in transferring funds between a liquid and illiquid account, there is also substantial inertia by households, on average, in rebalancing these accounts.³ We show that with this feature our model is able to account for the Bernanke and Kuttner (2005) evidence. In particular, a monetary easing in our model leads to a fall in real interest rates and a reduction in the equity premium. For reasonable calibrations of the model, the reduction in the equity premium is an important reason why stock prices rise in response to a monetary easing. In addition, we show that the model is able to account for the observed means on equity and risk-free rates with a power utility function with a reasonable degree of risk aversion.

Besides examining the positive implications of the model, we explore the normative implications of alternative monetary policy rules including inflation-targeting rules, a constant money growth rate, and rules that respond systematically to changes in aggregate activity driven by technology shocks. We find that the response of the equity premium to shocks depends critically on the systematic response of monetary policy. For inflation targeting rules or rules in which the monetary policy is procyclical, the equity premium moves countercyclically. However, for very aggressive countercyclical policies, the equity premium moves acyclically or even procyclically. A systematic change to monetary policy affects risk, because it influences a household's incentive to rebalance her portfolio, changing the behavior of households that matter for determining asset prices, and ultimately the amount of risk borne by these households.⁴

Changes in the systematic component of monetary policy have important distributional consequences in the model. Because of these effects, countercyclical monetary policies imply aggregate welfare gains over inflation targeting or constant money growth rules. Countercyclical policy works well, because it improves the welfare of the majority of households, who tend

³ Also, see Calvet, Campbell, and Sodini (2009) and Bilias, Georgarakos, and Haliassos (2008).

⁴ While systematic policy has an important influence on equity prices, monetary policy shocks, per se, account for only a small fraction of the average equity premium and the volatility of equity prices in the model.

to rebalance their portfolios infrequently. By transferring resources toward these households during booms, it allows them to raise their consumption without incurring the fixed costs associated with transferring funds across their brokerage and checking accounts. Thus, this policy effectively replicates how these households would respond, if they did not face these fixed costs.

Our paper is related to the literature on limited participation in financial markets in monetary economies (e.g., Lucas (1990)) and in particular to Alvarez, Atkeson, and Kehoe (2009), who use an endogenously segmented market model to study how time-varying risk can explain the forward premium anomaly in currency markets.⁵ Our paper departs in important ways from their analysis. In their model, a household that does not make a state-contingent transfer is better described as inactive rather than inattentive, since these households move no funds from financial to goods markets in the current period. By contrast, a household that chooses not to make a state-contingent transfer in our framework still transfers funds across markets; however, she does so in an inattentive manner, using a preset plan to allocate funds in the current period.⁶

Our paper is also related to Guvenen (2009) and Chien, Cole, and Lustig (2011). Guvenen (2009) develops a model of limited participation model for stocks and heterogeneous preferences in which non-stockholders have a smaller elasticity of intertemporal substitution than stock-holders. He shows that the model does relatively well in matching the mean and volatility of the equity premium, generates countercyclical risk, and has reasonable business cycle properties. Chien, Cole, and Lustig (2011) show how a model in which some households actively trade stocks and bonds and others hold either a fixed fraction or do not trade equity can account for the mean and volatility of the equity premium and risk-free rate and generates countercyclical risk. A major difference in this paper relative to Chien, Cole, and Lustig (2011) is that, contrary to these authors, we do not assume that investors fix their periods of inattention, but instead investors in our model choose their periods of inattention optimally in a state-contingent manner.

Moreover, we develop a monetary framework in which goods markets are segmented from

 $^{^5}$ See also Bacchetta and van Wincoop (2010) who study the forward premium anomaly in a model with infrequent portfolio adjustment.

 $^{^{6}}$ See Duffie (2010) for a recent survey emphasizing the importance of modelling inattentive behavior in financial markets.

asset markets. In contrast, both Guvenen (2009) and Chien, Cole, and Lustig (2011) emphasize segmentation between stocks and bonds in a real economy context.⁷ This allows us to use the model as a laboratory to provide novel insights about the positive and normative implications of monetary policy and its role in contributing to time-varying risk premia.

The rest of this paper proceeds as follows. The next section describes the model. In Section 3 we use the model to explore some positive and normative effects of alternative monetary policy rules. We pay special attention to the effects of monetary policy on endogenous fluctuations in risk. Section 4 concludes.

2 The Model

The economy is populated by a large number of households, firms, and a government sector. Trade occurs in financial and goods markets in separate locations so that they are segmented from each other. There are two sources of uncertainty in our economy — aggregate shocks to technology, θ_t , and money growth, μ_t . We let $s_t = (\theta_t, \mu_t)$ index the aggregate event in period twith s_0 given, and $s^t = (s_1, ..., s_t)$ denote the state, which consists of the aggregate shocks that have occurred through period t.

2.1 Firms

There is a large number of perfectly competitive firms. Each of these firms has access to the following technology for converting capital, $K(s^{t-1})$, and labor, $L(s^t)$, into output, $Y(s^t)$ at dates $t \ge 1$:

$$Y(s^{t}) = \exp(\theta_{t} + \eta t) K(s^{t-1})^{\alpha} L(s^{t})^{1-\alpha}.$$
(1)

⁷ Chien, Cole, and Lustig (2011) do not consider growth and introduce an exogenous amount of leverage to explain the equity premium. Our analysis abstracts from the presence of leverage but incorporate growth into the analysis, which has important implications for asset prices and in particular for the average level of the risk-free rate.

The variable η determines the economy's growth rate and θ_t is an aggregate technology shock which follows a first-order autoregressive process:

$$\theta_t = \rho_\theta \theta_{t-1} + \epsilon_{\theta t},\tag{2}$$

where $\epsilon_{\theta t} \sim N(0, \sigma_{\theta}^2)$ for all $t \geq 1$.

Capital does not depreciate, and there exists no technology for increasing or decreasing its magnitude. We adopt the normalization that the aggregate stock of capital is equal to one. Labor is supplied inelastically by households, and its supply is normalized to one. At each date $t \ge 1$, a firm hires labor, sells its output to the economy's households, sells its existing capital to other firms, and purchases new capital for production next period. Accordingly, its profits or dividends at date t are:

$$D(s^{t}) = Y(s^{t}) - w(s^{t})L(s^{t}) - p_{K}(s^{t})K(s^{t}) + p_{K}(s^{t})K(s^{t-1}),$$
(3)

where $w(s^t)$ and $p_k(s^t)$ denote the real wage and the real price of capital, respectively.

As in Jermann (1998), a firm is infinitely-lived and chooses $\{K(s^t), L(s^t)\}_{t=1}^{\infty}$ to maximize its discounted stream of profits:

$$\sum_{t=1}^{\infty} \int_{s^t} Q(s^t) D(s^t) ds^t, \tag{4}$$

subject to equations (1) and (3) taking real wages, the level of technology and $K(s_0)$ as given. A firm also takes its stochastic discount factor, $Q(s^t)$, as given, and later a firm's discount factor is related to the marginal rate of substitution of its owners.

2.2 Households

There are a large number of households of type γ , which denotes a household's fixed cost of making state contingent transfers from a brokerage account to a checking account. This cost is constant across time but differs across household types according to the probability density function $f(\gamma)$. Since this is the only difference across households, we index a household by γ . **Brokerage Account.** At date 0, a household learns her type and engages in an initial round of trade in the asset market, as goods markets do not open until date 1. With initial asset holdings, $\overline{B}(\gamma)$ in her brokerage account at date 0, the household trades equity shares in the firms, $S(s_0, \gamma)$, with other households and a complete set of one-period contingent claims, $B(s^1, \gamma)$, issued by the government. Accordingly, the flow of funds in a household's brokerage account at date 0 is given by:

$$\overline{B}(\gamma) = p_K(s_0)S(s_0,\gamma) + \int_{s_1} q(s^1)B(s^1,\gamma)ds_1,$$
(5)

where $q(s^1)$ is the price of the bond in state, s^1 .

For dates $t \ge 1$, a household's brokerage account evolves according to:

$$B(s^{t},\gamma) + P(s^{t})(p_{K}(s^{t}) + D(s^{t}))S(s^{t-1},\gamma) = \int_{s_{t+1}} q(s^{t},s_{t+1})B(s^{t},s_{t+1},\gamma)ds_{t+1} + (6)$$

$$P(s^{t})p_{K}(s^{t})S(s^{t},\gamma) + \exp(\eta t)P(s^{t})A(s_{0},\gamma) + P(s^{t})[x(s^{t},\gamma) + \exp(\eta t)\gamma]z(s^{t},\gamma),$$
(7)

where $P(s^t)$ is the aggregate price level and $A(s_0, \gamma)$ is a non-state contingent transfer of funds from a household's brokerage account to checking account at date t chosen at date 0. A household can alter this initial transfer plan by choosing $x(s^t, \gamma) \neq 0$, which requires paying γ , a fixed cost of foregone output. Accordingly, $z(s^t, \gamma)$ is an indicator variable equal to one if a household opts to pay her fixed cost and make a state-contingent transfer and equal to zero if a household does not.

Households that choose $z(s^t, \gamma) = 0$ are referred to as inattentive households, since their transfers are based on outdated plans that do not depend on the information about the current state. As in Reis (2006), we view the fixed cost, γ , and a household's reliance on a predetermined plan as reflecting the notion that it is costly for households to continually acquire and process information in forming expectations and making decisions. In principle, we could allow the predetermined plan to be time-dependent, updated infrequently by a household; however, for simplicity, we assume households choose this plan once and for all at date $0.^8$

⁸ We also assume that $A(s^0, \gamma)$ is fully indexed to inflation. Altering this assumption so that the non-state contingent plan is not fully indexed to inflation would tend to magnify the model's monetary non-neutrality.

A key difference between the households in our model and other market segmentation models is that we allow households to set up predetermined transfer plans. As we emphasize later, without these plans the model is unable to generate an average equity premia in line with the data. This feature also has an attractive interpretation, as it allows us to model inattentive behavior in a tractable way. Our approach in modelling inattentive households differs from Reis (2006) in that we focus on the decision to transfer funds across a brokerage and checking account – the key decision in a segmented market model – rather than on the amount of household consumption or savings.⁹ Moreover, Reis (2006) focuses on explaining puzzles regarding consumption behavior rather than asset pricing.

Checking Account. At each date $t \ge 1$, a household purchases goods for consumption, $c(s^t, \gamma)$, and works in the labor market. To purchase goods in period t, a household uses cash in her checking account:

$$P(s^{t})c(s^{t},\gamma) = M(s^{t-1},\gamma) + P(s^{t})x(s^{t},\gamma)z(s^{t},\gamma) + P(s^{t})\exp(\eta t)A(s_{0},\gamma).$$
(8)

At the beginning of period t, a household has $M(s^{t-1}, \gamma)$ dollars in her checking account with which to purchase goods. A household also receives cash from her non-state contingent transfer plan, and if she chooses to incur her fixed cost, she receives $P(s^t)x(s^t, \gamma)$.¹⁰

We have focused on transfers only between a checking account (i.e., more liquid assets) and a brokerage account (i.e., less liquid assets). In practice, a household has access to a wider range of financial products such as credit cards and other "near-money" assets that blur this distinction. In principle, one could incorporate such near-money assets by incorporating an additional account into the model whose assets can not directly be used to purchase goods but whose transaction cost is smaller than for the financial assets in the brokerage account. However, extending the model along these lines complicates the analysis and we abstract from

⁹ Our approach also differs from Reis (2006) in the details of how plans are set and revised. As in Reis (2006), a household pays a fixed cost to revise her plan; however, in Reis (2006) adjustments to a household's plan do not depend on the current state but on the state at a pre-set date and through these adjustments outdated plans are revised.

¹⁰ A household can reoptimize by setting $x(s^t, \gamma) < 0$, thereby transferring additional cash from her checking to brokerage account. Similarly, a household is free to choose $A(s_0, \gamma) < 0$.

this possibility.

Each household inelastically supplies her labor to the economy's firms. With a household's labor supply normalized to one, a household earns real wage income, $w(s^t)$. This wage income is received at the end of the period so it can not be used for current consumption. Accordingly, a household cash in its checking account at the end of period t is given by:¹¹

$$M(s^t, \gamma) = P(s^t)w(s^t).$$
(9)

A household's problem is to choose $A(s_0, \gamma)$ and $\{c(s^t, \gamma), x(s^t, \gamma), z(s^t, \gamma), M(s^t, \gamma), B(s^t, \gamma), S(s^{t-1}, \gamma)\}_{t=1}^{\infty}$ to maximize:

$$\sum_{t=1}^{\infty} \int_{s^t} \beta^t U(c(s^t, \gamma))g(s^t)ds^t,$$
(10)

subject to equations (5)-(9), taking prices and initial holdings of money, bonds, and stocks as given. In equation (10), the function $g(s^t)$ denotes the probability distribution over history s^t .

2.3 Monetary Policy

The government issues the economy's one-period state-contingent bonds and controls the economy's money stock, M_t . Its budget constraints at date 0 is $B = \int_{s_1} q(s_1)B(s^1)ds_1$ and at dates $t \ge 1$, its budget constraint is:

$$B(s^{t}) + M_{t-1} = M_t + \int_{s_{t+1}} q(s^{t}, s_{t+1}) B(s^{t}, s_{t+1}) ds_{t+1},$$
(11)

with $M_0 > 0$ given.¹² Monetary policy is specified to follow a rule for money growth, $\mu_t = log\left(\frac{M_t}{M_{t-1}}\right)$, of the form:

$$\mu_t = (1 - \rho_\mu)\mu + \rho_\mu\mu_{t-1} + d_\theta\theta_t + \epsilon_{\mu t}, \qquad (12)$$

¹¹ We have abstracted from the possibility that a household may want to save extra cash in their checking and/or brokerage accounts since equations (6) and (8) always bind. By doing so, we do not need to keep track of each individual's cash holdings, simplifying the analysis.

¹² To simplify the analysis, the government does not levy taxes or consume resources but only conducts open market operations with money and bonds.

where $\epsilon_{\mu t} \sim N(0, \sigma_{\mu}^2)$ for all $t \geq 1$. This rule allows for a systematic response of money to changes in technology (or equivalently output given that capital and labor are fixed).¹³ When $d_{\theta} > 0$, money growth is procyclical, and when $d_{\theta} < 0$, money growth is countercyclical. For our benchmark rule, we set $d_{\theta} = 0$.

The simple rules we evaluate include a constant money supply rule in which $\mu = \rho_{\mu} = d_{\theta} = \sigma_{\mu} = 0$, a procyclical rule in which $\mu = \rho_{\mu} = \sigma_{\mu} = 0$ and $d_{\theta} > 0$, and a countercyclical rule in which $\mu = \rho_{\mu} = \sigma_{\mu} = 0$ and $d_{\theta} < 0$. An additional rule that we consider that is not nested by equation (12) is a zero inflation or a price level targeting rule. This rule requires that μ_t be chosen such that inflation, $\pi(s^t) = \frac{P(s^t)}{P(s^{t-1})} = 1$, for all s^t .

2.4 Equilibrium Characterization

The economy's resource constraint is:

$$Y(s^{t}) = \exp(\theta_{t} + \eta t) = \int_{0}^{\infty} \left[c(s^{t}, \gamma) + \exp(\eta t) \gamma z(s^{t}, \gamma) \right] f(\gamma) d\gamma.$$
(13)

We normalize the supply of equity traded by households to one and require that $B(s^t) = \int_0^\infty B(s^t, \gamma) f(\gamma) d\gamma$ for all $t \ge 0$. The economy's price level and inflation rate can be obtained from:¹⁴

$$P(s^t) = M_t \exp(-\theta_t - \eta t), \tag{14}$$

which implies that velocity is constant and the economy's inflation is given by:

$$\pi(s^{t}) = \frac{P(s^{t})}{P(s^{t-1})} = \exp(\mu_{t} + \theta_{t-1} - \theta_{t} - \eta).$$
(15)

The consumption of an inattentive household (i.e., one that sets $z(s^t, \gamma) = 0$) is given by:

$$c_I(s^t, \gamma) = \frac{w(s^{t-1})}{\pi(s^t)} + e^{\eta t} A(s_0, \gamma) = (1 - \alpha) e^{\theta_t - \mu_t + \eta t} + e^{\eta t} A(s_0, \gamma),$$
(16)

 13 We study an exchange economy to focus on the distributional effects of monetary policy and its role in inducing movements in risk. In Gust and López-Salido (2011), we allow for endogenous labor and capital accumulation albeit we do not address monetary policy issues.

$$M_t = \int_0^\infty \left\{ M(s^{t-1}, \gamma) + P(s^t) \left[x(s^t, \gamma) + \exp(\eta t)\gamma \right] z(s^t, \gamma) + P(s^t) \exp(\eta t) A(s_0, \gamma) \right\} f(\gamma) d\gamma$$

 $^{^{14}}$ To derive equation (14), one needs to combine equations (8) and (13) with the money market clearing condition:

because optimization by firms implies $w(s^t) = (1 - \alpha)e^{\theta_t + \eta t}$ From this expression, we can see that inflation is distortionary, since, all else equal, it reduces the consumption of inattentive households. Accordingly, an unanticipated increase in money that raises inflation may induce a household to pay her fixed cost and become attentive.

In the Appendix A, we show that there is perfect risk-sharing amongst attentive households (i.e. those who choose $z(s^t, \gamma) = 1$) and that the initial asset holdings, $\bar{B}(\gamma)$ can be set so that:

$$c_A(s^t, \gamma) = c_A(s^t). \tag{17}$$

Accordingly, the consumption of attentive households, $c_A(s^t)$, is independent of γ . To further characterize, the consumption of attentive and inattentive households, we need to determine the flow of transfers from a household's initial plan, $A(s_0, \gamma)$. As derived in the Appendix A, household's choice of $A(s_0, \gamma)$ satisfies:

$$\sum_{t=1}^{\infty} \int_{s^t} \beta^t \left[U'(c_A(s^t)) - U'(c_I(s^t, \gamma)) \right] (1 - z(s^t, \gamma)) g(s^t) ds^t = 0.$$
(18)

This latter condition implies that in states of the world in which a household is inattentive (i.e., $z(s^t, \gamma) = 0$), the household chooses $A(s_0, \gamma)$ to equate her expected discounted value of marginal utility of its consumption to the expected discounted value of the marginal utility of consumption of the attentive households. Accordingly, the non-state contingent transfer plan provides some consumption insurance to households with large fixed costs.

We now characterize a household's decision for $z(s^t, \gamma)$ given optimal decisions for $c(s^t, \gamma)$, $x(s^t, \gamma)$, and $A(s_0, \gamma)$. As described in Appendix A, a household γ will choose to be attentive (i.e., $z(s^t, \gamma) = 1$) if $\gamma \leq \bar{\gamma}(s^t)$ where $\bar{\gamma}(s^t)$ is defined by:

$$U\left(c_A(s^t)\right) - U\left(c_I(s^t, \bar{\gamma}(s^t))\right) = U'\left(c_A(s^t)\right)\left[c_A(s^t) - c_I(s^t, \bar{\gamma}(s^t)) + \exp(\eta t)\bar{\gamma}(s^t)\right],\tag{19}$$

and inattentive otherwise. Equation (19) implies that there is a marginal household with fixed cost $\bar{\gamma}(s^t)$ whose net gain of being attentive is equal to the cost of transferring funds across the two markets. The net gain, $U(c_A(s^t)) - U(c_I(s^t, \gamma))$, is simply the difference in the level of utility from being attentive as opposed to inattentive. The net cost of making the statecontingent transfer comprises the fee γ and the amount transferred by the household, since $x(s^t, \gamma) = c_A(s^t) - c_I(s^t, \gamma)$. The asset pricing kernel in the economy depends on the consumption of attentive households and is given by:

$$m(s^{t+1}) = \frac{q(s^{t+1})}{g(s_{t+1}|s^t)} = \beta \frac{U'[c_A(s^{t+1})]}{U'[c_A(s^t)]},$$
(20)

where $g(s_{t+1}|s^t) = \frac{g(s^{t+1})}{g(s^t)}$ denotes the probability of state s_{t+1} conditional on state s^t . Accordingly, the pricing kernel is the state-contingent price of a security expressed in consumption units normalized by the probabilities of the state. We can also relate a firm's stochastic discount factor to the state-contingent prices of securities, as $Q(s^t) = \prod_{j=1}^t q(s^j)$.

To see how the extensive margin affects the pricing kernel, it is useful to rewrite the resource constraint as:

$$c_A(s^t) = F[\bar{\gamma}(s^t)]^{-1} \left[\exp(\theta_t + \eta t) - \int_{\bar{\gamma}(s^t)}^{\infty} c_I(s^t, \gamma) f(\gamma) d\gamma - \exp(\eta t) \int_0^{\bar{\gamma}(s^t)} \gamma f(\gamma) d\gamma \right], \quad (21)$$

where $F(\gamma)$ denotes the cumulative distribution function of γ . From expression (21), it is clear that $\bar{\gamma}(s^t)$, the fraction of attentive households, varies with the state and that changes in this fraction can alter the path of attentive consumption and thus the marginal utility of attentive consumption and the pricing kernel.

The pricing kernel can be used to determine the real risk-free rate (r^f) as well as the real return on equity (r^e) . These returns are given by:

$$[1 + r^{f}(s^{t})]^{-1} = \int_{s_{t+1}} m(s^{t}, s_{t+1})g(s_{t+1}|s^{t})ds_{t+1}, \qquad (22)$$

$$1 = \int_{s_{t+1}} m(s^t, s_{t+1}) [1 + r^e(s^t, s_{t+1})] g(s_{t+1}|s^t) ds_{t+1}.$$
 (23)

Optimization by firms implies that in equilibrium the return on equity satisfies:

$$(1 + r^e(s^{t+1})) = \frac{\alpha \frac{Y(s^{t+1})}{K(s^t)} + p_K(s^{t+1})}{p_K(s^t)}.$$
(24)

Using equations (22) and (23), we can then define the equity premium in our economy as:¹⁵

$$\frac{\mathcal{E}_t[1+r_{t+1}^e]}{1+r_t^f} = 1 - \operatorname{cov}_t\left(m_{t+1}, 1+r_{t+1}^e\right).$$
(25)

 $^{^{15}}$ For convenience we have switched notation to express both the expected return on equity and the covariance between the pricing kernel and the return on equity, which are both conditional on the state of the world at date t.

3 Quantitative Analysis

In this section, we show that the model has reasonable asset pricing properties and in line with empirical evidence generates a liquidity effect and a decline in the equity premium following an unanticipated monetary injection. We then use the model as a laboratory for evaluating the performance of alternative monetary policy rules. Before doing so, we briefly discuss the model's calibration and a deterministic version of the model.

3.1 Functional Forms and Calibration

Household's preferences are given by the isoelastic utility function, $U(c) = \frac{c^{1-\sigma}}{1-\sigma}$, where σ is the coefficient of relative risk aversion. We follow the discussion and the survey of the literature in Hall (2009) and Guvenen (2009), and set the relative risk aversion coefficient equal to 4. Consistent with a quarterly model, we set $\eta = 0.0047$, implying the economy grows at an annualized rate near 2%, and choose $\beta = 0.997$. The economy's capital share, α , is 0.36.

For the distribution of the fixed cost, $F(\gamma)$, we assume that there is some small positive mass of households with zero fixed costs and choose the remaining distribution, 1 - F(0), to be log-normal so that $\log(\gamma) \sim N(\log(\gamma_m), \sigma_m^2)$. We set F(0) = 0.0543, $\gamma_m = 2.9075$, and $\sigma_m = 1$, which imply that, on average, about 6 percent of households are attentive in a quarter with some households rebalancing frequently and a large mass of households rarely rebalancing. Such a calibration is broadly in line with evidence that household portfolio allocation displays substantial inertia.¹⁶

For the monetary policy shock, we set $d_{\theta} = 0$, $\rho_{\mu} = 0.95$ and $\sigma_{\mu} = 0.001$. This value for ρ_{μ} is in line with the value used by Alvarez, Atkeson, and Kehoe (2002). We set $\mu = 0.005$ so that average, annualized money growth rate is 2%. We calibrated the technology shocks based on the time series properties of aggregate consumption. We set $\rho_z = 0.985$ and chose $\sigma_z = 0.008$ so that the standard deviation for annualized consumption growth is 3 percent, consistent with annual data on U.S. consumption from 1889-2009. As discussed in Appendix B, the model is

¹⁶ See, for example, Brunnermeier and Nagel (2008) and Bonaparte and Cooper (2009).

solved numerically using global methods.¹⁷

3.2 Non-Stochastic Steady State

In a deterministic environment, the model reduces to a representative agent economy. According to equation (18), a household that chooses to be inattentive obtains the same level of consumption as an attentive household in non-stochastic steady state. An inattentive household can obtain such a level of consumption by choosing her initial plan such that $\tilde{c}_A = \tilde{c}_I = \frac{1-\alpha}{\exp(\mu)} + A$, where A takes on the same value across all inattentive households and the tildes over the variables reflect that these variables have been detrended by $\exp(\eta t)$. With consumption the same across households, all households with $\gamma > 0$ will be inattentive, and the households with $\gamma = 0$ will be indifferent between using x or the non-state contingent transfer, A.

The non-stochastic steady state highlights the important role that the initial transfer scheme plays in the model. Without this plan, the model is similar to other models of endogenous segmentation such as Alvarez, Atkeson, and Kehoe (2009). In that case, the model's interpretation is quite different, as a household that opts not to transfer funds is *inactive* rather than *inattentive*. This difference has important implications for the non-stochastic steady state, as an inactive household has a much greater incentive than an inattentive household (who makes use of her initial plan) to incur her fixed cost and transfer funds. Accordingly, the consumption of an active household exceeds the consumption of inactive households, who would only receive $\tilde{c}_I = \frac{1-\alpha}{\exp(\mu)}$. As a result, an increase in steady-state inflation, μ , would lower \tilde{c}_I and raise the number of active households. In contrast, in our model, inattentive households choose A > 0so that their consumption level reflects not only the proceeds from working but the proceeds from capital markets. An increase in μ induces inattentive households to choose a larger A and the degree of market segmentation remains unaffected: the inattentive households are still those with $\gamma > 0$.

¹⁷ We found that perturbation methods were only reliable for small shocks due to the state dependent behavior of the number of attentive households. In particular, for positive technology shocks, the number of attentive households is increasing in the level of technology. However, for large negative technology shocks, it is decreasing in the level of technology.

3.3 Asset Pricing Implications

Table 1 displays several statistics of interest from alternative versions of the model and compares them with their empirical counterparts taken from Guvenen (2009). As a reference point, the third column of the table reports the results from the economy with a single representative household.¹⁸ For our baseline calibration, with a relatively low coefficient of relative risk aversion, as discussed in Mehra and Prescott (1985), the representative agent model is unable to replicate prominent asset pricing features: the average equity premium is only 0.2% and the average (real) risk-free rate is 8.8% on an annualized basis. As discussed in Weil (1989), it is possible to match the observed equity premium in this model by increasing σ ; however, this comes at the cost of generating a counterfactual average risk-free rate.

The fourth column of Table 1 shows the results from the benchmark calibration of the model with inattentive behavior. This model is consistent with the high average equity premium and the low and smooth risk-free rate observed in the data. The model's Sharpe ratio at 0.18 is below the point estimate based on U.S. data, reflecting that the volatility of excess stock returns exceeds that observed in the data. Still, the Sharpe ratio is much higher than in the representative agent economy and lies within the 95 percent confidence interval.

A key reason the model can generate a large average equity premium with $\sigma = 4$ is that the volatility of consumption of attentive households is higher than average consumption volatility. As shown in Table 1, the volatility of consumption growth for households is 5.6 times greater than for average households. The consumption volatility of an attentive household is higher than an inattentive household, because the two aggregate shocks only affect the consumption of the latter type of household through changes in labor income. In contrast, attentive households experience fluctuations in both labor and capital income. A household that rebalances more frequently accepts this higher consumption volatility in return for a higher average level of consumption. This implication is in line with evidence of Parker and Vissing-Jorgensen (2009) provided that 'high consumption' households are in fact more likely to rebalance. In particular, these authors find that the exposure to changes in aggregate consumption growth of households

¹⁸ We use the calibrated parameters discussed above except that $F_L = 1$ in this case.

in the top 10 percent of the consumption distribution is about five times that of households in the bottom 80 percent.

As shown in Table 1, inattentive behavior plays a critical role in generating the average equity premium. When the households do not have access to the non-state contingent plan (i.e., $A(s_0, \gamma) = 0$), the asset pricing implications are similar to those of the representative agent model: the average equity premium is close to zero while the average risk-free rate is above 8 percent. Without the financial plan, the average fraction of active households is 29 percent. Even if we lower this average fraction to 6 percent by increasing the fixed costs, the asset pricing implications remain largely unchanged and the average cost of rebalancing is over 27 percent of GDP. In comparison, the average cost of rebalancing in the benchmark model with the financial plan is 0.2 percent of GDP.

In the model without inattentive behavior, the consumption of inactive household is equal to her labor income; thus, it inherits both the mean and volatility of this income. However, when a household has the ability to make transfers based on an initial plan, this household will transfer funds from her brokerage to checking account (i.e., $A(\theta_0, \gamma) > 0$), gaining access to capital income. Accordingly, the level of her consumption increases relative to the case in which she does not have access to this plan and the volatility of her consumption falls given the financial plan's non-state contingent nature. By reducing the consumption volatility of households that do not make state contingent transfers, the financial plan tends to increase the volatility of the consumption of the attentive households. This higher volatility arises because with an inattentive household's consumption partially buffered by the non-state contingent plan, an attentive household's consumption becomes more sensitive to capital income flows derived from equity markets.

To understand why the sensitivity of attentive consumption increases for larger values of A more formally, consider a version of the model in which there is no growth and there are only two types of households: an attentive household with $\gamma = 0$ and an inattentive household with $\gamma = \infty$ who receives $c_I = (1 - \alpha) \exp(\theta - \mu) + A$. From the resource constraint, it follows that $c_A = F(0)^{-1} [\exp(\theta) - (1 - F(0))c_I]$, where F(0) denotes the fraction of attentive

households. As shown in Appendix A, taking the derivative of attentive consumption with respect to technology shocks yields:

$$\left. \frac{d\log(c_A)}{d\theta} \right|_{ss} = F(0)^{-1} \left(\frac{\alpha}{\exp(\mu)} + \frac{\exp(\mu) - 1}{\exp(\mu)} \right) + \frac{1 - \alpha}{\exp(\mu)} > 1,$$
(26)

where the steady state level of technology has been normalized to one. Because F(0) < 1, it follows that $\frac{d \log(c_A)}{d\theta}\Big|_{ss} > 1$. The first term in equation (26) represents the effects of higher capital income from a technological increase, and the second term represents the effects of higher labor income on an individual's consumption. Because capital income is shared only amongst attentive households, it is scaled up by $F(0)^{-1}$, while labor income is shared amongst all households and need not be scaled; thus, expression (26) indicates that changes in capital income can potentially induce large fluctuations in the consumption of attentive households. With these households pricing assets in the economy, they require a relatively large average equity premium as compensation.

In the model without inattentive behavior (i.e., A = 0), c_A and c_I have the interpretation as active consumption and inactive consumption and $c_I = (1 - \alpha) \exp(\theta - \mu)$. In this case, the derivative of attentive consumption with respect to technology shocks is smaller since $\frac{d\log(c_A)}{d\theta}\Big|_{ss} = 1$. This smaller effect reflects that an increase in technology has a relatively large effect on inactive consumption, which rises one-for-one with the technological increase. Hence, even with F(0) < 1, the increase in technology is equally absorbed by both types of households. As a result, active consumption is less sensitive to changes in technology than in the model with inattentive behavior. Accordingly, inattentive behavior plays a crucial role in generating greater volatility of the consumption of attentive households to technology shocks, as it shifts aggregate risk away from inattentive households onto attentive households.

The fifth column of Table 1 displays the results using the benchmark calibration of the inattentive model except that there are no monetary shocks. The results in the table are very similar to the version of the model with monetary shocks, as the average equity premium, for instance, is 6.1% in the economy with both technology and monetary shocks and 5.8% in the economy with just technology shocks. Accordingly, monetary shocks only make a small contribution to asset pricing fluctuations in the model.

3.4 Monetary Policy Shocks and the Equity Premium

Besides having reasonable implications for the average risk premium and risk-free rate, the model generates a noticeable increase in the equity premium following a monetary contraction. This implication is in line with the evidence of Bernanke and Kuttner (2005), who find that a broad index of stock prices registers a gain of 1 percent in reaction to a 25 basis point easing of the federal funds rate. They decompose the response of stock prices into changes in current and expected future dividends, changes in current and expected future real interest rates, and changes in expected excess equity returns. They conclude that an important channel in which increases in stock prices occur is through changes in equity premia.

Figure 2 displays the impulse responses to an unanticipated decline in money growth in the inattentive model.¹⁹ As in the limited participation models of Lucas (1990) and Fuerst (1992), the model displays a noticeable liquidity effect, with the nominal interest rate increasing 25 basis points in response to the monetary tightening. Moreover, as in Alvarez, Atkeson, and Kehoe (2002), the effect is persistent. Equity prices fall about 2 percent on impact, with part of the decline reflecting a higher equity premium. On impact, the equity premium rises about 20 basis points. Such a response is in line with the empirical evidence presented in Bernanke and Kuttner (2005).

To understand why the model generates a rise in the equity premium, the bottom left panel of Figure 2 shows the response of the consumption of attentive households. The monetary contraction has no effect on output but has an important redistributive effect. It raises the consumption of inattentive households, whose real money balances available for consumption increase, and lowers the consumption of attentive households. As shown in the bottom right panel, this redistribution induces a fall in the fraction of attentive households. Accordingly, there is a reduction in the degree of risk-sharing amongst attentive households, which helps drive up the equity premium.²⁰

¹⁹ Following Hamilton (1994), we define the impulse response of variable, $y(s^t)$, at date t to a monetary innovation that occurs at date 1 as: $E[log(y(s^t)) | \epsilon_{\mu 1}, \mu_0, z_0] - E[log(y(s^t)) | \mu_0, z_0], \forall t \ge 1$, where E denotes the conditional expectations operator. See section D of the Appendix where we explain the computation of the impulse response functions.

²⁰ This intuition is perhaps easiest to understand for a large positive monetary shock which induces all

The model's ability to generate a liquidity effect and an increase in the equity premium after a monetary contraction is notable, especially when contrasted with New Keynesian models. These models as emphasized by Edge (2007) have difficulty producing a liquidity effect unless one incorporates additional real rigidities such as habit persistence in consumption and time to plan and build for investment projects. In addition, there is a limited role for monetary policy to influence conditional variances of variables in New Keynesian models, and, as a result, it difficult for these models to account for the evidence suggesting that the equity premium declines in response to a monetary injection.

3.5 Alternative Monetary Policy Rules and the Equity Premium

Given that the model is capable of accounting for some prominent empirical findings regarding interest rates and the equity premium, it is natural to use it as a laboratory for evaluating alternative policy rules. We begin by evaluating how changes in the systematic or anticipated component of the monetary policy rule affects the average equity premium and the risk free rate. In particular, we examine how changes in the average money growth rate, μ , the persistence of the money growth, ρ_{μ} , and the response of money to output, d_{θ} affect these variables.

Figure 1 shows how changes in these parameters affect the average equity premium and risk-free rate. The figure also displays the sample averages for the risk-free rate and the equity premium (see the black dot labeled "U.S. Data") and the 5% confidence ellipse based on the estimates from Guvenen (2009). The points along the red line with diamonds represent different combinations of the mean equity premium and risk-free rate for money growth rates ranging from 0 to 10 percent on an annualized basis. For all the average money growth rates in this range, the model yields a mean equity premium and risk-free rate within the 95% confidence region. Moreover, changes in average inflation rate have relatively little effect on the average equity premium and real risk-free rate.

As indicated in our discussion of the non-stochastic steady state, a higher average inflation

households to become attentive. For such a state of the world, the economy behaves similar to a world with a representative agent in which the equity premium is low. The equity premium is low, because aggregate risk is now spread over all the households rather than a small set of households.

rate increases the steady state value of the households' initial plan. This same consideration applies to the stochastic economy, as the function, $A(s_0, \gamma)$, shifts up when μ increases. As discussed above, an upward shift in $A(s_0, \gamma)$ increases the volatility of attentive consumption and the average equity premium.²¹

The purple line with triangles in Figure 1 displays the results from varying the persistence of the money growth process. For values of ρ_{μ} between 0.2 and 0.95, the combinations of mean equity premia and risk-free rates lie within the 95% confidence region. Raising the persistence of money growth shocks tends to reduce the average equity premium by driving up the incentive for a household to become attentive. This reflects that a higher value of ρ_{μ} makes the monetary shocks both larger and longer-lasting, benefitting attentive households. With more attentive households, risk in financial markets is spread over more households, attentive consumption growth becomes less volatile, and its covariance with the return on equity diminishes. Consequently, the average equity premium declines.²²

The green line with squares in Figure 1 shows the mean of the equity premium and risk-free rate for different values of d_{θ} . A countercyclical monetary policy rule (i.e., $d_{\theta} < 0$) tends to reduce the average risk premium, while a procyclical rule tends to raise it. Holding the fraction of attentive households fixed, a procyclical (countercyclical) rule tends to increase (decrease) the volatility of consumption growth of attentive households, as a monetary injection redistributes funds to attentive households during a boom when attentive consumption is already high. Conversely, in a downturn, a procyclical rule calls for lower money growth, redistributing cash away from attentive households, which exacerbates the fall in the consumption of attentive households.

²¹ This effect is partially offset by a small increase in the number of attentive households resulting from the higher value of μ .

²² Holding the fraction of attentive households fixed, an offsetting effect is that increasing ρ_{μ} raises the unconditional volatility of money growth, increasing the volatility of attentive consumption growth and hence the average equity premium.

3.6 Monetary Policy and the Cyclicality of Risk

Before discussing the normative implications of alternative policy rules, it is helpful to first examine how simple, systematic rules alter the transmission of technology shocks and affect the cyclicality of risk. Since our emphasis here is on systematic component of monetary policy, we only consider rules in which $\rho_{\mu} = \epsilon_{\mu} = 0$. The particular rules that we consider include a fixed money supply rule, ($\mu = d_{\theta} = 0$), a procyclical rule in which $d_{\theta} = 0.1$ and $\mu = 0$, and a countercyclical rule in which $d_{\theta} = -0.1$ and $\mu = 0$. Finally, we consider a price level or zero inflation targeting rule. From equation (15), this rule implies that $\mu_t = \theta_t - \theta_{t-1} + \eta$. Thus, in order to keep inflation constant in response to a highly persistent and positive technology shock, this rule will raise monetary growth initially but contract it in future periods as the shock gradually dies out.

Figure 3 displays the response of the economy following a positive technology shock for the constant money supply rule, the countercyclical rule, and the procyclical rule. In each case, output is exogenous and rises about 0.1 percent on impact (top left panel of the Figure) after which it gradually returns to its pre-shocked level. A key result that emerges from Figure 3 is that the equity premium moves countercyclically under all three rules.

To understand this result, consider first the constant money supply rule (the solid black line). A positive technology shock raises the consumption of attentive households more than inattentive households, since an attentive household changes her consumption in response to both the higher wage and capital income, while the consumption of inattentive household responds only to the higher wage income. This jump in capital income induces more households to become attentive, which in turn helps lower risk in equity markets. Under the constant money growth rule, the equity premium falls about 20 basis points, which helps push up equity prices.

The real interest rate falls on impact, reflecting intertemporal smoothing motives by attentive households. However, the decline in real interest rates is small because of a reduction in precautionary savings by attentive households. This decline is evident in the fall in conditional volatility of consumption growth for attentive households (the middle right panel). Finally, inflation falls sharply under the constant money growth rule but quickly falls back to its pre-shocked level.

The procyclical rule (the red line with circles) has similar qualitative effects on the equity premium than the constant money supply rule though the effects are larger. By increasing the money growth rate when technology is high, monetary policy in effect transfers cash away from inattentive households to attentive ones. Accordingly, there is a greater incentive to become attentive, and the fraction of attentive households rises more, helping induce a larger fall in the equity premium than under the constant money supply rule. There is a larger decline in precautionary savings under the procyclical rule than the constant money supply rule. Accordingly, the real interest rate rises instead of falls in this case, leading to a smaller increase in equity prices than under a constant money rule. The countercyclical rule (blue dashed line) works in reverse relative to the procyclical rule. In this case, the response of the equity premium is smaller and the real rate falls by more, reflecting a smaller change in precautionary savings.

Figure 4 shows the effects of a more aggressive countercyclical rule (blue dashed line). In this case, the equity premium rises a bit after the technology shock and is essentially acyclical. This response reflects that monetary policy now vigorously counteracts the rise in the consumption of attentive households driven by the technology shock by redistributing funds away from attentive to inattentive households, which spreads risk over a wider set of households. As shown in the top right panel of Figure 3, monetary policy achieves this redistribution by generating a persistent deflation.

Figure 4 also displays the case of a zero inflation targeting rule (red line with circles). The middle left panel shows that the real interest rate falls sharply under the zero inflation targeting rule, reflecting a large, temporary increase in money growth that is quickly reversed so that money growth becomes slightly negative in future periods. With the real interest rate falling sharply, the (real) price of equity jumps 2 percent and then declines to a level above its pre-shocked value.

The price of equity rises not only due to the fall in the real interest rate but also due to a sizeable decline in the equity premium. The equity premium moves countercyclically under a zero inflation targeting rule, because this rule calls for a large, temporary increase in the money growth rate after a positive technology shock. Consequently, there are increases in both the consumption of attentive households and the fraction of attentive households. In addition, there is a reduction in precautionary savings by attentive households.

3.7 Welfare Implications of Alternative Monetary Policy Rules

Table 2 compares aggregate welfare under alternative policy rules. We define aggregate welfare so that each household receives equal weight:

$$w(s_0) = \sum_{t=1}^{\infty} \int_{s^t} \int_{\gamma} \beta^t U(c(s^t, \gamma)) g(s^t) f(\gamma) d\gamma ds^t,$$
(27)

where $w(s_0)$ is conditional on the initial state of the world as well as the initial asset distribution. Hence, as discussed in Appendix C, to compare welfare across the different rules, we hold the initial asset distribution, $\bar{B}(\gamma) \forall \gamma$, fixed across policy rules. To do so, we replace equation (17) with

$$\frac{c(s^t,\gamma)}{c(s^1,\gamma)} = \frac{c(s^t,\gamma')}{c(s^1,\gamma')} \quad \text{for} \quad z(s^t,\gamma) = z(s^t,\gamma') = 1,$$

$$(28)$$

and use the given initial asset distribution to determine a household's initial consumption.²³

Table 2 provides a measure of the welfare gain in units of aggregate consumption by defining

$$\frac{w^A(s_0) - w^B(s_0)}{U'(c_s) c_s}$$

where $w^B(s_0)$ is welfare under fixed money supply rule, $w^A(s_0)$ denotes aggregate welfare under an alternative monetary policy rule, c_s is the level of aggregate consumption in nonstochastic steady state, and $U'(c_s)$ is its associated marginal utility. Accordingly, this index expresses the gain from adopting a particular policy rule instead of the constant money supply rule in terms of the permanent increase in steady state consumption.

From Table 2, it is clear that the countercyclical rule with $d_{\theta} = -0.5$ has the highest average welfare, as it would raise the level of steady state consumption about 0.25 percent relative to a

²³ We determine the function $\bar{B}(\gamma)$ using equation (17) under the constant money growth rule and use this distribution to compute welfare under the alternative policy rules shown in Table 2. We use the values of the shocks associated with the nonstochastic steady state for s_0 . For further details, see Appendix C.

fixed money supply rule. In contrast, the procyclical rules perform poorly, resulting in either a fall in welfare or only a small gain relative to the constant money supply rule.

To understand these results better, Figure 5 displays the effects of alternative policy rules on the welfare of individual households. A common feature of all the policy rules is that welfare is decreasing in the fixed cost of households so that households in lower percentiles of the distribution rebalance more frequently and have greater welfare. This reflects that the consumption level of these households is higher albeit more volatile.

The top panel of Figure 5 shows the welfare distribution for the fixed money supply rule (the solid black line), the procyclical rule with $d_{\theta} = 0.1$ (the red line with circles), and the countercyclical rule with $d_{\theta} = -0.1$ (the blue dashed line). Relative to the fixed money supply rule, the countercyclical policy improves the welfare of the majority of households, who are primarily inattentive, while modestly lowering the welfare of households that are frequently attentive. This improved welfare of the inattentive types reflects that a countercyclical policy transfers funds from attentive to inattentive households in productive times, allowing the inattentive ones to raise their consumption without incurring the fixed cost. Thus, this policy replicates what these households would do if they did not face a fixed cost of transferring funds from their brokerage account to their checking account. In contrast, a procyclical policy enacts the reverse redistribution plan: giving more funds to attentive households and less to inattentive ones during productive periods. While a small fraction of very frequent rebalancers are better off under the procyclical rule than the countercyclical rule, the majority of households are worse off.

The bottom panel of Figure 5 compares the zero inflation targeting rule (the magenta line with triangles) to the fixed money supply rule and the countercyclical rule with $d_{\theta} = -0.5$. As shown in Table 2, a zero inflation targeting rule improves the average welfare relative to a constant money supply rule but performs worse than the countercyclical rule with $d_{\theta} = -0.5$. The zero inflation targeting rule raises welfare relative to the constant money growth rule by sharply increasing the welfare of households that are frequently attentive while only slightly reducing the welfare of inattentive households. Households that frequently rebalance are better off, as the zero inflation targeting rule implies a large transfer to attentive households in the

initial period of a positive shock. Still, for average welfare, the countercyclical rule with $d_{\theta} = -0.5$ outperforms the zero inflation targeting rule and results in the highest average welfare of the rules that we considered.

4 Conclusions

We used a DSGE model that has reasonable implications for the equity premium and generates endogenous variations in risk to examine the positive and normative implications of alternative monetary policy rules. We showed that the response of the equity premium to shocks depends critically on the systematic response of monetary policy. Monetary policies primarily focused on inflation targeting induce procyclical movements in the equity premium, while very aggressive countercyclical policies induce acyclical movements. Countercyclical monetary policy can generate higher average welfare than constant money growth or inflation targeting rules by spreading consumption risk more broadly over households. A by-product of countercyclical policy is a sustained deflation, suggesting that the Friedman rule may also achieve superior outcomes. Thus, a natural extension of this paper is to compute optimal monetary policy and determine how well simple rules, the Friedman rule, or the countercyclical rule we emphasized here approximate it.

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	U	.S.	Representative	No Financial	Benchmark	Technology
$\mathrm{Statistic}^b$	$Data^{c}$		Agent	Plans	Calibration	Shocks Only
$E(r^e - r^f)$	6.2	(2.0)	0.2	0.2	6.1	5.8
$\sigma(r^e - r^f)$	19.4	(1.4)	7.8	7.9	33.6	32.2
$rac{E(r^e - r^f)}{\sigma(r^e - r^f)}$	0.32	(0.1)	0.04	0.03	0.18	0.18
$E(r^f)$	1.9	(5.4)	8.8	8.8	1.7	1.8
$\sigma(r^f)$	5.4	(0.6)	1.1	1.0	4.2	3.8
$\sigma(\Delta c)$	3.5	(0.4)	3.2	3.3	3.0	3.0
$\frac{\sigma(\Delta c_a)}{\sigma(\Delta c)}$			1	0.86	5.6	5.6
$E(\overline{F}(\overline{\gamma}))$			100	29	6	6
$\sigma(F(\overline{\gamma}))$			0	0.6	0.2	0.2
Avg.Cost of Reb.			0	21	0.2	0.2
(% of GDP)						

Table 1: Unconditional Moments of Asset $Returns^a$

^aResults for the models based on population moments.

^bThe symbol E denotes the unconditional mean of a variable and $\sigma(x)$ denotes the standard deviation of variable x. Rates of return are expressed in percent on an annualized basis.

^cThis column contains estimates (standard errors in parentheses) based on U.S. data for the period 1890-1991 and are taken from Guvenen (2009). The estimates for consumption are based on U.S. data for the period 1889-2009 and are available online at http://www.econ.yale.edu/ shiller/.

Rule	Parameters		Welfare	Avg. Fraction
	μ	$d_{ heta}$	Gain	of Attentive HHs
Fixed Money Supply	0	0	0.00	6.0
Procyclical	0 0 0	$\begin{array}{c} 0.1 \\ 0.5 \\ 1 \end{array}$	-0.052 0.020 0.032	$6.3 \\ 8.3 \\ 12.3$
Countercyclical	0 0 0	-0.1 -0.5 -1	0.053 0.258 -0.097	$5.8 \\ 5.4 \\ 5.7$
Zero Inflation Target	_	_	0.143	6.5

Table 2: Welfare Implications of Alternative Monetary Policy Rules*

*With the exception of the zero inflation target, the monetary policy rule is given by:

 $\mu_t = \mu + d_\theta \log(Y_t),$

where μ_t is the economy's money growth rate, Y_t is aggregate output, and μ denotes the average money growth rate. Under the zero inflation target, μ_t is chosen so that inflation is constant and equal to zero.

Figure 1: Monetary Policy and the Average Equity Premium



Note: The monetary policy rule is given by:

 $\mu_t = (1 - \rho_{\mu})\mu + \rho_{\mu}\mu_{t-1} + d_{\theta}\log(Y_t) + \epsilon_{\mu t},$

where μ_t is the economy's money growth rate, Y_t is aggregate output, μ denotes the average money growth rate, and $\epsilon_{\mu t} \sim N(0, \sigma_{\mu}^2)$.



Figure 2: Impulse Response to a Contractionary Monetary Shock (Deviation from Date 0 Expectation of a Variable)

Note: These impulse responses are from the benchmark calibration of the model with monetary policy specified as:

$$\mu_t = (1 - \rho_\mu)\mu + \rho_\mu\mu_{t-1} + \epsilon_{\mu t},$$

where μ_t is the economy's money growth rate, μ denotes the average money growth rate, $\epsilon_{\mu t} \sim N(0, \sigma_{\mu}^2)$, and $\rho_{\mu} = 0.95$.



Figure 3: Impulse Response to a Technology Shock for Alternative Policy Rules (Deviation from Date 0 Expectation of a Variable)

Note: The monetary policy rule is given by:

$$\mu_t = \mu + d_\theta \log(Y_t),$$

where μ_t is the economy's money growth rate, Y_t is aggregate output, and μ denotes the average money growth rate.



Figure 4: Impulse Response to a Technology Shock for a Zero Inflation Target (Deviation from Date 0 Expectation of a Variable)

Note: With the exception of the zero inflation target, the monetary policy rule is given by:

$$\mu_t = \mu + d_\theta \log(Y_t),$$

where μ_t is the economy's money growth rate, Y_t is aggregate output, and μ denotes the average money growth rate. Under the zero inflation target, μ_t is chosen so that inflation is constant and equal to zero.



Figure 5: Welfare Distribution for Alternative Monetary Policy Rules

Note: With the exception of the zero inflation target, the monetary policy rule is given by:

 $\mu_t = \mu + d_\theta \log(Y_t),$

where μ_t is the economy's money growth rate, Y_t is aggregate output, and μ denotes the average money growth rate. Under the zero inflation target, μ_t is chosen so that inflation is constant and equal to zero.

Appendix

This appendix is divided into four sections. In section A, we provide derivations for the equilibrium conditions of the model. Section B describes the solution algorithm. Section C describes the computation of welfare, and finally in section D we explain the computation of the impulse response functions.

A Theoretical Derivations

In this section of the appendix, we derive the model's equilibrium conditions. We also provide conditions under which a household's cash constraints in her checking and brokerage accounts hold with equality.

To show the sufficient condition for a household's cash constraint in her brokerage account to hold with equality, we define $N(s^t, \gamma)$ as a household's excess cash holdings in her brokerage account. Writing these cash holdings explicitly as part of a household's flow of funds in her brokerage accounts, a household's date 0 and date t constraints can be rewritten as:

$$\overline{B}(\gamma) = \int_{s_1} q(s_0, s_1) B(s_0, s_1, \gamma) ds_1 + N(s_0, \gamma) + p_K(s_0) S(s_0, \gamma),$$
(A.1)

and

$$B(s^{t},\gamma) + P(s^{t})(p_{K}(s^{t}) + D(s^{t}))S(s^{t-1},\gamma) + N(s^{t-1},\gamma) = \int_{s_{t+1}} q(s^{t},s_{t+1})B(s^{t},s_{t+1},\gamma)ds_{t+1} + P(s^{t})p_{K}(s^{t})S(s^{t},\gamma) + \exp(\eta t)P(s^{t})A(s_{0},\gamma) + P(s^{t})[x(s^{t},\gamma) + \exp(\eta t)\gamma]z(s^{t},\gamma) + N(s^{t},\gamma).$$
(A.2)

Let $a(s^t, \gamma)$ denote a household's excess cash holdings (in real terms) in her checking account and rewrite a household's cash constraint in her checking account as:

$$c(s^{t},\gamma) + a(s^{t},\gamma) = \frac{M(s^{t-1},\gamma)}{P(s^{t})} + x(s^{t},\gamma)z(s^{t},\gamma) + \exp(\eta t)A(s_{0},\gamma).$$
(A.3)

A household's end of the period cash is given by:

$$\frac{M(s^t,\gamma)}{P(s^t)} = w(s^t,\gamma) + a(s^t,\gamma).$$
(A.4)

We first characterize the household's optimal allocation of $c(s^t, \gamma)$ and $x(s^t, \gamma)$, taking as given $A(s_0, \gamma)$, and the state-contingent sequences for $M(s^{t-1}, \gamma)$, $a(s^t, \gamma)$, $N(s^t, \gamma)$, and $z(s^t, \gamma)$. We then characterize the optimal choice of $z(s^t, \gamma)$, $A(s_0, \gamma)$, $M(s^t, \gamma)$, $a(s^t, \gamma)$ and $N(s^t, \gamma)$ given the optimal rules for $c(s^t, \gamma)$ and $x(s^t, \gamma)$. Using the complete set of state-contingent securities, the asset market constraints can be written as a single date 0 constraint:

$$\overline{B}(\gamma) = \sum_{t=1}^{\infty} \int_{s^t} Q(s^t) \{ P(s^t) \left[x(s^t, \gamma) + \exp(\eta t) \gamma \right] z(s^t, \gamma) \\ N(s^t, \gamma) - N(s^{t-1}, \gamma) + P(s^t) [p_K(s^t)(S(s^t, \gamma) - S(s^{t-1}, \gamma)) - D(s^t)S(s^{t-1}, \gamma)] + \\ \exp(\eta t) P(s^t) A(s_0, \gamma) \} ds^t - p_K(s_0) S(s_0, \gamma) - N(s_0, \gamma).$$
(A.5)

where $Q(s^t) = \prod_{j=1}^t q(s^j)$. To obtain the previous expression we have imposed the transversality condition:

$$\lim_{t \to \infty} \int_{s^t} Q(s^t) B(s^t, \gamma) = 0 \tag{A.6}$$

With the date 0 asset market constraint, the household problem can be expressed as the following Lagrangian:

where $\nu(s^t, \gamma)$, $k(s^t, \gamma)$ and $v(\gamma)$ represent the Lagrange multipliers of the constraints. We use this Lagrangian to characterize the optimal choice of $x(s^t, \gamma)$, $c(s^t, \gamma)$ taking as given $A(s_0, \gamma)$ and the sequences, $\{z(s^t, \gamma)\}, M(s^{t-1}, \gamma), a(s^{t-1}, \gamma), N(s^{t-1}, \gamma), P(s^t), w(s^t)\}_{t=0}^{\infty}$. In doing so, we impose expression (A.6) and

$$\lim_{t \to \infty} \int_{s^t} Q(s^t) S(s^t, \gamma) = 0.$$

These two terminal conditions and $N(s^t, \gamma) > 0$, $a(s^t, \gamma) > 0 \quad \forall s^t$ rule out Ponzi schemes. We also require that $c(s^t, \gamma) > 0 \quad \forall s^t$. Given the transversality conditions and non-negativity constraints on money, expression (A.5) limits the size of the expected discounted value of future transfers, i.e. $x(s^t, \gamma)$ and $A(s_0, \gamma)$.

The first order conditions for $c(s^t, \gamma)$ and $x(s^t, \gamma)$ are:

$$\beta^t U'[c(s^t,\gamma)]g(s^t) = \nu(s^t,\gamma),$$

$$\nu(s^t,\gamma)z(s^t,\gamma) - v(\gamma)P(s^t)Q(s^t)z(s^t,\gamma) = 0.$$

Suppose that $z(s^t, \gamma) = 1$ for some state of the world s^t , then we can combine these two conditions to write:

$$\nu(s^t, \gamma) = \beta^t U'[c(s^t, \gamma)]g(s^t) = v(\gamma)P(s^t)Q(s^t).$$
(A.8)

Following Alvarez, Atkeson, and Kehoe (2002), we assume an initial wealth distribution, $\overline{B}(\gamma)$, such that $v(\gamma) = v$, $\forall \gamma$. Imposing $v(\gamma) = v$ in equation (A.8), it follows that $c_A(s^t, \gamma) = c_A(s^t)$.

The optimal transfer for a household when it is active is given by:

$$x(s^t, \gamma) = c_A(s^t) - c_I(s^t, \gamma), \tag{A.9}$$

where $c_I(s^t, \gamma)$ is the optimal consumption of household γ when she chooses to be inactive (i.e., $z(s^t, \gamma) = 0$). This level of consumption is given by:

$$c_I(s^t, \gamma) = \frac{M(s^{t-1}, \gamma)}{P(s^t)} + \exp(\eta t) A(s_0, \gamma) - a(s^t, \gamma)$$
(A.10)

Given the optimal choices of $c_A(s^t)$, $c_I(s^t, \gamma)$, and $x(s^t, \gamma)$, we now characterize the optimal rule for $z(s^t, \gamma)$ and the optimal choices for $A(s_0, \gamma)$, $M(s^t, \gamma)$, $a(s^t, \gamma)$, and $N(s^t, \gamma)$. Using the optimal rules for $c_A(s^t)$, $c_I(s^t, \gamma)$, and $x(s^t, \gamma)$, a household's problem can be rewritten:

$$\begin{split} &\operatorname{Max} \sum_{t=1}^{\infty} \int_{s^{t}} \beta^{t} U[c_{A}(s^{t})] z(s^{t}, \gamma) g(s^{t}) ds^{t} + \\ &\sum_{t=1}^{\infty} \int_{s^{t}} \beta^{t} U[\frac{M(s^{t-1}, \gamma)}{P(s^{t})} + \exp(\eta t) A(s_{0}, \gamma) - a(s^{t}, \gamma)] (1 - z(s^{t}, \gamma)) g(s^{t}) ds^{t} + \\ &\sum_{t=1}^{\infty} \int_{s^{t}} \Phi(s^{t}, \gamma) \left\{ w(s^{t}, \gamma) + a(s^{t}, \gamma) - \frac{M(s^{t}, \gamma)}{P(s^{t})} \right\} ds^{t} + \\ &v_{h}(\gamma) \{ \overline{B}(\gamma) - \sum_{t=1}^{\infty} \int_{s^{t}} Q(s^{t}) \{ P(s^{t}) [c_{A}(s^{t}) - \frac{M(s^{t-1}, \gamma)}{P(s^{t})} + \exp(\eta t) A(s_{0}, \gamma) - a(s^{t}, \gamma) + \exp(\eta t) \gamma] z(s^{t}, \gamma) + \\ &\exp(\eta t) P(s^{t}) A(s_{0}, \gamma) + P(s^{t}) [p_{K}(s^{t}) (S(s^{t}, \gamma) - S(s^{t-1}, \gamma)) - D(s^{t}) S(s^{t-1}, \gamma)] + N(s^{t}, \gamma) - N(s^{t-1}, \gamma) \right\} ds^{t} \\ &-p_{K}(s_{0}) S(s_{0}, \gamma) - N(s_{0}, \gamma) \} \,. \end{split}$$

We use a variational argument to characterize the optimal choice of $z(s^t, \gamma)$. The increment to the Lagrangian of setting $z(s^t, \gamma) = 1$ in the state of the world s^t for household γ is:

$$\beta^t U[c_A(s^t)]g(s^t) - v_h(\gamma)Q(s^t)P(s^t)[c_A(s^t) - c_I(s^t,\gamma) + \exp(\eta t)\gamma]ds^t$$
(A.11)

which corresponds to the difference between the utility gain minus the cost of transferring funds. The increment to the Lagrangian of setting $z(s^t, \gamma) = 0$ is:

$$\beta^t U[c_I(s^t, \gamma)]g(s^t)ds^t, \tag{A.12}$$

which represents the direct utility gain from remaining inactive. The first order condition with respect to $c_A(s^t, \gamma)$ is:

$$\beta^t U'[c_A(s^t)]g(s^t) = v_h(\gamma)P(s^t)Q(s^t), \qquad (A.13)$$

from which it follows that the Lagrange multiplier $v_h(\gamma) = v_h$, $\forall \gamma$. Substituting this expression into equation (A.11) and subtracting (A.12) from it yields expression (19).

To derive equation (18) of the main text, we need to take the first order conditions with respect to $A(s_0, \gamma)$. This yields:

$$\sum_{t=1}^{\infty} \int_{s^t} \left[\beta^t U'[c_I(s^t, \gamma)](1 - z(s^t, \gamma))g(s^t) - v_h P(s^t)Q(s^t)(1 - z(s^t, \gamma)) \right] \exp(\eta t) ds^t = 0 \quad (A.14)$$

Substituting equation (A.13) into equation (A.14) yields expression (18) in the text.

To derive sufficient conditions that imply that the two cash constraints hold with equality, note that the first order conditions with respect to $N(s^t, \gamma)$, $m(s^{t+1}, \gamma)$, $a(s^t, \gamma)$ and are given by:

$$\int_{s^{t+1}} Q(s^{t+1}) ds^{t+1} - Q(s^t) ds^t \le 0$$

$$\int_{s^{t+1}} \beta^{t+1} \frac{U' \left[c_I(s^{t+1}, \gamma) \right]}{P(s^{t+1})} (1 - z(s^{t+1}, \gamma)) g(s^{t+1}) ds^{t+1} -$$
(A.15)

$$\frac{\Phi(s^t,\gamma)}{P(s^t)}ds^t + v_h \int_{s^{t+1}} Q(s^{t+1})z(s^{t+1},\gamma)ds^{t+1} = 0$$
(A.16)

$$-\beta^t U'\left[c_I(s^t,\gamma)\right]\left(1-z(s^t,\gamma)\right)g(s^t) + \Phi(s^t,\gamma) + v_h Q(s^t)P(s^t)z(s^t,\gamma) \le 0$$
(A.17)

Using the definition of $Q(s^t)$, equation (A.15) can be used to define the nominal interest as follows:

$$(1 + R(s^t))^{-1} \equiv \int_{s_{t+1}} q(s^{t+1}) ds_{t+1} \le 1.$$
 (A.18)

When the nominal interest rate is strictly positive, then $N(s^t, \gamma) = 0$ and equation (8) holds as an equality.

To derive the sufficient condition for a household's checking account to hold with equality, we follow a similar strategy. In particular, using equation (A.13) and substituting equation (A.17) into equation (A.16) yields:

$$\frac{\beta \int_{s_{t+1}} \left[U'[c_A(s^{t+1})] z(s^{t+1}, \gamma) + U'[c_I(s^{t+1}, \gamma)] \left(1 - z(s^{t+1}, \gamma)\right) \right] \frac{g(s_{t+1}|s^t)}{\pi(s^{t+1})} ds_{t+1}}{U'[c_A(s^t)] z(s^t, \gamma) + U'[c_I(s^t, \gamma)] \left(1 - z(s^t, \gamma)\right)} \le 1.$$

If the expected discounted marginal utility of future consumption is strictly less than a household's current marginal utility of consumption, then $a(s^t, \gamma) = 0$, and as a result equation (8) in the main text holds as an equality.

A.1 The Attentiveness of Households with Zero Fixed Costs

To show that a household with zero fixed cost is always attentive, it is convenient to use equation (19) and define:

$$h(s^{t},\gamma) = U\left(c_{A}(s^{t})\right) - U\left(c_{I}(s^{t},\gamma)\right) - U_{c}^{A}\left(s^{t}\right)\left[c_{A}(s^{t}) - c_{I}(s^{t},\gamma) + \exp(\eta t)\gamma\right], \qquad (A.19)$$

so that $h(s^t, \gamma) > 0$ implies that the household with fixed cost γ is attentive. For households with $\gamma = 0$ to be attentive, we need to show that:

$$h(s^{t},0) = U(c_{A}(s^{t})) - U(c_{I}(s^{t},0)) - U_{c}^{A}(s^{t})[c_{A}(s^{t}) - c_{I}(s^{t},0)] > 0, \quad \forall s^{t}.$$

Substituting the expression for the preferences into the previous expression for $h(s^t, 0)$ yields:

$$h(0) = \frac{c_A^{1-\sigma}}{1-\sigma} - \frac{c_I(0)^{1-\sigma}}{1-\sigma} - \frac{c_A - c_I(0)}{c_A^{-\sigma}}$$

where, for simplicity and without loss of generality, we have suppressed the dependence on the state s^t and set $\eta = 0$.

state s^t and set $\eta = 0$. Let $\tilde{h}(0) = \frac{h(0)}{c_A^{-\sigma}}$ and rewrite the previous expression as:

$$\tilde{h}(s^t, 0) = F_1 - F_2$$

where $F_1 = \frac{1}{\sigma - 1} \left[\left(\frac{c_A}{c_I} \right)^{\sigma - 1} - 1 \right]$ and $F_2 = \left[1 - \left(\frac{c_A}{c_I} \right)^{-1} \right]$.

For any $c_A > 0$ and $c_I > 0$, then $\tilde{h}(0) \ge 0$. To see this, we need to assume that $\sigma > 1$ and consider two cases. First, suppose $c_A > c_I$ then it is possible to show that $F_1 > F_2$. In the case, a household with $\gamma = 0$ will choose to transfer funds from her brokerage account to her checking account. Now suppose $c_A < c_I$, then the household wants to put extra funds into her brokerage account, since the loss in utility from doing so (F_1) is more than offsets the gain of having extra funds in that account (F_2) .

A.2 The Responsiveness of Attentive Consumption

The purpose of this section is to derive expression (26) of the main text, i.e. the response of consumption of attentive households to technology shocks. To do so, we focus on a two agent economy in which the number of attentive households is fixed, and we abstract from growth (i.e. $\eta = 0$). Under these assumptions, we can use expression (21), the aggregate resource constraint, to write the consumption of attentive consumers as follows:

$$c_A(\theta^t) = \frac{1}{F(0)} \left[\exp(\theta^t) - (1 - F(0))c_I(\theta^t) \right]$$

Differentiating the previous expression with respect to θ_t yields:

$$\frac{dc_A(\theta^t)}{d\theta_t} = \frac{1}{F(0)} \left[\exp(\theta^t) - (1 - F(0)) \frac{dc_I(\theta^t)}{ds_t} \right].$$
(A.20)

To obtain the expression for $\frac{dc_I(\theta^t)}{d\theta_t}$ we use expression (16):

$$c_I(\theta^t) = (1 - \alpha) \left[\exp(\theta^t) - \mu_t \right] + A,$$

which can be differentiated to obtain:

$$\frac{dc_I(\theta^t)}{d\theta_t} = (1 - \alpha) \left[\exp(\theta^t) - \mu_t \right] = c_I(\theta^t) - A.$$

Substituting this expression into (A.20) yields:

$$\frac{dc_A(\theta^t)}{d\theta_t} = \frac{1}{F(0)} [\exp(\theta^t) - (1 - F(0))(c_I(\theta^t) - A)] = \frac{1}{F(0)} [\exp(\theta^t) - (1 - f(0))c_I(\theta^t)] + \frac{1 - F(0)}{F(0)}A \\
\frac{dc_A(\theta^t)}{d\theta_t} = c_A(\theta^t) + \frac{1 - F(0)}{F(0)}A.$$
(A.21)

Notice that, dividing by $c_A(\theta^t)$ both sides of the previous expression yields:

$$\frac{dc_A(\theta^t)}{c_A(\theta^t)d\theta_t} = \frac{d\log c_A(\theta^t)}{d\theta_t} = 1 + \frac{1 - F(0)}{F(0)}\frac{A}{c_A(\theta^t)}$$
(A.22)

As discussed in the text, in the model with A = 0, the previous elasticity is equal to one. The presence of A > 0, makes this elasticity greater than one (i.e., greater than in the model without the financial plan).

We know proceed to evaluate the elasticity at the steady state. To do so, we notice that in the non-stochastic steady state, $c_I = c_A = \frac{(1-\alpha)}{\exp(\mu)} + A = 1$, which implies that $A = 1 - \frac{(1-\alpha)}{\exp(\mu)} = \frac{\alpha}{\exp(\mu)} + \frac{\exp(\mu)-1}{\exp(\mu)}$. To obtain the expression of the elasticity used in the main text we need to rewrite expression (A.22) as follows:

$$\frac{d\log c_A(\theta^t)}{d\theta_t} = \frac{1}{F(0)} \frac{A}{c_A(\theta^t)} + (1 - \frac{A}{c_A(\theta^t)}),$$

which, using the previous expressions, can be evaluated at the steady state yielding:

$$\frac{d\log c_A(\theta^t)}{d\theta_t} = \frac{1}{F(0)} \left(\frac{\alpha}{\exp(\mu)} + \frac{\exp(\mu) - 1}{\exp(\mu)} \right) + \frac{1 - \alpha}{\exp(\mu)},$$

that corresponds to expression (26) of the main text.

B Solving the Model

This appendix describes the solution algorithm used to compute the model's equilibrium.

B.1 Characterizing the Equilibrium with Policy Functions

Before describing the solution algorithm, we characterize a solution to our model using the time-invariant function, $A(\gamma)$, $\forall \gamma \in (0, \infty)$. The function $A(\gamma)$ denotes the level of non-state contingent transfers for household γ , $A(\gamma, s_0)$, evaluated at the initial level of technology and money growth, which we set equal to their non-stochastic steady state values. (We do not solve for A(0) given that, as shown earlier, households with $\gamma = 0$ always chooses to be attentive.)

The function, $A(\gamma)$, satisfies the Euler equation:

$$V(s,\gamma;A(\gamma)) = 0 \text{ for } s = \bar{s}, \tag{B.1}$$

where $\bar{s} = (0, \mu_{ss})$ and μ_{ss} refers to the rate of money growth in non-stochastic steady state. The value function, V in accordance with expression (22) satisfies:

$$V(s,\gamma;A(\gamma)) = \widetilde{\beta} \int_{s'} \{U'[c_A(s')] - U'[c_I(s';A(\gamma))]\} (1 - z(s',\gamma))g(s'|s) + \widetilde{\beta} \int_{s'} V(s',\gamma;A(\gamma))g(s'|s).$$
(B.2)

where $\tilde{\beta} = \frac{\beta}{\exp(\sigma\eta)}$, $U'(c) = c^{-\sigma}$ and g(s'|s) is a bivariate normal distribution for s' conditional on state s in which the technology shocks are independent from monetary shocks. (The discount factor is scaled by the economy's growth rate, because below we describe the solution to the economy in which the relevant variables have been scaled by the economy's growth rate.)

Given the function, $A(\gamma)$, the consumption of inattentive households satisfies:

$$c_I(s; A(\gamma)) = (1 - \alpha) \exp(\theta - \mu) + A(\gamma).$$
(B.3)

The consumption of attentive households and the marginal household type, $\overline{\gamma} = \overline{\gamma}(s)$, are determined simultaneously by the resource constraint and expression (19). Specifically, c_A and $\overline{\gamma}$ satisfy:

$$c_A(s) = \left(F(\bar{\gamma})\right)^{-1} \left(\exp(\theta) - \int_{\bar{\gamma}}^{\infty} c_I(s; A(\gamma)) f(\gamma) d\gamma - \int_0^{\bar{\gamma}} \gamma f(\gamma) d\gamma\right),\tag{B.4}$$

and

$$U[c_A(s)] - U[c_I(s; A(\bar{\gamma}))] - U'[c_A(s)] \{c_A(s) - c_I(s; A(\bar{\gamma})) + \bar{\gamma}(s)\} = 0,$$
(B.5)

where $\gamma \leq \overline{\gamma}(s)$ implies $z(s, \gamma) = 1$ and $\gamma > \overline{\gamma}(s)$ implies $z(s, \gamma) = 0$.

From equation (B.2), it follows that if $z(s, \gamma) = 0 \forall s$, then the value-function becomes independent of γ except for its dependence on $A(\gamma)$. Accordingly, it is convenient to define γ_M and $A(\gamma_M)$ such that $A(\gamma) = A(\gamma_M)$ for all $\gamma \geq \gamma_M$ where $A(\gamma_M)$ satisfies:

$$V(s; A(\gamma_M)) = \widetilde{\beta} \int_{s'} \{ U'[c_A(s')] - U'[c_I(s'; A(\gamma))] \} g(s'|s)$$

+ $\widetilde{\beta} \int_{s'} V(s'; A(\gamma_M)) g(s'|s).$ (B.6)

In other words, γ_M is defined so that households with a larger fixed cost are always inattentive, and all such households choose the same level of the non-state contingent plan regardless of type. Because of this property, the function $A(\gamma)$ can be written as:

$$A(\gamma) = A_L(\gamma), \text{ for } \gamma \in (0, \gamma_M]$$

$$A(\gamma_M), \text{ for } \gamma \ge \gamma_M$$
(B.7)

B.2 Solution Algorithm

To find the solution of the model we use a piecewise linear interpolation to approximate $A_L(\gamma)$:

$$A_L(\gamma) \approx \widehat{A}_L(\gamma) \equiv A(\gamma_i) + \frac{\gamma - \gamma_i}{\gamma_{i+1} - \gamma_i} \left[A(\gamma_{i+1}) - A(\gamma_i) \right], \text{ for } \gamma \in [\gamma_i, \gamma_{i+1}]$$

where $\gamma_i \in \{\gamma_1, \gamma_2, ..., \gamma_M\}$. To determine $\widehat{A}_L(\gamma)$ we need to find the values $A(\gamma_i)$ and γ_M such that expression (B.1) holds for $\gamma_i \in \{\gamma_1, \gamma_2, ..., \gamma_M\}$, and $\gamma_M = \arg\min\{\gamma : z(s, \gamma) = 0, \forall s\}$. To do so we proceed as follows.

- 1. For a given initial guess of $A(\gamma_i)$, i = 1, ..., M, and γ_M , we obtain $\widehat{A}(\gamma)$, where $\widehat{A}(\gamma) = \widehat{A}_L(\gamma)$ for $\gamma \in (0, \gamma_M]$ and $\widehat{A}(\gamma) = A(\gamma_M)$ for $\gamma \geq \gamma_M$. Using this, we can determine $\{c_I(s; \widehat{A}(\gamma)), c_A(s), \overline{\gamma}(s)\}$ from expressions (B.3), (B.4), and (B.5). Given $\overline{\gamma}(s)$ we obtain $z(s, \gamma), \forall \gamma$.
- 2. Using the decision rules in step 1, we compute $V(s, \gamma_i; A(\gamma_i))$ recognizing that expression (B.2) is a linear Fredholm equation of the second kind. Following Judd (1999) we use quadrature methods to solve this equation by writing:

$$\tilde{\beta} \int_{s'} \{ U'[c_A(s')] - U'[c_I(s'; A(\gamma_i))] \} (1 - z(s', \gamma_i))g(s'|s) \doteq f(s)$$

$$f(s) = \tilde{\beta} \sum_{j=1}^{N_s} \omega_j \{ U'[c_A(s_j)] - U'[c_I(s_j; A(\gamma_i))] \} (1 - z(s_j, \gamma_i))g(s_j|s)$$

$$\int_{s'} V(s', \gamma_i; A(\gamma_i))g(s'|s) \doteq \sum_{j=1}^{N_s} \omega_j V(s_j, \gamma_i; A(\gamma_i))g(s_j|s)$$

where $s_j \in [-l, l]$, for $j = 1, ..., N_s$, with $l = \{\kappa \sigma_{\theta}, \kappa \sigma_{\mu}\}$ and $\kappa > 0$. The elements which make up s_j are equally spaced and the weights, ω_j , are chosen according to Simpson's rule. With this quadrature scheme, we can write expression (B.2) as the linear system:

$$V(s_k, \gamma_i; A(\gamma_i)) = f(s_k) + \tilde{\beta} \sum_{j=1}^{N_s} \omega_j V(s_j, \gamma_i; A(\gamma_i)) g(s_j|s), \text{ for } k = 1, \dots, N_s$$

- 3. From this linear system of equations we can determine $V(s_k, \gamma_i; A(\gamma_i))$. Notice that we construct the grid so that the \overline{s} is one of the points, yielding $V(\overline{s}, \gamma_i; A(\gamma_i))$. We repeat this procedure for each of the household types, i = 1, ..., M, that we use to characterize $A(\gamma)$. For the given initial guess of $A(\gamma_i)$, i = 1, ..., M, and γ_M ; we also compute $\arg\min\{\gamma : z(s_k, \gamma) = 0, \forall s_k\}$. In this way, we can update $A(\gamma_i)$, i = 1, ..., M and γ_M . To do so, we use a non-linear equation solver to find $A(\gamma_i)$ for i = 1, ..., M and γ_M such that $V(\overline{s}, \gamma_i; A(\gamma_i)) = 0$, for i = 1, ..., M, and $\gamma_M = \arg\min\{\gamma : z(s_k, \gamma) = 0, \forall s_k\}$.
- 4. To compute the (detrended) price of equity, $p_k(s)$, we use the solution for $c_A(s)$ and

$$p_k(s) = \frac{\beta}{\exp((\sigma - 1)\eta)} \int_{s'} \frac{U'[c_A(s')]}{U'[c_A(s)]} [\alpha \exp(\theta') + p_k(s')]g(s'|s)ds'$$

which combines expressions (23) and (24) of the main text. This expression is also a linear Fredholm equation of the second kind. Thus, we can follow the strategy used in step 2 above to define a linear system of equations from which we can determine $p_k(s_j)$ for $j = 1, ..., N_s$. In simulating the model we approximate $p_k(s)$ using the Nystrom extension, where we form $\hat{p}_k(s)$:

$$\widehat{p}_k(s) = \frac{\beta}{\exp((\sigma - 1)\eta)} \sum_{j=1}^{N_s} \omega_j \frac{U'[c_A(s_j)]}{U'[c_A(s)]} [\alpha \exp(\theta_j) + p_k(s_j)].$$

C Computing Welfare

C.1 Welfare under Constant Money Growth

To compute welfare under a fix money supply rule, we follow the procedure described above to solve the model. This produces the decision rules for $A(\gamma)$ and consumption: $\{c_I(\theta; A(\gamma)), c_A(\theta)\}$. Note that the assumption about money supply implies that the only exogenous state variable is technology and hence $s = \theta$.

Let $w^B(\theta)$ denote welfare under the fixed money supply rule, which can be determined from:

$$w^{B}(\theta) = \widetilde{\beta} \int_{\theta'} \int_{\gamma} U(c(\theta', \gamma)) g(\theta'|\theta) f(\gamma) d\gamma d\theta' + \widetilde{\beta} \int_{\theta'} w^{B}(\theta') g(\theta'|\theta) d\theta',$$

where $g(\theta)$ refers to the univariate normal distribution for technology shocks. We compute $w^B(\theta)$ recognizing that this expression is also a linear Fredholm equation of the second kind. Accordingly, we can follow the strategy used in step 2 of Appendix B.2 to define a linear system of equations from which we can determine $w^B(\theta_k)$ for $k = 1, ..., N_{\theta}$. We construct the grid so that the $\overline{\theta} = 0$ is one of the points, and the numbers in Table 2 use $w^B(0)$.

C.2 Initial Asset Distribution under Constant Money Growth

Average household welfare implicitly depends on the initial asset position of the households. To compare welfare across alternative monetary policy rules, we keep the initial distribution of assets fixed at the distribution associated with the constant money growth rule. To determine this initial asset position, $\bar{B}(\gamma)$, we set $N(\theta^t) = 0$ and focus on a symmetric equilibrium where $S(\theta^t, \gamma) = 1$. Accordingly, expression (A.5) implies:

$$v_h\left(\overline{B}(\gamma) - p_K(\theta_0)\right) = \sum_{t=1}^{\infty} \widetilde{\beta}^t \int_{\theta^t} U'[c_A(\theta^t)] \left\{ c^B(\theta^t, \gamma) - \alpha Y(\theta^t) \right\} g(\theta^t) d\theta^t, \quad (C.1)$$
$$c^B(\theta^t, \gamma) = \left[c_A(\theta^t) - \frac{(1-\alpha)\exp(\theta_t)}{\mu} + \gamma \right] z(\theta^t, \gamma) + A(\theta_0, \gamma)(1-z(\theta^t, \gamma)),$$

where $c^B(\theta^t, \gamma)$ represents the consumption of household γ under the money growth rule and $\tilde{\beta}^t = \frac{\beta^t}{\exp(\sigma \eta t)}$. In the above, we have used that in equilibrium, $D(\theta^t) = \alpha Y(\theta^t)$. To determine $\overline{B}(\gamma)$ and the Lagrange multiplier v_h , we also use:

$$\int_0^\infty \overline{B}(\gamma) f(\gamma) d\gamma = B,$$
(C.2)

where B satisfies the government's intertemporal budget constraint: $B = \sum_{t=1}^{\infty} \int_{\theta^t} Q(\theta^t) [1 - \exp(-\mu)] \exp(\mu t) M_0$. Note that given $M_0 > 0$ this expression uniquely pins down the initial level of government debt, B. Let

$$\Lambda^{B}(\theta,\gamma) = U'[c_{A}(\theta)] \left\{ c_{A}^{B}(\theta,\gamma) - \alpha Y(\theta) \right\},\,$$

then we can write expression (C.1) as follows:

$$v_h\left(\overline{B}(\gamma) - p_K(\theta_0)\right) = V^B(\theta, \gamma), \text{ where } V^B(\theta, \gamma) = \widetilde{\beta} \int_{\theta'} \left[\Lambda^B(\theta', \gamma) + V^B(\theta', \gamma)\right] g(\theta'|\theta) d\theta'.$$
(C.3)

Given a value of B consistent with the intertemporal budget constraint, expressions (C.2) and (C.3) are a system of equations determining v_h and $\overline{B}(\gamma)$ such that:

$$\overline{B}(\gamma) = \overline{B}_L(\gamma), \text{ for } \gamma \in (0, \gamma_M]$$
$$\overline{B}(\gamma_M), \text{ for } \gamma \ge \gamma_M.$$

To determine $\overline{B}(\gamma)$, we use the decision rules computed in Appendix B. We approximate $\overline{B}_L(\gamma)$ using piecewise linear interpolation:

$$\overline{B}_L(\gamma) \approx \overline{B}(\gamma_i) + \frac{\gamma - \gamma_i}{\gamma_{i+1} - \gamma_i} \left[\overline{B}(\gamma_{i+1}) - \overline{B}(\gamma_i) \right], \text{ for } \gamma \in [\gamma_i, \gamma_{i+1}],$$

where $\gamma_i \in {\gamma_1, \gamma_2, ..., \gamma_M}$. Also, $\overline{B}(\gamma_i)$ is determined using a similar procedure as the one described in steps 2 and 3 of Appendix B.2 for $A(\gamma_i)$.

C.3 Solving the Model Given an Initial Distribution of Assets

In this section we describe how to solve the model for a given distribution of assets, $\overline{B}(\gamma)$. By doing so, we are able to make welfare comparisons across alternative monetary policy rules, holding fixed this initial distribution.

C.3.1 Equilibrium Characterization

An important element of the theoretical derivation of the equilibrium conditions in the benchmark model (as described in Appendix A) is that the initial distribution of assets $\overline{B}(\gamma)$ is chosen so that the Lagrange multiplier $v(\gamma) = v$. This condition implies that the consumption among attentive consumers was equalized. However, consumption need not be equalized across attentive households once we fix the initial distribution of assets, $\overline{B}(\gamma)$ to the one that corresponds constant money growth rule. In this section, we re-characterize the equilibrium conditions of the model in order to compute welfare for alternative policy rules holding fixed $\overline{B}(\gamma)$ at the distribution computed in Appendix C.2. Because the rules we consider imply that money growth is a function of the technology shock, the exogenous state can again by characterized using θ^t .

To characterize a household's optimal allocation of $c(\theta^t, \gamma)$, it is convenient to define $c_A(\theta^t, \gamma)$ when a household chooses $z(\theta^t, \gamma) = 1$ and $c_I(\theta^t, \gamma)$ when $z(\theta^t, \gamma) = 0$. Using the Lagrangian defined in expression (A.7) and taking the first order conditions for c and x, it follows that:

$$P(\theta^t)Q(\theta^t) = \frac{\beta^t U'[c_A(\theta^t, \gamma)]g(\theta^t)}{v(\gamma)},$$
(C.4)

when $z(\theta^t, \gamma) = 1$. This expression holds for the household with $\gamma = 0$. As shown earlier, this household will always be attentive and

$$P(\theta^t)Q(\theta^t) = \frac{\beta^t U'[c_A(\theta^t, 0)]g(\theta^t)}{v(0)},$$
(C.5)

holds for all θ^t .

In periods in which a household with $\gamma > 0$ chooses to be attentive, her consumption will be proportional to the other attentive households. To see this, combine expressions (C.4) and (C.5):

$$\frac{U'[c_A(\theta^t,\gamma)]}{v(\gamma)} = \frac{U'[c_A(\theta^t,0)]}{v(0)},\tag{C.6}$$

 $\forall \gamma$ such that $z(\theta^t, \gamma) = 1$. Using the households with $\gamma = 0$ as a reference, expression (C.6) allows us to characterize the function $c_A(\theta^t, \gamma)$ given the function $v(\gamma)$. In particular, when a household chooses to be attentive its consumption relative to the household with $\gamma = 0$ is given by the ratio of the Lagrange multipliers $\frac{v(\gamma)}{v(0)}$.

Assuming a household does not carry excess cash in its checking account, it follows from

expressions (A.4) and (A.10) that:

$$c_I(\theta^t, \gamma) = \frac{w(\theta^{t-1})}{\pi(\theta^t)} + \exp(\eta t) A(\theta_0, \gamma)$$
(C.7)

$$x(\theta^{t},\gamma) = c_{A}(\theta^{t},\gamma) - \frac{w(s^{t-1})}{\pi(\theta^{t})} + \exp(\eta t)A(\theta_{0},\gamma)$$
(C.8)

From the Lagrangian (A.7), it follows that the optimal choices for $A(\theta_0, \gamma)$ satisfies:

$$\sum_{t=1}^{\infty} \int_{\theta^t} \beta^t \exp(\eta t) \left[U'(c_A(\theta^t, 0)) - U'(c_I(\theta^t, \gamma)) \right] (1 - z(\theta^t, \gamma)) g(\theta^t) d\theta^t = 0,$$
(C.9)

Using a similar variational argument as described earlier, the optimal choice of $z(\theta^t, \gamma)$ is determined from:

$$U\left(c_A(\theta^t, \bar{\gamma}(\theta^t))\right) - U\left(c_I(\theta^t, \bar{\gamma}(\theta^t))\right) = U\left(c_A(\theta^t, \bar{\gamma}(\theta^t))\right) \left[c_A(\theta^t, \bar{\gamma}(\theta^t)) - c_I(\theta^t, \bar{\gamma}(\theta^t)) + \exp(\eta t)\bar{\gamma}(\theta^t)\right], \quad (C.10)$$

where $z(\theta^t, \gamma) = 1$ for $\gamma \leq \bar{\gamma}(\theta^t)$ and $z(\theta^t, \gamma) = 0$ for $\gamma > \bar{\gamma}(\theta^t)$.

Given $\overline{B}(\gamma)$, the intertemporal asset market constraint, expression (A.5), can be used to determine $v(\gamma)$. To do so, we focus on a symmetric equilibrium in which $S(\theta^t, \gamma) = 1 \forall \gamma$ and assume that $N(\theta^t) = 0 \forall t \ge 1$. Accordingly, we can write expression (A.5) as:

$$v(\gamma) \left(\overline{B}(\gamma) - p_K(\theta_0)\right) = V^A(\theta^t, \gamma).$$
(C.11)

where the value-function, $V^A(\theta^t, \gamma)$, is given by:

$$V^{A}(\theta^{t},\gamma) = \sum_{t=1}^{\infty} \beta^{t} \int_{\theta^{t}} U'[c_{A}(\theta^{t},\gamma)] \left\{ c^{A}_{A}(\theta^{t},\gamma) - \alpha Y(\theta^{t}) \right\} g(\theta^{t}) d\theta^{t}, \qquad (C.12)$$

$$c^{A}(\theta^{t},\gamma) = \left[c_{A}(\theta^{t}) - \frac{(1-\alpha)\exp(\theta_{t}+\eta t)}{\mu(\theta_{t})} + \exp(\eta t)\gamma \right] z(\theta^{t},\gamma) + \exp(\eta t)A(\theta_{0},\gamma)(1-z(\theta^{t},\gamma)),$$

where $c^{A}(\theta^{t}, \gamma)$ represents the consumption of household γ under one of the alternative monetary policy rules: the procyclical, the countercyclical, or the zero inflation policy rule.

Given the functions $v(\gamma)$ and $A(\theta_0, \gamma)$, expressions (C.6), (C.7), (C.10), and the resource constraint:

$$Y(\theta^t) = \int_0^{\bar{\gamma}(\theta^t)} c_A(\theta^t, \gamma) f(\gamma) d\gamma + \int_{\bar{\gamma}(\theta^t)}^\infty c_I(\theta^t, \gamma) f(\gamma) d\gamma + \exp(\eta t) \int_0^{\bar{\gamma}(\theta^t)} \gamma f(\gamma) d\gamma, \qquad (C.13)$$

characterize the optimal choices of $c_A(\theta^t, 0)$, $c_A(\theta^t, \gamma)$ for $0 < \gamma \leq \overline{\gamma}(\theta^t)$, $c_I(\theta^t, \gamma)$ for $\gamma > \overline{\gamma}(\theta^t)$ and $\overline{\gamma}(\theta^t)$. Equations (C.9) and (C.11) and can be used to determine the functions $A(\theta_0, \gamma)$ and $v(\gamma)$, taking $\overline{B}(\gamma)$ as given. As before, we define γ_M so that households with a larger fixed cost are always inattentive. From these two expressions, it follows that households with $\gamma \geq \gamma_M$ choose the same level of the non-state contingent plan and and the same value of the Lagrange multiplier $(v(\gamma_M))$. Because of this property, the function $v(\gamma)$ can be written as:

$$v(\gamma) = v_L(\gamma), \text{ for } \gamma \in (0, \gamma_M]$$

$$v(\gamma_M), \text{ for } \gamma \ge \gamma_M$$

(C.14)

C.3.2 Solution Procedure

To solve the model with a fix distribution of initial assets we modify the solution procedure discussed in Appendix B.2. In particular, $V^A(\theta, \gamma)$ and the value function associated with (C.9) can be determined by recognizing that they can be expressed as linear Fredholm equations and applying piecewise linear interpolation to approximate $A_L(\gamma)$ and $v_L(\gamma)$. With these functions available, $w^A(\theta)$ is computed for each of the alternative monetary policy rules—the procyclical, countercyclical, and zero inflation policy rules—in an analogous manner to welfare under the constant money supply rule.

D Impulse Responses

Following Hamilton (1994), an impulse response of variable, $y(s^t)$, at date t to a monetary innovation that occurs at date 1 is defined as:

$$E\left[\log\left(y(s^t)\right) \mid \mu_1^r, \mu_0, \theta_0\right] - E\left[\log\left(y(s^t)\right) \mid \mu_0, \theta_0\right], \forall t \ge 1,$$
(D.1)

where μ_1^r is a random variable governing the size of the innovation at date 1 and μ_0 and θ_0 are the initial levels of money growth and technology. (The impulse response to a technology shock is defined analogously, and we set μ_0 and θ_0 equal to their steady state values in Figures 2, 3, and 4). Accordingly, the impulse responses displayed in Figure 2 show the effects of a revision in expectations about a monetary innovation that occurs at date 1. For log-linear models, equation (D.1) simplifies to the usual analytical representation in which (up to a scaling factor) the model's linear coefficients characterize the impulse response function. Since in our context evaluating the expectations in equation (D.1) involves multidimensional integrals, we use Monte Carlo integration to compute these expectations.